Three Essays on Monopolist Second-degree Discrimination Strategies in the Presence of Positive Network Effects

by

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Abstract

This thesis consists of three papers examining price and quality discrimination strategies in the presence of positive network effects.

The first paper develops a general framework based on the critical assumption of strategic complementarity to describe and solve the screening problem faced by the monopolist seller of a network good. By applying monotone comparative static tools to compare first- and second-best allocations, it demonstrates that the joint presence of asymmetric information and positive network effects leads to a strict downward distortion for all consumers in the quantities provided. Despite this overall downward distortion result, it is shown that the equilibrium allocation is an increasing function of the network effect intensity, and that a discriminating monopoly may supply larger quantities than a perfectly competitive industry.

The second paper examines compatibility and coordination issues in a general model of screening with positive network effects. It shows how pessimistic expectations can decrease a monopoly's power in practicing second-degree discrimination, and demonstrates how divide-and-conquer techniques may solve the consumers' coordination problem and uniquely implement a screening mechanism.

The third paper provides a pure network effect based explanation of a common practice in software markets, which is that firms remove some functions of their original products and sell a functionally-downgraded version at a lower or zero price. Building a functional degradation model with asymmetric network effects, it investigates when and why firms have incentives to introduce a functionally-degraded good and discusses the welfare implications.

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I have also benefitted from comments from participants at conferences and seminars where I presented different versions of these papers. Needless to say, all remaining errors are mine.

Finally, let me put a disclaimer on the style of the thesis here: all of the following Chapters

[1] The paper is now forthcoming in the Journal of Industrial Economics.

use the plural “we” instead of “I”. My feeling is that doing so results in a more coherent style, partly because one of the Chapters is coauthored, and second, it facilitates conversation between the author and the reader.

Chapter 1

Motivation and overview

This thesis examines monopolist price and quality discrimination strategies in the presence of positive network effects. Positive network effects are present if an economic agent's utility derived from the consumption of a good is positively affected by the consumption level (or number) of other agents consuming the same or compatible products.[1] These effects commonly arise in various modern industries, like telecommunications, hardware and software, and these industries share similar properties such as they are highly concentrated and firms charge different nonlinear tariffs. The goal is to build a general model of second-degree discrimination practiced by a monopolist seller of a network good, to give a complete characterization of the optimal contracts it can use, and to examine whether the results are in line with or against the classical results of the literature on incentive theory.[2]

The underlying motivation of this thesis is the lack of a general model on screening in the presence of positive network effects. In one of the most recent survey on network effects, Margolis and Liebowitz (2002) demonstrate the underconsumption result, i.e. each consumer is supplied with smaller than optimal quantities, in a very simple model of symmetric information. They then note that "the network model assumes that potential network participants are homogenous. [...] Absent this assumption, monopoly pricing, including price discrimination,

[1] Network effects are called network externalities in many papers, on the discussion of this distinction see Margolis and Liebowitz (1995, 2002).

[2] When we mention classical second-degree discrimination results, we refer to the works of Mussa and Rosen (1978) and Maskin and Riley (1984), summarized for example in Fudenberg and Tirole (1991) and Laffont and Martimort (2002).

could be brought into either model",[3] but to our knowledge there exists no paper accomplishing this work.

The main part of the thesis builds on the minimal assumption of complementarities, an inherent characteristic of positive network effects, that allows us to use monotone comparative static tools.[4] The major advantage of this approach is that instead of solving the model explicitly for the different equilibrium allocations, it gives a simple method to compare them by developing a parametrized functional form that encompasses the regimes to be compared as optimal solutions for different parameter values, and then shows that the optimal solution is a strictly monotone function of this parameter.[5]

1.1 Screening contracts in the presence of positive network effects

Chapter 2 builds a unifying framework to combine two well-known results. The first is one of the main conclusions in second-degree discrimination models, namely that the incentive problem due to information asymmetry makes the monopoly distort the quantity supplied to consumers with smaller willingness to pay for the good, but creates 'no distortion at the top': consumers with the largest willingness to pay are provided with the first-best optimal quantities. The second result is due to the externality literature: once a consumer's utility is not only the function of his own consumption level, but of the others' consumption levels as well, all economic agents end up with socially suboptimal quantities.[6] Network effects, which are generally assumed to be positive, result in underconsumption of the network good for all consumers. A natural conjecture would be that if asymmetric information and positive network effects are both present, these two results reinforce each other.

We develop a general framework based on the critical assumption of strategic complementarity to describe and solve the screening problem faced by the monopolist seller of a network good.

[3] Margolis and Liebowitz (2002), p. 82.

[4] Complementarities can arise in production as well, if the technology exhibits economies of scope, which is also frequently observed in network industries.

[5] The use of monotone comparative statics in economics was pioneered by Topkis (1978), Vives (1990) and Milgrom and Shannon (1994). The summary and applications of these results can be found in Topkis (1998) and Vives (1999).

[6] For a general overview on externalities, see for example Laffont (1988).

By applying monotone comparative static tools to compare first- and second- best allocations, we demonstrate that the joint presence of asymmetric information and positive network effects leads to a strict downward distortion for all consumers in the quantities provided. Despite this overall downward distortion result, we show that the equilibrium allocation is an increasing function of the network effect intensity, so positive network effects always lead to higher consumption levels for all consumers than in the standard models without network effects.

Complementarity also allows us to compare the allocations in different discrimination regimes to the one under perfect competition. We find that the perfectly discriminating monopoly always supplies higher quantities for all consumers than a perfectly competitive industry, since it can set the socially optimal allocation and reap the increased surplus of each consumer. This is because under perfect competition the network effects cannot be fully internalized, and the monopoly may perform better, since by controlling both the price and the quantity it may solve this type of coordination problem. On the other hand, the comparison of second-best discrimination and perfect competition allocations does not give unambiguous results, since we are comparing two outcomes, which fail to be the first-best because of two different reasons: incentive problems due to information asymmetry and the incapability of internalizing network effects. However, if the intensity of network effects is high enough, then a screening monopoly might perform strictly better from an allocation point of view.

1.2 Compatibility strategies and coordinations problems in contracting with network effects

The model given in Chapter 2 relies on two somehow restrictive assumptions: the good sold to different consumer groups are fully compatible with each other and consumers can coordinate on the Pareto-superior equilibrium, which is perfectly in line with the mechanism designer's incentives. Therefore, more pessimistic expectations can decrease a monopoly's power in practicing second-degree discrimination techniques, so firms might bear additional costs in influencing the formation of these expectations in order to avoid unfavorable outcomes. In Chapter 3 we show that compatibility and coordination questions are connected to each other, and we extend the previous model to include these features and compare the resulting equilibria.

In the first part, we examine the natural question whether the mechanism designer can decrease the costs of deterring consumers from switching by making some of its goods partially incompatible to others in a static screening setting. We show that in our benchmark model incompatibility strategies cannot be used to increase profits. The underlying reason behind this feature is that consumers have a one-dimensional type, so it is sufficient to screen them only in dimension, which is the quantity variable. However, incompatibility strategies may have a potential use in forming expectations that are a crucial factor in network models.

In the second part, we demonstrate how combining screening with divide-and-conquer techniques may solve the consumers' coordination problem and uniquely implement a screening mechanism. We show that with sequential contracting in the presence of asymmetric information and complementarities (i.e. network effects), different expectations about future network sizes will be relevant in the incentive constraints of different consumer types. The screening motive is much stronger in designing the contracts for consumers at later stages for two reasons. First, in the beginning the gap between the most optimistic and pessimistic expectations may be so big that it is too costly to separate the different types, and parallelly, for consumers at later stages a higher network size is already assured, and the monopoly can exploit the differences in network valuation more effectively. So the monopoly might prefer to pool the different types in the early contracting stages, and concentrate only on building a large installed base.

1.3 Functional degradation and asymmetric network effects

The general model developed in previous Chapters can be simply applied to study quality discrimination strategies in network industries. In the software industry, different versions of the same software are frequently encountered. A typical versioning policy we observe is the production of so called read-only versions, when the firm removes the writing function of the full version, and sells the degraded version at a lower price, often for free by making it downloadable from the Internet. What is particularly interesting in these versioning strategies is that users of different versions benefit from networks of different sizes and also create different network effects.

In Chapter 4, we build a functional degradation model, aiming to explain when and why firms have incentives to introduce a functionally-degraded good in markets subject to significant network effects. In our model it is assumed that consumers differ only in their valuation of network effects, which allows us to focus on the pure impact network effects have on the profitability of the degradation strategy, abstracting from quality differentiation based motives. Moreover, in order to capture the nature of functional degradation more accurately, we model network effects to be specific to individual functions imbedded in the good (e.g. writing and reading in a word-processor example), and also allow the intensity of network effects to be asymmetric across different functions. Our analysis shows that introducing a functionally-degraded good can be profitable for the firm if the consumers' preference structure in terms of the valuations of networks is biased towards the function removed in the process of creating the degraded version (e.g. the writing function in the read-only version), and that the firm may wish to offer the degraded version free of charge if the bias in network valuations is sufficiently large.

We also examine the welfare consequences of versioning (taking the case of selling only the full version as a benchmark), and establish sufficient conditions under which the functional degradation strategy leads to a Pareto-improvement. We find that the firm's private incentive for introducing a degraded good is very much aligned with a social planner's objective, i.e. the introduction of damaged network goods tends to improve social welfare and therefore should not be prohibited by public policy.

Chapter 2

Screening contracts in the presence of positive network effects

2.1 Introduction

This Chapter derives a general model of second-degree discrimination in the presence of positive network effects. Positive network effects are also called complementarities in consumption, and are present if an economic agent's utility derived from the consumption of the good is positively affected by the consumption level (or number) of other agents consuming the same or compatible products.[1] These effects commonly arise in various modern industries, like telecommunications, hardware and software or banking, and these industries are similar in the following properties: they are highly concentrated and the firms use a wide variety of nonlinear tariffs or very detailed contracts. Our goal in this Chapter is to use the tools of monotone comparative statics to describe the screening problem faced by a monopolist seller of a network good, and to give a complete characterization of the optimal contracts it can use.

We build a unifying framework to examine two well-known results, which are usually referred separately in the analysis of network economics. The first is one of the main conclusions in second-degree discrimination models, namely that the incentive problem due to information asymmetry makes the monopoly distort the quantity supplied to consumers with smaller will-

[1] For the implications of strategic complementarities on the production side (positive production externalities) in principal-agent models, see Lockwood (2000).

ingness to pay for the good, but makes 'no distortion at the top': consumers with the largest willingness to pay are provided with the first-best optimal quantities.[2] The second result is due to the externality literature: once a consumer's utility is not only the function of her own consumption level, but of the others' consumption levels as well, then in equilibrium all economic agents end up with socially suboptimal quantities.[3] Network effects, which are generally assumed to be positive, result in underconsumption of the network good for all consumers.

The main aim of this Chapter is to show that if asymmetric information and positive network effects are both present, these two impacts reinforce each other, so there will be a strict downward distortion for all consumers in the quantities provided. We also find that despite the downward distorting impact of positive network effects, the equilibrium outcome is an increasing function of the intensity of the network effects, no matter which type of discrimination we consider. The first result is a theoretical contribution to the literature on optimal screening, and together with the second it has important implications for optimal pricing policies in network economies. Last, we show that in some cases the discriminating monopoly supplies larger quantities for all consumers than a perfectly competitive industry, which result may be relevant for regulatory economics in these industries.

Let us demonstrate the strict downward distortion result by a simple example. Suppose there are only two consumers of a network good, let us call them sophisticated and normal. Assume that the sophisticated consumer benefits more both from her individual consumption and from network size, where the latter is now identified as total consumption level. Whenever the monopoly is capable of perfectly discriminating between the two consumers, it grasps both consumers' surplus and supplies the welfare-maximizing quantities. However, when the monopoly is restricted to offer the same menu of contracts to both consumers, standard incentive theory tells us that it should distort the quantity devoted to the normal consumer downwards in order to make switching less attractive to the sophisticated consumer. Now if the normal consumer's quantity decreases, so does network size, and since positive network effects are present, the sophisticated consumer's utility from her individual consumption is negatively affected. Thus,

[2] The seminal results of the second-degree discrimination literature were derived by Mussa and Rosen (1978) and Maskin and Riley (1984), summarized for example in Fudenberg and Tirole (1991) and Laffont and Martimort (2002).

[3] For a general overview on externalities, see for example Laffont (1988).

it is no more feasible to offer her the first-best optimal quantity, and her consumption should be distorted downwards as well.

Before starting with the main model, we briefly discuss the related literature. The "old literature" on network effects, which was basically on telecommunications pricing, focused on the question whether the network size would be the one that maximizes social welfare.[4] It was found that in perfect competition, the network effects cannot be fully internalized, and a monopoly may perform better since it controls both the price and the quantity, and may use cross-subsidization policies more effectively.[5] Our analysis reinforces the result that the perfectly discriminating monopoly always supplies larger quantities for all consumers than a perfectly competitive industry, since it may set the socially optimal allocation and reap the increased surplus of each consumer. However, the comparison of second-best discrimination and perfect competition allocations does not give unambiguous results, since we are comparing two outcomes that fail to be the first-best for two different reasons: incentive problems due to information asymmetry and the incapability of internalizing network effects.

The "new literature" on network effects considered mainly homogeneous types of consumers and concentrated on the multiple equilibria problem created by different (rational) expectations.[6] Models with heterogeneous types of consumers were only recently used by Fudenberg and Tirole (2000) and Ellison and Fudenberg (2001). However, these models analyze the effectiveness of dynamic strategies, like entry deterrence or software upgrades, in the presence of network effects, hence they concentrate on intertemporal discrimination aspects. Hahn (2003) builds a special model of telecommunication to examine the role of call and network externalities in nonlinear pricing. He establishes the result that in equilibrium all types end up with suboptimal quantities, so the 'no distortion on the top' result does not hold. Nevertheless, since he works with a special utility structure, he attributes this result to the existence of call externalities.

Two works closely related to ours are Segal (1999, 2003), which develop a general model

[4] Seminal papers include Rohlfs (1974), Littlechild (1975) and Oren et al. (1982). This classification between the old and new literature on network effects is based on Liebowitz and Margolis (2002).

[5] Similar conclusions have been derived in the macroeconomics literature on imperfect competition, for example in Cooper and John (1988, p. 454): "a demand externality may arise, though, in market structures where agents require information on both prices and quantities in making choices [...] In these cases quantities matter to individual decision makers, and prices do not completely decentralize allocations".

[6] Farrell and Saloner (1985) and Katz and Shapiro (1985) were the first to raise this problem.

of contracting with externalities and characterize the nature of the arising inefficiencies. When externalities are positive, Segal shows that each agent's consumption level is smaller in the resulting equilibrium allocation than in the socially efficient one. Strategic complementarity is identified as the factor accounting for this general feature;[7] however, the analysis names two additional assumptions that are useful in identifying the direction of distortions: first, the consumers are identical (hence there are no information asymmetries), and second, total welfare depends only on aggregate trade (that is the network size), and not on its allocation across consumers.[8]

Our main contribution to the Segal (1999) framework is to show that the underconsumption result holds without these simplifying assumptions if externalities are positive. In our model consumers are heterogenous in two respects: first, they have different (exogenously given) valuations towards the same menus, and second, depending on their (endogenous) choices, they may have different valuations for the same network.[9] This setting can be applied both to networks where the agents are screened by the different consumption or usage level (such as in telecommunication), and to networks where agents have unit demand for the good and are screened by the quality of the service (such as in software markets).

Strategic complementarity, which is an inherent characteristic of positive network effects, is the "critical assumption" in the terminology of Milgrom and Roberts (1994) that drives our results. It allows us to characterize the optimal contracts in a general setting by applying monotone comparative static tools, pioneered by Topkis (1978) and Milgrom and Shannon (1994). The main advantage of our approach is that instead of solving the model explicitly for the different first- and second-best allocations, it gives a simple method to compare the equilibrium allocations. We develop a parametrized functional form that encompasses both regimes as optimal solutions for different parameter values, and then show that the optimal solution is a strictly monotone function of this parameter.

The Chapter is organized as follows. Section 2 presents a discrete model of two types, where most of our qualitative results are already derived. This simplified model will be the base for the

[7] Segal (1999, p. 356) names this key property "increasing externality".

[8] See Segal (1999, p. 341).

[9] The second property works for the other direction as well: different networks sizes result in different marginal utilities of the same individual consumption for different consumers.

two extensions developed in the next Chapter as well, and in this part we give an introduction to the tools of monotone comparative statics we intend to use. Section 3 and 4 generalize the model for any discrete number and for a continuum of types, respectively, and discuss the differences and difficulties arising from the more complicated setup. In a nutshell, in the general discrete setup we should be more careful with the possible pooling of some consumer types, but the strict downward distortion result still holds. In the continuous setup, the characterization of optimal contracts will be completely the same, but on the other hand, as strict monotone comparative statics are designed mainly for decision problems in finite dimensions, we can establish only the downward distortion result, but not in the strict sense. Finally, Section 5 concludes and discusses the possible extensions of the model. The proofs of more technical Lemmas in this Chapter are put in Appendix A at the end of the thesis.

2.2 A discrete two-type model

Consider a monopoly that produces a good exhibiting positive network effects at a constant marginal cost c. Consumers have heterogenous preferences for the good, which heterogeneity is captured by a one-dimensional type parameter θ_i. In this Section we will work with only two types of consumers, so $\theta \in \{\theta_1, \theta_2\}$ where $0 < \theta_1 < \theta_2$. The probability for a consumer being of a type θ_1 and θ_2 is p and $(1-p)$, respectively, and this distribution function is common knowledge for all consumers and the monopoly. A consumer with the type parameter θ_2 (respectively, θ_1) will be usually referred as a high- (respectively, low-) type consumer. To facilitate notations, let us denote $(\theta_2 - \theta_1)$ by $\Delta\theta$. We assume that there is a continuum of consumers in each type, so a single consumer's contribution to the network is negligible.

Suppose that if consumer of type θ_i purchases q_i amount of the network good at the tariff t_i charged by the monopoly, then her utility will be defined as

$$\theta_i V(q_i, \overline{q}) - t_i,$$

where $\overline{q}$ is defined as the total amount of network good in the economy, usually referred as network size. So if low-type consumers buy q_1 and all high -type consumers purchase q_2, then $\overline{q} = pq_1 + (1-p)q_2$.

Three remarks are in order. First, in this Chapter we assume that the network goods sold to the consumers are perfectly compatible with each other, the possible use of incompatibility strategies will be explored in the next two Chapters. Second, the network size could be defined more generally such that consumers give different weights to the packages purchased by other consumers. For example, if consumers look at the usage levels of different packages as perfect substitutes in the network size, but give a higher weight to the packages similar to her own, then the "personal" network size of the consumer buying q_1 will be $\overline{q}_1 = \alpha p q_1 + \beta(1-p)q_2$, where $\alpha > \beta$. However, as we will argue later, as long as $\overline{q}_i$ is a strictly positive function of all q_i-s, our comparative statics results remain the same. Third, instead of the classical quantity discrimination approach, we could give another interpretation of this problem, where consumers have unit demands for the network good, and the goods differ in their quality q_i. If in this case we normalize the mass of consumers to 1, $\overline{q}$ can be seen as average quality level in the network.

Suppose that there are no externalities on non-traders: for all $\overline{q}, V(0,\overline{q}) = 0$, so the outside option is zero for all consumers.[10] However, we do not restrict our analysis to pure network goods, i.e. the stand-alone utility $V(q,0)$ may differ from 0. We assume that $V(\cdot)$ is twice continuously differentiable and that $V_1 > 0$ and $V_{11} \leq 0$, so the marginal utility of individual consumption level is positive and decreasing.[11]

The positivity of network effects is captured by the following key assumptions. First, any increase in network size increases the consumer's utility, so the marginal utility is always positive: $V_2 > 0$. Second, we assume that if network size becomes larger, each consumer's marginal utility of consuming an additional network good increases as well. Therefore the individual and total consumption levels are complements, so $V_{12} \geq 0$.

The timing of the model is as follows:

1. The monopoly offers a menu of contracts $\{(q,t)\} = \{(q_1,t_1),(q_2,t_2)\}$.

2. Consumers observe all possible contracts, and form their expectations $\overline{q}^e$ about the network size $\overline{q}$.

[10] This property ensures that consumers' reservation value is type-independent. See Jullien (2000) for a general model presenting the complications arising from type-dependent reservation values.

[11] In what follows, lower indexes always refer to partial derivatives of the function $V(\cdot)$ in its respective argument. Second, the words increasing (decreasing) and bigger (smaller) are used in the weak sense.

3. Each consumer decides which package to purchase or buys nothing, and payoffs are made.

We require that consumers' expectations are rational, so they should be fulfilled in equilibrium.

2.2.1 The first-best optimal contract

As a benchmark case, suppose that the monopoly knows each consumer's type, and based on this information it can discriminate among them. Then the contracts (q_1, t_1) and (q_2, t_2) will be designed such that in equilibrium each type expects to realize non-negative utility:

$$\theta_1 V(q_1, \overline{q}^e) - t_1 \geq 0, \text{ and} \tag{2.1}$$

$$\theta_2 V(q_2, \overline{q}^e) - t_2 \geq 0. \tag{2.2}$$

Monopoly profit is given by

$$\Pi = p(t_1 - cq_1) + (1-p)(t_2 - cq_2),$$

which should be maximized such that participation constraints (2.1) and (2.2) are satisfied and expectations are fulfilled: $\overline{q}^e = \overline{q} = pq_1 + (1-p)q_2$, since all consumers of the same type will behave in the same way. Naturally, it is optimal to make both of these constraints binding, which leaves no rent for the consumers:

$$\begin{aligned} t_1 &= \theta_1 V(q_1, \overline{q}), \text{ and} \\ t_2 &= \theta_2 V(q_2, \overline{q}). \end{aligned}$$

If these contracts are offered, then there exists only two rational expectations equilibria: either all types θ_i choose the contract (q_i, t_i) or all consumers choose the contract $(0, 0)$.[12] Since the monopoly is always able to get rid of the indifference by leaving an infinitesimally small rent to the consumers if choosing the contract (q_i, t_i), the first equilibrium can be made Pareto-preferred to the second one by virtually no cost, so we will concentrate only on the first

[12] If any consumer group (of measurable size) is expected to choose the null contract, then all other consumers have a strict incentive to follow.

equilibrium.[13] There are many concerns about the Pareto-criterion as an equilibrium selection device, as it will be discussed later, but in this Chapter we will use exclusively this one. The next Chapter deals with the costs the mechanism designer should bear if it wants to engage in unique implementation.

Therefore, the key decision variable for the monopoly is $q = (q_1, q_2)$, and the optimal quantity schedule determines the optimal tariff schedule. After substituting the binding participation constraints, we have the following profit-maximization problem:

$$\begin{aligned} \Pi^{FB}(q) &= p[\theta_1 V(q_1, \overline{q}) - cq_1] + (1-p)[\theta_2 V(q_2, \overline{q}) - cq_2] = \\ &= p\theta_1 V(q_1, \overline{q}) + (1-p)\theta_2 V(q_2, \overline{q}) - c\overline{q} \end{aligned} \tag{2.3}$$

Since there are no externalities on non-traders, the perfectly discriminating monopoly internalizes all network effects. Thus the monopoly's problem is equivalent with the welfare-maximizing one, and the optimal allocation produces no social inefficiency. This result is in line with Segal (2001), since so far the heterogeneity of consumers played no role.

In this Chapter, since we concentrate on the critical assumptions allowing us to compare the first- and second best outcomes, we assume that the respective profit function has a positive bounded maximum, without imposing any sufficient conditions guaranteeing this property. This approach is not as restrictive as it may sound. First, if a first-best maximum does not exist, the problem does not make much economic sense. Second, we will show that the first-best maximum provides an upper bound for the second-best maximum, so in the second-best case we are searching for a maximizer of a continuous function on a compact set, which certainly exists.

The following first-order conditions characterize the first-best allocation $q^{FB} = (q_1^{FB}, q_2^{FB})$:[14]

$$\theta_1 V_1(q_1, \overline{q}) + p\theta_1 V_2(q_1, \overline{q}) + (1-p)\theta_2 V_2(q_2, \overline{q}) = c, \text{ and} \tag{2.4}$$

$$\theta_2 V_1(q_2, \overline{q}) + p\theta_1 V_2(q_1, \overline{q}) + (1-p)\theta_2 V_2(q_2, \overline{q}) = c. \tag{2.5}$$

[13] An alternative way to secure the first equilibrium if the mechanism designer can make a credible commitment to a "money-back-gurantee" in case a measurable size of consumers fails to coordinate on the first equilibrium.

[14] Throughout this and the following Chapter, the final forms of the first-order conditions are always derived after dividing the equation by the density of the respective type, which is now p and $(1-p)$.

In both of these equations, the first term measures the marginal utility of individual consumption, we will call it first-best individual effect. The second term and the third term sums the marginal utility increases of all consumers due to the increased consumption of consumer group i, which will be called network effect. Note that the network effects are the same in all equations. If there are no network effects, we are back to the standard result of first-best implementation: individual effect should equal marginal cost.

By combining the two first-order conditions (2.4) and (2.5), we have that

$$\theta_1 V_1(q_1, \overline{q}) = \theta_2 V_1(q_2, \overline{q}). \tag{2.6}$$

Since $V_{11} \leq 0$, this equation implies that $q_1^{FB} < q_2^{FB}$, so in the first-best case high-types' consumption level is higher than low-types'.

2.2.2 The second-best optimal contract

If the monopoly should offer the same menu of contracts for all consumers, the first-best optimal menu of contract cannot generate a rational expectations equilibrium, since all high-type consumers would expect to have a positive surplus of $\Delta\theta V(q_1^{FB}, \overline{q}^{FB})$ by individually switching to the contract (q_1^{FB}, t_1^{FB}), which destroys the first-best equilibrium. In order to deter individual deviations, the second-best case the optimal menu $\{(q_1, t_1), (q_2, t_2)\}$ should satisfy the following incentive constraints:

$$\begin{aligned} \theta_1 V(q_1, \overline{q}) - t_1 &\geq \theta_1 V(q_2, \overline{q}) - t_2, \text{ and} & (2.7)\\ \theta_2 V(q_2, \overline{q}) - t_2 &\geq \theta_2 V(q_1, \overline{q}) - t_1, & (2.8) \end{aligned}$$

where we have already imposed the rational expectations condition $\overline{q}^e = \overline{q} = pq_1 + (1-p)q_2$. Since we have assumed a continuum of consumers in each type, a single consumer's choice cannot have a significant effect on network size, so $\overline{q}$ remains unchanged if other consumers stick to their equilibrium choice.

If we add the two incentive constraints (2.7) and (2.8), we have that

$$\Delta\theta[V(q_2, \overline{q}) - V(q_1, \overline{q})] \geq 0 \tag{2.9}$$

should hold. Since $V_1 > 0$, in order to have an implementable mechanism, we should have $q_2 \geq q_1$. We will refer to this condition as the monotonicity constraint. The reason why we end up with exactly the same implementability conditions as in the standard screening problem without network effects is because in the incentive constraints we require only that no consumer has any incentive to deviate individually from her equilibrium choice.

Now, as standard in incentive theory literature, we try to reduce the number of relevant constraints. First, the participation constraint for high-type consumers will be automatically satisfied if constraints (2.1) and (2.8) are satisfied, since

$$\theta_2 V(q_2, \overline{q}) - t_2 \geq \theta_2 V(q_1, \overline{q}) - t_1 \geq \theta_1 V(q_1, \overline{q}) - t_1 \geq 0.$$

Second, we expect that the incentive constraint for the low-type consumers holds with an inequality in equilibrium, so we ignore it for the moment and check at the end whether it is satisfied. Then by standard arguments the two relevant constraints (2.1) and (2.8) should bind in optimum and we can write down the tariffs charged by the monopoly as functions of the quantities:

$$t_1 = \theta_1 V(q_1, \overline{q}), \text{ and} \tag{2.10}$$

$$t_2 = \theta_2 V(q_2, \overline{q}) - \Delta\theta V(q_1, \overline{q}). \tag{2.11}$$

These two equations demonstrate the standard intuition of second-degree discrimination: the surplus of the low-type consumers is fully grasped, while high-type consumers get their information rent of $\Delta\theta V(q_1, \overline{q})$ in equilibrium. By substituting this results into the low-type consumers' incentive constraint (2.7), we see that it is satisfied if

$$\Delta\theta[V(q_2, \overline{q}) - V(q_1, \overline{q})] \geq 0,$$

which is exactly the monotonicity constraint derived before.

The question of multiple rational expectations equilibria

Note that this is not the only rational expectations equilibrium (REE) that can arise given $\{(q_1, t_1), (q_2, t_2)\}$ satisfying (2.10) and (2.11). Let us consider only the pure strategy equilibria, i.e. when all consumers of the same type choose the same contract $c_i = (q_i, t_i)$, where $c_0 = (0, 0)$. The following table examines the nine possibilities that may arise.

		θ_2		
		c_0	c_1	c_2
	c_0	always REE	possible REE	never REE
θ_1	c_1	never REE	never REE	always REE
	c_2	never REE	never REE	possible REE

If in any outcome the expected network size $\overline{q}'$ is smaller than $\overline{q} = pq_1 + (1-p)\, q_2$ then low-types expect to have negative utility. In this case they will always have an individual incentive to switch to c_0, so this property definitely rules out (c_1, c_0) and (c_1, c_1) as REE and also (c_2, c_0) if $pq_2 < \overline{q}$. However, if $pq_2 > \overline{q}$, then (c_2, c_0) cannot be an REE since high-types would have an individual incentive to switch to c_2. This is because c_2 was preferred to them at expected network size $\overline{q}$, so it is more the case by complementarity if expected network size is higher.

Let us now examine the outcome (c_0, c_2), where $\overline{q}' = (1-p)\, q_2$. In this outcome a high-type consumer expects $U_2 = \theta_2 V(q_2, \overline{q}') - t_2$, while by individually switching to c_1 she expects $U_2' = \theta_2 V(q_1, \overline{q}') - t_1$. After substituting tariff functions (2.10) and (2.11) we have that the expected surplus of switching is

$$\Delta U_2 = U_2' - U_2 = \theta_2 \{[V\,(q_2, \overline{q}) - V\,(q_1, \overline{q})] - \left[V\,\left(q_2, \overline{q}'\right) - V\,\left(q_1, \overline{q}'\right)\right]\}.$$

ΔU_2 is for sure positive given that $V_{12} \geq 0, q_2 \geq q_1$ and $\overline{q} \geq \overline{q}'$, so this outcome is not self-fulfilling either.

Next, in the outcome (c_2, c_1), any low-type consumer expects $U_1 = \theta_1 V(q_2, \overline{q}') - t_2$ and a high-type expects $U_2 = \theta_2 V(q_1, \overline{q}') - t_1$, where $\overline{q}' = pq_2 + (1-p)\, q_1$. If a low-type considers to individually switch to c_1, then she expects $U_1' = \theta_1 V(q_1, \overline{q}') - t_1$, while if a high-type considers to individually switch to c_2, she expects $U_2' = U_2 = \theta_2 V(q_2, \overline{q}') - t_2$. Let us sum the expected surpluses of switching:

$$\begin{aligned}\Delta U &= \left(U_2' - U_2\right) + \left(U_1' - U_1\right) = \\ &= \theta_2[V\left(q_2, \overline{q}'\right) - V\left(q_1, \overline{q}'\right)] - \theta_1[V\left(q_2, \overline{q}'\right) - V\left(q_1, \overline{q}'\right)].\end{aligned}$$

This term is again positive by $V_{12} \geq 0$ and $q_2 \geq q_1$, so at least one type would have an individual incentive to switch.

The outcome (c_1, c_2) is by construction an REE, and the same is true about (c_0, c_0) unless the stand-alone value $\theta_i V(q_i, 0)$ exceed t_i, which case we discard (if this happens, there are no coordination problems at all). However, complementarity itself does not rule out the possibility that $(0, c_1)$ and (c_2, c_2) are REEs. In the first outcome low-types will stick to c_0 since expected network size $\overline{q}' = (1-p)\, q_1$ is smaller than $\overline{q}$, and high-types have no incentive to switch to c_0 if

$$\theta_2 V(q_1, \overline{q}') - t_1 > 0,$$

which case we cannot exclude without making other restrictions. (A high-type will never have an individual incentive to switch to c_2 since she is indifferent between c_2 and c_1 at network size $\overline{q}$, and now expected network size is smaller.) Similarly, (c_2, c_2) can be an REE if low-types have no incentive to switch to c_1:

$$\theta_1 V(q_2, \overline{q}') - t_2 > \theta_1 V(q_1, \overline{q}') - t_1,$$

where now $\overline{q}' = q_2 > \overline{q}$.

Although we have shown there might be multiple equilibria, we have reasons to concentrate on the outcome (c_1, c_2). First, this is the only REE that leads to the screening of consumers, which is our central topic. Second, even if we have the maximum number of four equilibria, they can be Pareto-ordered. It can be easily checked that in this case

$$\begin{aligned}0 &= U_1(c_0, c_0) = U_1(c_0, c_1) = U_1(c_1, c_2) < U_1(c_2, c_2), \text{ and} \\ 0 &= U_2(c_0, c_0) < U_2(c_0, c_1) < U_2(c_1, c_2) < U_2(c_2, c_2),\end{aligned}$$

so if we apply the Pareto-criterion, the consumers always coordinate on the largest equilibrium. If (c_2, c_2) is not an REE, this means that the screening outcome (c_1, c_2) will be realized, and if (c_2, c_2) is an REE, the monopoly just makes more profit if consumers "fail" to coordinate on (c_2, c_2) instead of (c_1, c_2). Additionally, in the latter case the monopoly's profit-maximizing screening problem should also lead to offer a pooling menu with $c_1 = c_2$.

The result that equilibria are ordered in games with strategic complementarities is widely known (Theorem 7 in Milgrom-Roberts (1990)), and an additional reason to concentrate on the Pareto-superior equilibrium in these games is that it is the only coalition-proof (correlated) equilibrium under any admissible communication structure.[15] However, our game is not one with strategic complementarities in the strict sense, since as we will define and examine later in Subsection 2.2.3, the utility functions of the players are only quasisupermodular, but not supermodular in the quantity decision variables.

The characterization of optimal screening contracts

Substituting equations (2.10) and (2.11) leads to the following profit function:

$$\begin{aligned}\Pi^{SB}(q) &= p[\theta_1 V(q_1, \overline{q}) - cq_1] + (1-p)[\theta_2 V(q_2, \overline{q}) - \Delta\theta V(q_1, \overline{q}) - cq_2] = \\ &= p\left(\theta_1 - \frac{1-p}{p}\Delta\theta\right) V(q_1, \overline{q}) + (1-p)\theta_2 V(q_2, \overline{q}) - c\overline{q}. \end{aligned} \tag{2.12}$$

This function should be maximized with respect to the monotonicity constraint (2.9) and the non-negativity constraint $q \geq 0$. We ignore these constraints for the moment, and check at the end whether they are satisfied in equilibrium.

The optimal allocation $q^{SB} = (q_1^{SB}, q_2^{SB})$ can be characterized by the first-order conditions. First, it is more instructive to analyze the first-order condition in respect of q_2:

$$\theta_2 V_1(q_2, \overline{q}) + p\theta_1 V_2(q_1, \overline{q}) + (1-p)\theta_2 V_2(q_2, \overline{q}) - (1-p)\Delta\theta V_2(q_1, \overline{q}) = c. \tag{2.13}$$

The first term is the first-best individual effect, the second and the third terms are the first-best network effects. However, there is an additional term, which will be called second-best network

[15] Note, however, that this latter result has been proved only for a discrete number of players, see Milgrom-Roberts (1996).

effect. The presence of a second-best term in the optimum condition for the high-type consumers is due to the fact that the information rent $\Delta\theta V(q_1, \overline{q})$ is affected by total network size $\overline{q}$ as well, which contains q_2, while in standard incentive theory the information rent depends only on q_1. The emergence of a second-best term in the first-order condition of high-type consumers already foreshadows the result that in the presence of network effects the 'no distortion at the top' result will no longer hold.

The other first-order condition in respect of q_1 is the following:

$$\theta_1 V_1(q_1, \overline{q}) + p\theta_1 V_2(q_1, \overline{q}) + (1-p)\theta_2 V_2(q_2, \overline{q}) -$$
$$-(1-p)\Delta\theta V_2(q_1, \overline{q}) - \frac{1-p}{p}\Delta\theta V_1(q_1, \overline{q}) = c. \tag{2.14}$$

The first three terms are the first-best effects, the fourth one is the second-best network effect. The fifth term will be called standard second-best effect, and has exactly the same form as the effect creating the downward distortion for the low-type consumers in the standard discrete-type model without network effects.

If we combine the two first-order conditions (2.13) and (2.14), we arrive to the following condition:

$$\theta_1' V_1(q_1, \overline{q}) = \theta_2 V_1(q_2, \overline{q}), \tag{2.15}$$

where $\theta_1' = \theta_1 - \frac{1-p}{p}\Delta\theta$. Now let us examine the implications of the omitted constraints.

First, in order to have a positive solution for q_1, θ_1' should be positive, since V_1 is always positive. If θ_1' were non-positive, it would be optimal for the monopoly to exclude low-type consumers by setting $q_1^{SB} = 0$ (and then naturally $t_1^{SB} = 0$ as well), since for any positive q_1 the information rent given away for high-type consumers would be higher then the surplus collected from low-type consumers. We will refer to the condition $\theta_1' \geq 0$ as the no shut-down condition, which has again the same form as in the classical screening literature without network effects. Second, since $\theta_1' < \theta_2$ and $V_{11} \leq 0$, it follows that $q_1 < q_2$, so the monotonicity constraint is always satisfied.

In the following discussion we can assume without any loss of generality that in the optimal mechanism both types are served and discriminated. Otherwise, if low types were shut down, we would trivially have the strict downward distortion result: low-type consumers' consumption is decreased to zero, and although high-types are supplied a first-best quantity, this should be calculated at a smaller network size of $\overline{q} = (1-p)q_2^{FB}$, so they end up with smaller quantities as well.

An illustrative example

In order to demonstrate the general results derived later, let us solve the problem for a specific utility structure. Let the valuation function $V(q_i, \overline{q})$ be multiplicatively separable in q_i and $\overline{q}$, specifically we assume that $V(q_i, \overline{q}) = q_i^a \overline{q}^b$, where a and b are positive constants. In order to simplify notations, let us normalize θ_2 to 1, so $0 < \theta_1' < \theta_1 < 1$. Note that the valuation function is homogenous of degree $(a+b)$ in (q_1, q_2), while the linear cost function is homogenous of degree 1. So if we want to have a bounded maximum, we need to assume that $a + b < 1$.

We can write up the following profit function:

$$\Pi(q, \theta) = \left[p\theta q_1^a + (1-p)q_2^a\right] \overline{q}^b - c\overline{q}.$$

This general formula encompasses both the first-best profit function given in (2.3) for $\theta = \theta_1$ and the second-best profit function given in (2.12) for $\theta = \theta_1'$.

Both in the first-best and second-best regime we have derived the general result that the individual effects of increasing respectively q_1 and q_2 equal each other (see conditions (2.6) and (2.15)), so

$$\theta a q_1^{a-1} \overline{q}^b = a q_2^{a-1} \overline{q}^b.$$

This equation can be used to express the optimal q_1 as a function of the optimal q_2:

$$q_1(q_2, \theta) = q_2 \theta^{\frac{1}{1-a}} \tag{2.16}$$

Since $\theta < 1$ and $\frac{1}{1-a} \in (0, 1)$, q_1 should be smaller than q_2.

Another interesting property is that in optimum

$$\frac{q_1(\theta)}{q_2(\theta)} = \theta^{\frac{1}{1-a}}.$$

Since $\frac{1}{1-a}$ is positive, the ratio $\frac{q_1}{q_2}$ is a strictly increasing function of θ. Since θ is smaller under the second-best regime, we see that the incentive problem creates a downward distortion in the relative magnitudes of the optimal quantities compared to the first-best regime.

We can now evaluate the marginal revenue from increasing q_2, which (after dividing by the respective density $1-p$) gives us the sum of individual and network effects at the left-hand side of first-order conditions (2.5) and (2.13) for first- and second-best regime. The discounted marginal revenue denoted by MR_2 is a function of q_1 and q_2, but we can use the optimum condition (2.16) to reduce it to a one-dimensional function of q_2. After tedious computations (see the steps to be followed in the continuous-type example of Subsection 2.4.4), this expression simplifies to

$$MR_2(q_2, \theta) = (a+b)\left(p\theta^{\frac{1}{1-a}} + (1-p)\right)^b q_2^{a+b-1}.$$

We can now summarize the main results of this example. First, there exists a unique optimum (q_1, q_2) for each θ with $q_i > 0$. This is because $a + b < 1$, so $MR_2(q_2)$ is a strictly decreasing function of q_2, multiplied by a positive constant, while $\lim_{q_2 \to 0} MR_2(q_2) = \infty$ and $\lim_{q_1 \to \infty} MR_2(q_2) = 0$. Therefore, there exists a unique positive q_2 where $MR_2(q_2) = c$.

Second, the first-best optimum q^{FB} is strictly larger than the second-best optimum q^{SB}. This is because $MR_2(q_2, \theta)$ is a strictly increasing function of θ, so $q_2(\theta_1') < q_2(\theta_1)$ by $\theta_1' < \theta_1$, and by condition (2.16) q_1 is a strictly increasing function of q_2, hence $q_1(\theta_1') < q_1(\theta_1)$.

2.2.3 Comparison of equilibria

We have derived the first-order conditions for the perfect (first-degree) and incentive (second-best) discrimination cases, which characterize the equilibrium quantities, and thereby the equilibrium tariffs in the respective regimes. We have seen that in the second-best case the presence of network effects distorts the first-best allocations for all consumers, and we have demonstrated by an example that this is a strict downward distortion for all consumers types. However, as our illustrative example showed, solving the equation system of interrelated first-order conditions and then comparing directly the equilibrium allocations involve tedious computations for a simplified utility structure as well.

For deriving our main result of strict downward distortion in quantities in general, we develop a parametrized functional form that encompasses both regimes as optimal solutions for different parameter values, and then use monotone comparative statics tools to show that the optimal solution is a strictly monotone function of this parameter. We have already followed this approach in our illustrative approach, but we were able to check the impact of changing the model parameter only after solving the system of first-order conditions.

A primer on monotone comparative statics

First, we review several definitions and results which will be used in the following discussion.[16] Let X denote a choice set which is partially ordered by the relation $\geqslant$. For any $x', x'' \in X$ we define $x' \vee x''$ as the least upper bound (so called join) of x' and x'' and $x' \wedge x''$ as the greatest lower bound (so called meet) of x' and x''. Then X is a lattice if it contains $x' \vee x''$ and $x' \wedge x''$ for any $x', x'' \in X$. In our case, $X = R^2$, the partial order is the componentwise order, and $x' \vee x'' = (\max\{x_1', x_1''\}, \max\{x_2', x_2''\})$ and $x' \wedge x'' = (\min\{x_1', x_1''\}, \min\{x_2', x_2''\})$.

The property that the decision variables are complementary can be expressed in a strong and a weak form. Suppose that X is a lattice. Then the objective function $f(x) : X \to R$ is supermodular in x on X if for any $x', x'' \in X$ we have $f(x') + f(x'') \leq f(x' \vee x'') + f(x' \wedge x'')$. It can be shown that in the case of smooth objective functions in R^n, this condition is equivalent to $\frac{\partial^2 f(x)}{\partial x_i \partial x_j} \geq 0$ for all $x_i \neq x_j$ and x,[17] so the marginal return of any choice variable is increasing in all other choice variables.

A weaker definition is the following. The objective function $f(x) : X \to R$ is quasisupermodular in x on X if for any $x', x'' \in X, f(x' \wedge x'') \leq f(x')$ implies $f(x'') \leq f(x' \vee x'')$ and $f(x' \wedge x'') < f(x')$ implies $f(x'') < f(x' \vee x'')$. In the words of Milgrom and Roberts (1994, p. 162), "if an increase in some subset of the choice variables is desirable at some level of the remaining choice variables, it will remain desirable as the remaining variables also increase". It is easy to see that supermodularity implies quasisupermodularity, but not vice versa.

Finally, we want to examine the relation between the choice variables x and parameters t. Let X be a lattice and T is a partially ordered set, and the objective function $f(x,t) : X \times T \to R$. Then $f(x,t)$ satisfies the single-crossing property in (x,t) on $X \times T$ if for any $x', x'' \in X$ and $t', t'' \in T$ such that $x' > x''$ and $t' > t'', f(x', t'') \geq f(x'', t'')$ implies $f(x', t') \geq f(x'', t')$

[16] This review follows closely the comprehensive book of Topkis (1998), and we refer to this book for all proofs of the results stated.

[17] See Theorem 2.6.1 and 2.6.2 in Topkis (1998).

and $f(x', t'') > f(x'', t'')$ implies $f(x', t') > f(x'', t')$. This property can be also stated in strict form: $f(x, t)$ satisfies the strict single-crossing property in (x, t) on $X \times T$ if for any $x', x'' \in X$ and $t', t'' \in T$ such that $x' > x''$ and $t' > t''$, $f(x', t'') \geq f(x'', t'')$ implies $f(x', t') > f(x'', t')$.[18] We will also use a stronger cardinal property concerning the relation between a choice variable and the parameters: we will say that x_i has increasing marginal returns if $\frac{\partial f(x,t)}{\partial x_i}$ is strictly increasing in t.

Now let us state the main theorems of monotone comparative statics that we intend to build on.

1. Monotonicity Theorem (Milgrom and Shannon (1994), Theorem 4): $\arg\max_{x \in X} f(x, t)$ is increasing in t on T if and only if $f(x, t)$ is quasisupermodular in x on X for all $t \in T$ and satisfies the single-crossing property in (x, t) on $X \times T$

2. Monotone Selection Theorem (Milgrom and Shannon (1994), Theorem 4'): if $f(x, t)$ is quasisupermodular in x on X for all $t \in T$ and satisfies the strict single-crossing property in (x, t) on $X \times T$ then every selection $x^*(t) \in \arg\max_{x \in X} f(x, t)$ is increasing in t on T.

Note the Monotone Selection Theorem identifies the condition only for non-decreasing optimal response to an increase in the exogenous parameter(s). However, in some cases (as in our problem) it might be desirable to examine whether the maximizer is a strictly increasing function of the model parameters. By arguing from first-order conditions, Edlin and Shannon (1998) extend this result by showing that under some conditions the maximizer should be strictly increasing in at least one dimension. In the next Lemma we extend their result even further by identifying a sufficient condition so that the maximizer is strictly increasing in all dimensions.

Lemma 1 *Let $X \subset R^n$ be a lattice and T partially ordered set. Let $f : X \times T \to R$ be a continuously differentiable function that is quasisupermodular in x on X for all $t \in T$ and that has increasing marginal returns for all choice variables. Then if $x^*(t) \in \arg\max_{x \in X} f(x, t)$ and $x^*(t) \in int(X)$, $t'' > t'$ implies $x^*(t'') > x^*(t')$ for every selection of maximizers.*

Proof. *See Appendix A.1* ■

[18] This property has strong connections to the so-called Spence-Mirrlees condition widely used in the screening literature, see Milgrom-Shannon (1994) for more details.

Main comparative statics results

Now let us check how our model setting satisfies the definitions given above. The objective functions are the monopoly's profit functions, which for the respective regimes are

$$\begin{aligned}\Pi^{FB}(q) &= p\theta_1 V(q_1, \overline{q}) + (1-p)\theta_2 V(q_2, \overline{q}) - c\overline{q}, \text{ and} \\ \Pi^{SB}(q) &= p\theta_1' V(q_1, \overline{q}) + (1-p)\theta_2 V(q_2, \overline{q}) - c\overline{q},\end{aligned}$$

where $\theta_1' = \theta_1 - \frac{1-p}{p}\Delta\theta \in (0, \theta)$.

We should first deal with the problem that consumers' utility was defined for each pair of individual consumption and network size, which was more convenient to formalize the consumers' decision problem, so the complementarity definitions were also given for this functional form. However, the mechanism designer's key decision variables are the quantities supplied for the different types, so we should examine which properties of the utility functions are inherited to the profit function.

For this purpose, let us define $U_i(\theta, q) = \theta V(q_i, \overline{q})$, where $\overline{q} = pq_1 + (1-p)q_2$. Note that we again impose that in equilibrium all consumers of the same type θ_i consume the same amount of q_i, so we can refer to q_i as the group consumption level of type-i consumers. First, the marginal returns of all q_i-s are positive, since the marginal utility of both individual and network consumption levels are assumed to be positive. Second, this property implies that $U_i(\theta, q)$ is strictly increasing in q, and a strictly increasing function always satisfies $U_i(\theta, q'') < U_i(\theta, q' \vee q'')$ for any q', q''. Therefore $U_i(\theta, q)$ is quasisupermodular in q for all θ.

Both profit functions are a weighted sum of $U_i(\theta, q)$-s, where the weights are all strictly positive, minus a linear term in q, which is always submodular in q.[19] Since quasisupermodularity is preserved for monotone transformations,[20], both profit functions are quasisupermodular in q all $t \in T$.

Note that neither the individual utility functions, nor their weighted sum satisfies necessarily the stronger form of complementarity, namely supermodularity on q. For example, in the case

[19] A function is submodular if its negative is supermodular. A linear function is naturally both super- and submodular.

[20] See Lemma 2.6.5 in Topkis (1998).

of $U_1(\theta_1, q) = \theta_1 V(q_1, \overline{q})$ this would require that

$$\frac{\partial^2 U_1(q)}{\partial q_1 \partial q_2} = \theta_1 (1-p) \left[V_{12}(q_1, \overline{q}) + p V_{22}(q_1, \overline{q}) \right] \geq 0,$$

and in the case of the first-best profit function that

$$\frac{\partial^2 \Pi^{FB}(q)}{\partial q_1 \partial q_2} = p(1-p) \{ \theta_1 \left[V_{12}(q_1, \overline{q}) + p V_{22}(q_1, \overline{q}) \right] +$$

$$\theta_2 \left[V_{12}(q_1, \overline{q}) + (1-p) V_{22}(q_1, \overline{q}) \right] \} \geq 0,$$

hold for all q and parameter values $p \in (0,1)$ and θ_1. However, in the case of weak complementarity between individual consumption and network size (i.e. when $V_{12}(\cdot)$ is close to zero), these conditions could be guaranteed only if $V_{22}(\cdot)$ were assumed to be positive. But under this assumption the utility functions $U_i(\theta_1, q)$ have "increasing returns to scale" in q: for any $t > 1, tq$ would generate a surplus more than $tU_i(\theta_1, q)$, and since the technology has constant returns to scale, the monopoly's optimal profit would not have a finite positive value.

Armed with these properties, we are now able to give a simple proof for our main theorem.

Proposition 2 *The second-best allocation is strictly smaller than the first best allocation, that is $q^{SB} < q^{FB}$.*

Proof. *Consider the following parametrized form $\Pi : Q \times T \rightarrow R$, where $T = R_+$ is the parameter space:*

$$\Pi(q, \alpha) = \alpha p V(q_1, \overline{q}) + (1-p)\theta_2 V(q_2, \overline{q}) - c\overline{q}.$$

When $\alpha = \theta_1$, we have the first-best profit function $\Pi^{FB}(q)$, while for $\alpha = \theta_1' \in (0, \theta_1)$ we have the second-best profit function $\Pi^{SB}(q)$. First, we have already seen that the function $\Pi(q, \alpha)$ is quasisupermodular in q on Q for all α. Second, since $U_i(\theta, q)$ is multiplicatively separable in θ and q, the marginal returns of all q_i-s are increasing in α, so $\Pi(q, \alpha)$ also should have increasing marginal returns for all q_i-s. Therefore, since $\alpha^{FB} > \alpha^{SB}$, Lemma 1 ensures that $q(\alpha^{SB}) < q(\alpha^{FB})$. ■

Two remarks are in order. First, in the analysis consumers of different types faced the same network size $\overline{q} = pq_1 + (1-p)q_2$. However, if we define the "personal" network size as $\overline{q}_i = g(q_i, q_{-i})$ such that $g(\cdot)$ is strictly increasing in both dimensions, then $U_i(\theta, q)$ is still quasisupermodular in q on Q, and so is $\Pi(q, \alpha)$, so our comparative statics results remain the same. Second, we have chosen constant marginal costs only for expositional simplicity. Our qualitative results remain unchanged if we allow for a cost function exhibiting weak cost complementarities (or so called economies of scope), a property that generally fits the structure of network industries.[21] This is because weak cost complementarities imply a submodular cost function, as shown by Sharkey (1982), and then $\Pi(q, \alpha)$ remains quasisupermodular in q on Q.

Now we compare the equilibrium allocations of first- and second-best discrimination regimes to the perfectly competitive case, which we define as identical firms supplying the network good at a price equal to marginal cost c. Then each consumer of type θ_i derives a utility of

$$\theta_i V(q_i, \overline{q}^{PC}) - cq_i,$$

where $\overline{q}^{PC}$ is the expected network size under perfect competition, which should be fulfilled in equilibrium.

Maximizing utility in q_i results in the first-order condition of

$$\theta_i V_1(q_i, \overline{q}^{PC}) = c, \text{ for all } i \in N. \tag{2.17}$$

Let us denote the solution of this equation system by q^{PC}. Again, by combining two first-order conditions, we see that $q_i^{PC} > q_j^{PC}$ for $\theta_i > \theta_j$.

Proposition 3 *The equilibrium allocation under perfect competition is strictly smaller than in the first-best discrimination case, that is* $q^{PC} < q^{FB}$.

Proof. *If we compare first-order conditions (2.17) with the first-order conditions (2.4) and (2.5) of the first-best discrimination case, we see that*

$$\theta_i V_1(q_i^{FB}, \overline{q}^{FB}) < \theta_i V_1(q_i^{PC}, \overline{q}^{PC})$$

for all $i \in N$. *In the first-best case, the monopoly is supplying the welfare-maximizing allocation, so* $\overline{q}^{FB}$ *cannot be smaller than* $\overline{q}^{PC}$, *since the externalities are positive. Then since* $V_{12} \geq 0$,

[21] It is also a sufficient condition for the firm being a natural monopoly.

for all q_i^{FB}

$$V_1(q_i^{FB}, \overline{q}^{PC}) \leq V_1(q_i^{FB}, \overline{q}^{FB}).$$

Combining this inequality with the former one, we have that

$$\theta_i V_1(q_i^{FB}, \overline{q}^{PC}) < \theta_i V_1(q_i^{PC}, \overline{q}^{PC}),$$

which yields that $q_i^{FB} > q_i^{PC}$ *for all* $i \in N$*, since* $V_{11} \leq 0$. ∎

The underlying reason behind this result is fairly intuitive. Perfectly competitive firms cannot internalize the network effects implied by larger allocations, since the linear pricing scheme does not allow to influence the quantity choice of the consumers. On the other hand, the perfectly discriminating monopoly can set the (larger) socially optimal allocation, and reap the increased surplus of each consumer.

The comparison of the allocations under second-best discrimination and perfect competition (defined by equations (2.13), (2.14) and (2.17)) does not give unambiguous results, since we are comparing two outcomes, which fail to be the first-best for two different reasons: incentive problems due to information asymmetry and the incapability of internalizing network effects. However, if the impact of network effects is large enough to offset the effect due to the decrease in individual consumption (loosely speaking, if V_2 is sufficiently larger than $p\theta_1' V_1$), then we conjecture to have a larger allocation in the screening monopoly regime than under perfect competition.

Last, we show that the equilibrium outcome is an increasing function of the intensity of the network effects, no matter which type of discrimination we consider. As a corollary, we can state that discrimination in the presence of network effects always leads to a larger allocation than in the standard screening case (i.e. without network effects). This result is natural in the case of first-best discrimination, but it also shows that despite the downward distorting factor from the first-best allocation, the presence of network effects has a positive impact in total on the resulting allocation in the second-best case as well.

Let us slightly modify the utility function to $\theta_i V(q_i, \beta\overline{q}) - t_i$, where $\beta \geq 0$ measures the intensity of network size. If $\beta = 0$, we are back to the standard discrimination case without positive network effects, while our original model refers to $\beta = 1$.

Proposition 4 *Both for first- and second-best discrimination in the presence of network effects, an increase in the network effect intensity* β *results in*

1. *a weak increase of the equilibrium allocation* $q(\beta)$,

2. *a strict increase of the quantity supplied for at least one consumer type, and*

3. *a strict increase of the equilibrium network size* $\overline{q}(\beta)$.

Proof. *Let us define the following function* $\Pi : Q \times T \to R$, *where* $T = R_+^2$:

$$\Pi(q, \alpha, \beta) = \alpha p V(q_1, \beta\overline{q}) + (1-p)\theta_2 V(q_2, \beta\overline{q}) - c\overline{q}.$$

The parameter α *still stands for the regime to be analyzed:* $\alpha = \theta_1$ *in first-best discrimination and* $\alpha = \theta_1'$ *in second-best discrimination. We have seen that the function* $\Pi(q, \alpha, \beta)$ *is quasisupermodular in* q *on* Q *for each* α *and* β. *We also have that* $U_i(\theta_i, q, \beta) = \theta_i V(q_i, \beta\overline{q})$ *satisfies the strict single-crossing property in* (q, β) *for all* $\theta_i > 0$, *since if* $q' > q''$ *and* $\beta' > \beta''$, $\theta_i V(q_i', \beta''\overline{q}') \geq \theta_i V(q_i'', \beta''\overline{q}'')$ *implies* $\theta_i V(q_i', \beta'\overline{q}') > \theta_i V(q_i'', \beta'\overline{q}'')$ *by* $V_2 > 0$ *- if a larger allocation is weakly preferred at smaller network intensity then the larger allocation should be strictly preferred at larger network intensity. Since the strict single-crossing is preserved under monotone transformation and all the multipliers are positive constants,* $\Pi(q, \alpha, \beta)$ *satisfies the strict single-crossing property in* (q, β) *as well. We can therefore apply the Monotone Selection Theorem to the equilibrium allocation* $q(\beta) \in \arg\max \Pi(q, \alpha, \beta)$*:* $\beta' > \beta''$ *implies* $q(\beta') \geq q(\beta'')$.

Since we are considering interior solutions, the first-order conditions for this profit-maximizing problem are respectively

$$\begin{aligned} \alpha V_1(q_1, \beta\overline{q}) + \alpha p V_2(q_1, \beta\overline{q}) + (1-p)\theta_2 V_2(q_2, \beta\overline{q}) &= c, \text{ and} \\ \theta_2 V_1(q_2, \beta\overline{q}) + \alpha p V_2(q_1, \beta\overline{q}) + (1-p)\theta_2 V_2(q_2, \beta\overline{q}) &= c. \end{aligned}$$

Combining these equations leads to $\alpha V_1(q_1, \beta\overline{q}) = \theta_2 V_1(q_2, \beta\overline{q})$. *As* q'' *is optimal given* β'', *we have* $\alpha V_1(q_1'', \beta''\overline{q}'') = \theta_2 V_1(q_2'', \beta''\overline{q}'')$. *Now if we look at this equation by changing only* β'' *to* β', *we have* $\alpha V_1(q_1'', \beta'\overline{q}'') < \theta_2 V_1(q_2'', \beta'\overline{q}'')$, *since* $q_1 < q_2, V_{12} > 0$ *and* $\alpha < \theta_2$. *But this means that*

$q\left(\beta''\right) \notin \arg\max \Pi(q,\alpha,\beta')$, so $q\left(\beta'\right)$ should be larger then $q\left(\beta''\right)$ in at least one dimension. Finally, this property trivially implies that $\overline{q}(\beta) = pq_1(\beta) + (1-p)\, q_2(\beta)$ should strictly increase in β. ■

The reason why we cannot apply our Strict Monotone Selection Theorem in this case is that the choice variables do not necessarily have increasing marginal returns, as

$$\frac{\partial^2 \Pi^j}{\partial q_1 \partial \beta} = p\overline{q}\left\{\alpha^j p\left[V_{12}(q_1, \beta\overline{q}) + pV_{22}(q_1, \beta\overline{q})\right] + (1-p)\theta_2 V_{22}(q_2, \beta\overline{q})\right\}$$

are not necessarily positive in general. It is natural to conjecture a strict increase of the allocation in the network intensity parameter, but it can be shown only in specific examples, as shown in the next Subsection.

Illustrative example - continued

We now turn back to our example with the functional form $V(q_i, \overline{q}) = q_i^a \left(\overline{q}\right)^b$ to illustrate the results stated in Propositions 3 and 4 and examine the validity of our conjectures made.

The first-order condition 2.17 under perfect competition writes as

$$\begin{aligned}
\theta_1 a \left(q_1\right)^{a-1} \left(pq_1 + (1-p)q_2\right)^b &= c, \text{ and} \\
a \left(q_2\right)^{a-1} \left(pq_1 + (1-p)q_2\right)^b &= c
\end{aligned}$$

for low- and high-type consumers, respectively. By combining the two conditions, we have that

$$\theta_1 a \left(q_1\right)^{a-1} \left(\overline{q}\right)^b = a \left(q_2\right)^{a-1} \left(\overline{q}\right)^b,$$

so after rearranging we have

$$q_1 = q_2 \left(\theta_1\right)^{\frac{1}{1-a}}.$$

Note that the functional relationship between the optimal q_1 and q_2 under perfect competition is the very same as under first-best discrimination (see condition 2.16), and since it is described by a strictly increasing function, we have that if q_2^{PC} is larger (smaller) than q_2^{FB} then q_1^{PC} should be larger (smaller) than q_1^{FB} as well.

By substituting $q_2(q_1)$ into the discounted marginal revenue of q_2, after some simplifications it simplifies to

$$MR_2^{PC}(q_2) = a\left(p\left(\theta_1\right)^{\frac{1}{1-a}} + (1-p)\right)^b (q_1)^{a+b-1},$$

while for the discrimination case analyzed before we have derived that

$$MR_2(q_2,\theta) = (a+b)\left(p\theta^{\frac{1}{1-a}} + (1-p)\right)^b (q_1)^{a+b-1},$$

where $\theta = \theta_1$ for the first-best regime, and $\theta = \theta_1' < \theta_1$ for the second-best regime. We have already seen that both functions are decreasing in q_2 and increasing in θ. The respective allocation is found where the decreasing discounted marginal revenue equals the constant marginal cost.

First, it is easy to see that $MR_1^{PC}(q_1)$ is always smaller than $MR_1(q_1,\theta_1)$ for all positive values of b, so if positive network effects are present, the perfectly competitive is indeed strictly smaller than under first-best discrimination.

Second, comparing $MR_2^{PC}(q_2)$ and $MR_2(q_2,\theta')$ justifies the discussion we made after discussing the ordering of the competitive and second-best discrimination outcome. We have that q_2^{SB} is smaller than q_2^{PC} only if

$$(a+b)\left(p\left(\theta_1'\right)^{\frac{1}{1-a}} + (1-p)\right)^b > a\left(p\left(\theta_1\right)^{\frac{1}{1-a}} + (1-p)\right)^b,$$

so the network effects captured by b is not strong enough to offset the effect of $\theta_1' < \theta_1$. Since $q_1^{SB}(q_2) = q_2^{SB}\left(\theta_1'\right)^{\frac{1}{1-a}}$ and $\frac{1}{a-1} < 0$, $q_2^{SB} < q_2^{PC}$ implies $q_1^{SB} > q_2^{PC}$ as well. However, if $q_2^{SB} > q_2^{PC}$, then the ordering of q_1^{SB} and q_1^{PC} is ambiguous.

In order to examine the impact of network effect intensity, we redefine the valuation function as $V(q_i,\overline{q}) = q_i^a\left(\beta\overline{q}\right)^b$, where $\beta \geq 0$ measures the intensity of network effects. Now the profit function writes as

$$\Pi(q,\theta,\beta) = \beta^b\left[p\theta q_1^a + (1-p)q_2^a\right]\overline{q}^b - c\overline{q},$$

so the discounted marginal revenue is

$$MR_2(q_2,\theta,\beta) = \beta^b\,(a+b)\left(p\theta^{\frac{1}{1-a}} + 1 - p\right)^b q_1^{a+b-1}.$$

Since $MR_2(q_2, \theta, \beta)$ is strictly increasing in β, $q_2(\theta, \beta)$ and therefore $q_1(\theta, \beta)$ are both strictly increasing functions of the network effect intensity parameter β.

2.3 The general discrete type case

Now we generalize the model given above to any discrete number of types. Since most the results of the previous Section generalize for this case, we will be as brief as possible and engage in intuitive discussion only where it has not been given before.

There are n different types of consumers such that $0 < \theta_1 < \theta_2 < ... < \theta_n$, and let N denote the set of different types. There is a continuum of consumers in each type, and the types are independently distributed by a given cumulative distribution function $F(\theta)$ and a respective density function $f(\theta)$. The distribution function is common knowledge for all consumers and the monopoly.

2.3.1 The first-best optimal contract

If the monopoly can offer personalized contracts to each consumer, then each type should realize non-negative utility, that gives us the participation constraints

$$\theta_i V(q_i, \overline{q}) - t_i \geq 0 \qquad (P_i)$$

for all $i \in N$. Since the distribution function is common knowledge, each consumer rationally expects the network size $\overline{q}$ to be $\sum_{i \in N} q_i f(\theta_i)$.

The profit of the monopoly is given by

$$\Pi = \sum_{i \in N} (t_i - cq_i) f(\theta_i),$$

and it has to be maximized such that participation constraints (P_i) are satisfied. Naturally, all constraints will be binding in optimum, so the optimal quantity schedule determines the optimal tariff schedule. Therefore, the key decision variable for the monopoly is $q = (q_1, ..., q_n) \in Q =$

R^n, and the final form of the profit function is

$$\Pi^{FB} = \sum_{i \in N} \theta_i V\left(q_i, \overline{q}\right) f(\theta_i) - c\overline{q}, \tag{2.18}$$

which equals social surplus.

Assuming that Π^{FB} has a bounded maximum in Q, it will be characterized by the following set of first-order conditions characterize the first-best allocation q^{FB}:

$$\theta_i V_1(q_i, \overline{q}) + \sum_{j \in N} \theta_j V_2(q_j, \overline{q}) f(\theta_j) = c, \text{ for all } i \in N. \tag{2.19}$$

The first term is the now familiar (first-best) individual effect, while the second term sums the marginal utility increases of all consumers due to the increased consumption of consumer group i. Again, the latter is called the (first-best) network effect, and it is the same in all equations.

By combining any two first-order conditions, we have that

$$\theta_i V_1(q_i, \overline{q}) = \theta_j V_1(q_j, \overline{q})$$

for all $i, j \in N$. Since $V_{11} \leq 0$, $\theta_i < \theta_j$ implies $q_i < q_j$.

2.3.2 The second-best optimal contract

If the monopoly should offer the same menu of contracts for all consumers, then an incentive-compatible menu structure $\{(q_i, t_i)\}_{i=1}^n$ should satisfy participation constraints (P_i) and the following set of incentive constraints:

$$\theta_i V(q_i, \overline{q}) - t_i \geq \theta_i V(q_j, \overline{q}) - t_j \tag{IC_{ij}}$$

for all $i, j \in N$, where $\overline{q} = \sum_{i \in N} q_i f(\theta_i)$ is the rationally expected equilibrium network size. Again, in the incentive constraints we require only that no consumer has any incentive to deviate individually from her equilibrium choice.

By adding incentive constraints (IC_{ij}) and (IC_{ji}), we see that

$$(\theta_i - \theta_j)[V(q_i, \overline{q}) - V(q_j, \overline{q})] \geq 0 \tag{2.20}$$

should hold for all $i, j \in N$. Since $V_1 > 0$, in order to have an implementable mechanism, the quantity scheme $q(\theta)$ should be a non-decreasing function of the type, which is the generalization of the monotonicity constraint.

The following Lemma analyzes the set of constraints to find the relevant ones.

Lemma 5 *In the second-best optimum there are n binding constraints: (P_1), the participation constraint of the lowest-type consumer, and $(IC_{i(i-1)})$ for $i = 2, ..., n$, the downward local incentive constraints, the other constraints are all slack. Therefore, the optimal tariffs are*

$$\begin{aligned} t_1 &= \theta_1 V(q_1, \overline{q}), \text{ and} \\ t_i &= \theta_i V(q_i, \overline{q}) - \sum_{j=1}^{i-1} \Delta\theta_j V(q_j, \overline{q}), \text{ for } i = 2, ..., n, \end{aligned}$$

where $\Delta\theta_j = \theta_{j+1} - \theta_j$.

Proof. *See Appendix A.2.* ■

Again, the optimal menu leaves the lowest type consumer with no surplus, while consumers of higher types should get an information rent of $\sum_{j=1}^{i-1} \Delta\theta_j V(q_j, \overline{q})$ in order to satisfy incentive compatibility.

As we have seen before in the two-type case, choosing $c_i = (q_i, t_i)$ for each consumer of type θ_i is not the only rational expectations equilibrium at the optimal tariff structure. If we assume that the stand-alone value of using the network good is not too high, everybody choosing the contract $c_0 = (0, 0)$ always remains a rational expectations equilibrium. Again, we focus only on the pure strategy equilibria, when each consumer of the same type chooses the same contract. The following Lemma summarizes some useful properties the rational expectations equilibria possess.

Lemma 6 *In any rational expectations equilibria at the optimal tariff structure given in Lemma 5* *i) consumers cannot violate monotonicity: if $\theta_i < \theta_j$ and $k > l$, then it cannot happen that θ_i-types choose c_k and θ_j-types choose c_l,*

ii) if expected network size $\overline{q}'$ is smaller then $\overline{q}$ then every type θ_i should "jump down", i.e. choose a contract c_k such that $k < i$,

iii) if expected network size $\overline{q}'$ is higher then $\overline{q}$ then no type can "jump down", i.e. every type θ_i choose a contract c_k such that $k \geq i$.

Proof. *See Appendix A.3.* ■

This Lemma has two important consequences. The first property ensures that the only screening equilibrium where each type selects a different contract that is not the null contract is the screening outcome we have concentrated on so far. Second, these three properties together imply that the possible REEs that are not larger then screening outcome can be ordered in the quantity space, and by the complementarity it results in the same ordering in the utility space as well.[22] Note that this Lemma does not imply that all outcomes satisfying these properties are necessarily REEs (as we have seen in the two-type case, that requires additional conditions to hold as well), but gives reasons to focus on the screening outcome discussed so far.

By substituting the optimal tariff functions into the profit function, it simplifies to the following form:[23]

$$\begin{aligned}
\Pi^{SB} &= \theta_1 V(q_1,\overline{q}) f(\theta_1) + \sum_{i=2}^{n} \left[\theta_i V(q_i,\overline{q}) - \sum_{j=1}^{i-1} \Delta\theta_j V(q_j,\overline{q}) \right] f(\theta_i) - c\overline{q} = \\
&= \sum_{i \in N} \left(\theta_i - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)} \right) V(q_i,\overline{q}) f(\theta_i) - c\overline{q}. \qquad (2.21)
\end{aligned}$$

The function Π^{SB} should be maximized in q, with respect to the monotonicity constraint and $q \geq 0$. We ignore these constraints for the moment, and check at the end whether they are

[22] For example, in the 3-type case the only possible equilibra satisfying the properties above are: $(c_0,c_0,c_0), (c_0,c_0,c_1), (c_0,c_0,c_2), (c_0,c_1,c_2), (c_1,c_2,c_3), (c_1,c_3,c_3), (c_2,c_2,c_3), (c_2,c_3,c_3)$ and (c_3,c_3,c_3). The first four outcomes are all smaller than the screening outcome (c_1,c_2,c_3), and they can be ordered indeed. The only two allocations that cannot be ordered are (c_1,c_3,c_3) and (c_2,c_2,c_3).

[23] This simplified form contains a non-defined type parameter, θ_{n+1} in $\Delta\theta_n$. However, it does not play any role, since it is multiplied by $1-F(\theta_n)=0$.

satisfied in equilibrium. Then the optimal allocation q^{SB} is characterized by the following first-order conditions:

$$\theta_i V_1(q_i,\overline{q}) + \sum_{j\in N}\theta_j V_2(q_j,\overline{q}) f(\theta_j) - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)} V_1(q_i,\overline{q}) -$$

$$-\sum_{j\in N}\Delta\theta_j \left[1-F(\theta_j)\right] V_2(q_j,\overline{q}) = c, \text{ for all } i\in N. \tag{2.22}$$

We see again that the first term is the first-best individual effect, the second is the first-best network effect and the third subtracted term is the second-best individual effect. The latter effect equals zero for consumers of type θ_n, so produces 'no individual distortion at the top', and is positive for all other types. The final sum to be subtracted is the second-best network effect, and is strictly positive for all consumers.

Since the network effects are the same in all first-order conditions, combining two of them gives

$$\left(\theta_i - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)}\right) V_1(q_i,\overline{q}) = \left(\theta_j - \Delta\theta_j \frac{1-F(\theta_j)}{f(\theta_j)}\right) V_1(q_j,\overline{q})$$

for all $i,j\in N$. Now let us examine the implications of the omitted constraints.

First, $\theta_i - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)}$ should be positive for all consumers supplied with a positive quantity, since $\theta_n - \Delta\theta_n \frac{1-F(\theta_n)}{f(\theta_n)} = \theta_n$ and V_1 are both positive. If $\theta_i - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)} \leq 0$, the ith type (and by the monotonicity constraint all lower types) will be shut down, so $q_i = 0$. Second, since $V_{11} \leq 0$, in order to satisfy the monotonicity constraint, $\theta_i - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)} > \theta_j - \Delta\theta_j \frac{1-F(\theta_i)}{f(\theta_i)}$ should be satisfied for $i > j$, if the monopoly wants to separate type-i and type-j consumers. If $\theta_i - \Delta\theta_i \frac{1-F(\theta_i)}{f(\theta_i)} \leq \theta_j - \Delta\theta_j \frac{1-F(\theta_i)}{f(\theta_i)}$, the two types will be bunched, that is $q_i = q_j$. A possible sufficient condition to avoid bunching is that $\Delta\theta_i$ is the same for all i and $F(\theta)$ satisfies the monotone hazard rate property: $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) \leq 0$.

In the following discussion we assume that in the optimal mechanism at least two different types are served and discriminated.

2.3.3 Comparison of equilibria

Again, instead of comparing the allocations defined by the first-order conditions, we start by looking at the profit function in the respective regimes:

$$\begin{aligned}\Pi^{FB} &= \sum_{i\in N}\theta_i V\left(q_i,\overline{q}\right)f(\theta_i) - c\overline{q}, \text{ and}\\ \Pi^{SB} &= \sum_{i\in N}\left(\theta_i - \Delta\theta_i\frac{1-F(\theta_i)}{f(\theta_i)}\right)V(q_i,\overline{q})f(\theta_i) - c\overline{q}.\end{aligned}$$

Both functions are defined on $Q = R^n_+$, a set which forms a lattice. Since we have assumed that q_i and $\overline{q} = \sum\limits_{i\in N} q_i f(\theta_i)$ are complements, it implies that $V(\cdot)$ is quasisupermodular in q on Q, so we can use monotone comparative static tools to compare the optimal allocations.

Proposition 7 *The second-best allocation is strictly smaller than the first best allocation, that is $q^{SB} < q^{FB}$.*

Proof. *Consider only the types that are not shut down in the second-best regime (set S), since for the others the strict downward distortion holds trivially. Let us take the following parametrized form $\Pi : Q \times T \to R$:*

$$\Pi(q,\alpha) = \sum_{i\in S}\left(\theta_i + \alpha\Delta\theta_i\frac{1-F(\theta_i)}{f(\theta_i)}\right)V(q_i,\overline{q})f(\theta_i) - c\overline{q},$$

where $T = [-1, 0]$. When $\alpha = 0$, we have the first-best profit function, while $\alpha = -1$ yields the second-best profit function. Then the function $\Pi(q,\alpha)$ is quasisupermodular in q on Q for all α, since by the no shut-down condition the multipliers of $V(\cdot)$ are always positive for all $i \in S$. Moreover, the marginal returns of all q_i-s are increasing in α, since $\frac{\partial^2\Pi(q,\alpha)}{\partial q_i \partial\alpha} > 0$ for all q_i. Therefore, the conditions of Lemma 1 are all satisfied, so $\alpha^{FB} > \alpha^{SB}$ implies $q(\alpha^{FB}) > q(\alpha^{SB})$.

■

The comparison of the first- and second best allocation to the perfectly competitive outcome in the two-type case was already given for the general case, so all the results derived there applies here as well.

Last, we show that the equilibrium outcome is a weakly increasing function of network effect intensity, and equilibrium network size strictly increases. For this goal we use again the utility function $U(\theta_i) = \theta_i V(q_i, \beta\overline{q}) - t_i$, where $\beta \geq 0$ stands for network effect intensity.

Proposition 8 *Both for first- and second-best discrimination in the presence of network effects, an increase in the network effect intensity β results in*

1. *a weak increase of the equilibrium allocation $q(\beta)$,*
2. *a strict increase of the quantity supplied for at least one consumer type, and*
3. *a strict increase of the equilibrium network size $\overline{q}(\beta)$.*

Proof. *Let the profit function be $\Pi : Q \times T \to R$, where $T = R_+^2$ and S is the set of consumers supplied with a positive quantity in the respective regime:*

$$\Pi(q, \alpha, \beta) = \sum_{i \in S} \left(\theta_i + \alpha \Delta\theta_i \frac{1 - F(\theta_i)}{f(\theta_i)} \right) V(q_i, \beta\overline{q}) f(\theta_i) - c\overline{q}.$$

The parameter α still stands for the regime to be analyzed: $\alpha = 0$ in first-best discrimination and $\alpha = -1$ in second-best discrimination. We have seen that the function $\Pi(q, \alpha, \beta)$ is quasi-supermodular in q on Q for each α and β and since $U_i(\theta_i, q, \beta) = \theta_i V(q_i, \beta\overline{q})$ satisfies the strict single-crossing property in (q, β) for all $\theta_i > 0$ and all the multipliers are positive constants by the no shut-down condition, $\Pi(q, \alpha, \beta)$ satisfies the strict single-crossing property in (q, β) as well. We can therefore apply the Monotone Selection Theorem to the equilibrium allocation $q(\beta) \in \arg\max \Pi(q, \alpha, \beta)$: $\beta' > \beta''$ implies $q(\beta') \geq q(\beta'')$.

If we combine two first-order conditions for $i, j \in S$ such that $i < j$, we have

$$\gamma_i V_1(q_i, \beta\overline{q}) = \gamma_j V_1(q_j, \beta\overline{q})$$

for all $i, j \in S$, where $\gamma_k = \theta_k + \alpha\Delta\theta_k \frac{1-F(\theta_k)}{f(\theta_k)}$. As $q(\beta'') = q''$ is optimal given β'', we have $\gamma_i V_1(q_i'', \beta''\overline{q}'') = \gamma_j V_1(q_j'', \beta''\overline{q}'')$. Now if we change only β'' to β', we should have $\gamma_i V_1(q_i'', \beta'\overline{q}'') < \gamma_j V_1(q_j'', \beta'\overline{q}'')$, since $q_i \leq q_j, V_{12} > 0$ and $\gamma_i < \gamma_j$ by the monotonicity constraint. But this means that $q(\beta'') \notin \arg\max \Pi(q, \alpha, \beta')$, so $q(\beta')$ should be larger then $q(\beta'')$ in at least one dimension. Finally, this property trivially implies that $\overline{q}(\beta)$ should strictly increase in β. ■

2.4 Discussion of the continuous-type case

In the screening literature it is common to discuss the models in a continuous-type framework, since it considerably simplifies the exposition of the implementation problem.[24] Indeed, the description of first- and second best optimal contracts presented in the previous Section can be easily modified to account for a continuum of types, and the characterizing conditions will have exactly the same form stated in integrals instead of sums. However, as strict monotone comparative statics are designed mainly for decision problems in finite dimensions, our analysis leads to slightly weaker results.

Suppose now that θ is distributed on $[\underline{\theta}, \overline{\theta}]$ according to a distribution function $F(\theta)$, with a positive density $f(\theta)$ at each point. Since we want that every consumer is able to get a positive utility, we set $\underline{\theta} > 0$. However, we might even allow for $\underline{\theta} = 0$, although in this case the lowest-type consumers always want to choose the null contract, since this group is of zero measure, so their choices do not affect the rationally expected equilibrium network size. Additionally, we assume that $F(\cdot)$ satisfies the monotone hazard rate property, namely that $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) \leq 0$. As we have seen before, we may assume that all types are supplied with positive quantities in both the first- and second-best regime.

As in the discrete-type case, the relevant decision variables are the quantity choices of the monopoly, since the optimal tariff schedule $t(\theta)$ will be determined by the optimal quantity schedule $q(\theta)$. Now let Q be the partially ordered set of bounded functions $q(\theta)$ defined on $[\underline{\theta}, \overline{\theta}]$. Let us define the join and meet of two elements $q'(\theta)$ and $q''(\theta)$ as the upper and lower envelope of the two functions: $q'(\theta) \vee q''(\theta) = \max\{q'(\theta), q''(\theta)\}$ and $q'(\theta) \wedge q''(\theta) = \min\{q'(\theta), q''(\theta)\}$. Then Q is a lattice since it always contains the join and meet of any two elements.

2.4.1 The first-best case

In perfect discrimination, the monopoly offers a contract $(q(\theta), t(\theta))$ for each type θ that grasps its whole:

$$t(\theta) = \theta V(q(\theta), \overline{q}), \tag{2.23}$$

[24] See for example Section 7.3 of Fudenberg and Tirole (1991) or Appendix 3.1 of Laffont and Martimort (2002).

where $\overline{q} = \int_{\underline{\theta}}^{\overline{\theta}} q(u)f(u)du$. Monopoly profit is given by

$$\Pi = \int_{\underline{\theta}}^{\overline{\theta}} [t(\theta) - cq(\theta)]f(\theta)d\theta,$$

and after substitution of equation (2.23), the monopoly's profit-maximization problem is the following:

$$\max_{q(\theta)} \Pi = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta V(q(\theta), \int_{\underline{\theta}}^{\overline{\theta}} q(u)f(u)du) - cq(\theta)\right] f(\theta)d\theta. \tag{2.24}$$

Pointwise maximization in $q(\theta_i)$ gives the following first-order condition for each $\theta_i \in [\underline{\theta}, \overline{\theta}]$:

$$\theta_i V_1(q(\theta_i), \overline{q}) + \int_{\underline{\theta}}^{\overline{\theta}} \theta V_2(q(\theta), \overline{q})f(\theta)d\theta = c. \tag{2.25}$$

The continuum of these first-order conditions define the first-best quantity profile $q^{FB}(\theta)$. Again, we see the intuition presented in the discrete-type case: the first term is the individual effect and the second term is the sum of network effects, which is the same in all first-order conditions. These two effects together should equal the marginal cost of the network good.

By combining two first-order conditions for θ_i and θ_j, we have that

$$\theta_i V_1[q(\theta_i), \overline{q}] = \theta_j V_1[q(\theta_j), \overline{q}]. \tag{2.26}$$

Since $V_{11} \leq 0$, if $\theta_i > \theta$ then $q(\theta_i) > q(\theta_j)$.

2.4.2 The second-best case: implementability

We now turn to the second-best case, and first we derive necessary conditions for implementability. Based on the Revelation Principle we may restrict ourselves to truthful direct revelation mechanisms $\{(q(\widetilde{\theta}), t(\widetilde{\theta}))\}$, where $q(\widetilde{\theta})$ and $t(\widetilde{\theta})$ are assumed to be piecewise differentiable functions. If every consumer truthfully reveals her type (as it will be the case in equilibrium), total quantity purchased will be $\overline{q} = \int_{\underline{\theta}}^{\overline{\theta}} q(u)f(u)du$.

Each consumer of type θ maximizes her utility in answer $\widetilde{\theta}$, while she is conjecturing that every other consumer truthfully reveals her type:

$$\max_{\{\widetilde{\theta}\}} U(\theta, \widetilde{\theta}) = \theta V(q(\widetilde{\theta}), \overline{q}) - t(\widetilde{\theta}).$$

Truthtelling is optimal for all θ if the following first-order condition holds:

$$\theta V_1(q(\theta), \overline{q})\dot{q}(\theta) - \dot{t}(\theta) = 0. \tag{2.27}$$

If we differentiate this equation by θ and combine it with the local second-order condition, we have

$$V_1(q(\theta), \overline{q})\dot{q}(\theta) \geq 0, \tag{2.28}$$

and since $V_1 > 0$, it leads to the necessary condition of

$$\dot{q}(\theta) \geq 0. \tag{2.29}$$

So in order to have an implementable mechanism, the quantity scheme (and according to equation (2.27) the tariff scheme as well) has to be a non-decreasing function of the type.

2.4.3 The second-best optimal contract

To solve the optimal program for the monopolist it is easier to consider the rent of the θ-type consumer:

$$U(\theta) = \theta V(q(\theta), \overline{q}) - t(\theta). \tag{2.30}$$

If we differentiate this equation by θ and use equation (2.27), we arrive at the differential equation

$$\dot{U}(\theta) = \theta V(q(\theta), \overline{q}),$$

which has a solution of

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V(q(v), \overline{q})dv.$$

By substituting this result into equation (2.30) and using that $U(\underline{\theta}) = 0$, we can express the optimal tariff function for each type:

$$t(\theta) = \theta V(q(\theta), \overline{q}) - \int_{\underline{\theta}}^{\theta} V(q(v), \overline{q}) dv.$$

Again, we concentrate on the equilibrium when all consumers are differentiated. Now, we can write down the monopoly's profit-maximization problem as

$$\max_{q(\theta)} \Pi = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta V(q(\theta), \overline{q}) - \int_{\underline{\theta}}^{\theta} V(q(v), \overline{q}) dv - cq(\theta) \right] f(\theta) d\theta,$$

$$\text{s.t.} \dot{q}(\theta) \geq 0.$$

First, we ignore the monotonicity constraint and check at the end whether it is satisfied in equilibrium. Second, if we integrate the second part of the integrand by parts, we have

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta} V(q(v), \overline{q}) dv \right] f(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} V(q(\theta), \overline{q}) [1 - F(\theta)] d\theta.$$

By using this result the reduced problem is

$$\max_{q(\theta)} \Pi = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) V(q(\theta), \int_{\underline{\theta}}^{\overline{\theta}} q(u) f(u) du) - cq(\theta) \right] f(\theta) d\theta. \tag{2.31}$$

Pointwise maximization in $q(\theta_i)$ gives the following first-order condition for each θ_i:

$$\theta_i V_1[q(\theta_i), \overline{q}] + \int_{\underline{\theta}}^{\overline{\theta}} \theta V_2(q(\theta), \overline{q}) f(\theta) d\theta -$$

$$- \frac{1 - F(\theta_i)}{f(\theta_i)} V_1(q(\theta_i), \overline{q}) - \int_{\underline{\theta}}^{\overline{\theta}} [1 - F(\theta)] V_2(q(\theta), \overline{q}) d\theta = c, \tag{2.32}$$

which can be interpreted in the same way as in the discrete-type case (see equation (2.22)).

Combining two first-order conditions for θ_i and θ_j gives

$$\left(\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \right) V_1(q(\theta_i), \overline{q}) = \left(\theta_j - \frac{1 - F(\theta_j)}{f(\theta_j)} \right) V_1(q(\theta_j), \overline{q}). \tag{2.33}$$

If $\theta_i > \theta_j$, by monotone hazard rate property $\left(\theta_i - \frac{1-F(\theta_i)}{f(\theta')}\right) < \left(\theta_j - \frac{1-F(\theta_j)}{f(\theta_j)}\right)$ holds, and since $V_{11} \leq 0$, it follows that $q(\theta_i) > q(\theta_j)$ and so the omitted necessary condition on monotonicity is fulfilled in equilibrium.

2.4.4 Comparison of equilibria

Now we briefly discuss out two main Propositions using the Monotone Selection Theorem.

Proposition 9 *The second-best allocation is smaller than the first best allocation, that is $q^{SB} \leq q^{FB}$.*

Proof. *Let us take the following parametrized form $\Pi : Q \times T \to R$:*

$$\Pi(q, \alpha) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta + \alpha \frac{1 - F(\theta)}{f(\theta)} \right) V(q(\theta), \overline{q}) - cq(\theta) \right] f(\theta) d\theta,$$

where $T = [-1, 0]$. When $\alpha = 0$, we have the first-best profit function, while $\alpha = -1$ yields the second-best profit function. Then the function $\Pi(q, \alpha)$ is quasisupermodular in q on Q for all α, since by the no shut-down condition the multipliers of $V(\cdot)$ are always positive for all $i \in S$ and quasisupermodularity is preserved under integration.[25] *Therefore, the conditions of the Monotone Selection Theorem are all satisfied, so $\alpha^{FB} > \alpha^{SB}$ implies $q(\alpha^{FB}) \geq q(\alpha^{SB})$.* ∎

Note that we can show the downward distortion result, but not in the strict sense. This is because the proof of Lemma 1 relies on the finite dimension of the choice space, so we cannot apply it in the present context. However, the example in the next Subsection shows that the strict downward distortion result may occur in the continuous type case as well.

Proposition 10 *Both for first- and second-best discrimination in the presence of network effects, an increase in the network effect intensity β results in*

1. *a weak increase of the equilibrium allocation $q(\beta)$, and*

2. *a strict increase of the quantity supplied for at least one consumer type.*

Proof. *The proof follows the same lines that in Proposition 8, therefore it is omitted.* ∎

[25] *See Vives (1990) for the proof.*

Again, the continuum of types leads to a slightly weaker result. This is because the strict increase can be shown to hold in general only for one type as β increases, and that does not imply an increase in $\overline{q}$ as the distribution is atomless. However, we again show an example where an increase in the network effect intensity results in a strict increase in the quantity supplied for all types, and then naturally in network size as well.

Illustrative example

Once again, we work with the utility function $\theta_i V(q_i, \overline{q}) = (q_i)^a (\overline{q})^b$, where $\overline{q} = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta) d\theta$. As before, $a, b \in (0,1)$ and $a + b < 1$. In order to simplify notations, let us normalize $\overline{\theta}$ to 1.

The general profit function can be written as

$$\Pi = \left(\int_{\underline{\theta}}^{\overline{\theta}} \gamma(\theta) \left[q(\theta) \right]^a f(\theta) d\theta \right) \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta) d\theta \right)^b - c\overline{q},$$

where $\gamma(\theta) = \theta$ in the first-best regime and $\gamma(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$ in the second-best regime.

In both regimes, combining first-order condition for any θ and $\overline{\theta} = 1$ (conditions (2.26) and (2.33))

$$\begin{aligned} \gamma(\theta) \left[q(\theta) \right]^{a-1} &= \left[q(1) \right]^{a-1} \\ q(\theta) &= \left[\gamma(\theta) \right]^{\frac{1}{1-a}} q(1) \end{aligned} \tag{2.34}$$

The discounted marginal revenue of the quantity supplied to $\overline{\theta} = 1$ is

$$MR(1) = V_1(q(1), \overline{q}) + \int_{\underline{\theta}}^{\overline{\theta}} \gamma(\theta) V_2(q(\theta), \overline{q}) f(\theta) d\theta,$$

which should equal marginal cost c by first-order conditions (2.25) and (2.32). By using equation

(2.34), the respective terms are

$$
\begin{aligned}
V_1(q(1),\overline{q}) &= a\,[q(1)]^{a-1}\left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)f(\theta)d\theta\right)^{b} = \\
&= a\,[q(1)]^{a-1}\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,q(1)f(\theta)d\theta\right)^{b} = \\
&= a\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)d\theta\right)^{b}[q(1)]^{a+b-1},
\end{aligned}
$$

$$
\gamma(\theta)V_2(q(\theta),\overline{q})f(\theta) = b\gamma(\theta)[q(\theta)]^{a}\left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)f(\theta)d\theta\right)^{b-1} f(\theta) =
$$

$$
\begin{aligned}
&= b\gamma(\theta)[[\gamma(\theta)]^{\frac{1}{1-a}}\,q(1)]^{a}\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,q(1)f(\theta)d\theta\right)^{b-1} f(\theta) = \\
&= b\left([\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)\right)\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)d\theta\right)^{b-1}[q(1)]^{a+b-1}, \text{ and}
\end{aligned}
$$

$$
\int_{\underline{\theta}}^{\overline{\theta}} \gamma(\theta)V_2(q(\theta),\overline{q})f(\theta)d\theta =
$$

$$
\begin{aligned}
&= \int_{\underline{\theta}}^{\overline{\theta}} b\left([\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)\right)\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(u)]^{\frac{1}{1-a}}\,f(u)du\right)^{b-1}[q(1)]^{a+b-1}\,d\theta \\
&= b\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)d\theta\right)^{b}[q(1)]^{a+b-1}.
\end{aligned}
$$

Combining the terms gives

$$
MR(1) = (a+b)\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)d\theta\right)^{b}[q(1)]^{a+b-1}.
$$

This function is decreasing in $q(1)$ from infinity towards zero, so it defines a unique $q(1)$ where it equals c. If we switch from the second-best regime to the first-best regime, then all $\gamma(\theta)$-s increase, which increases $\left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}}\,f(\theta)d\theta\right)^{b}$, so the optimal $q(1)$ strictly increases. Since by equation (2.34) all other $q(\theta)$-s are strictly increasing function of $q(1)$, the first-best allocation should be strictly larger than the second-best.

In order to examine the impact of network effect intensity, we redefine the valuation function as $V(q_i, \overline{q}) = q_i^a (\beta\overline{q})^b$, where $\beta \geq 0$ measures the intensity of network effects. Now the profit function writes as

$$\Pi(q, \theta, \beta) = \beta^b \left(\int_{\underline{\theta}}^{\overline{\theta}} \gamma(\theta) [q(\theta)]^a f(\theta) d\theta \right) \left(\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) f(\theta) d\theta \right)^b - c\overline{q},$$

so the discounted marginal revenue of the quantity supplied for the highest type is

$$MR(1) = \beta^b (a+b) \left(\int_{\underline{\theta}}^{\overline{\theta}} [\gamma(\theta)]^{\frac{1}{1-a}} f(\theta) d\theta \right)^b [q(1)]^{a+b-1}.$$

Since $MR(1)$ is strictly in increasing in β, $q(1)$ and therefore all other $q(\theta)$-s are strictly increasing functions of the network effect intensity parameter β.

2.5 Conclusion

In this Chapter we have derived a general model to analyze the second-degree price discrimination problem of a monopoly selling a network good exhibiting strategic complementarities. By using the tools of monotone comparative statics, we were able to give a full characterization of screening contracts. We have seen that strategic complementarities and asymmetric information together lead to a strict downward distortion for all consumers, and the equilibrium outcome is an increasing function of the intensity of the network effects. Additionally, we have shown that a discriminating monopoly may supply larger quantities for all consumers than a perfectly competitive industry.

A crucial feature of our model was that the optimal contracts are designed such that it is individually not profitable for deviating from the truthtelling equilibrium. However, the natural question arises whether it could be advantageous for some consumers to form a coalition to coordinate their decisions and then reallocate the goods among themselves. Jeon and Menicucci (2003) show that there is no loss of generality in restricting our attention to contracts that satisfy only individually incentive compatibility constraints, if the coalitions are formed under asymmetric information. This is because buyers fail to realize the gains from joint deviations due to the transaction costs of asymmetric information among themselves, and the monopoly can use this fact to construct a menu of contracts, by which it can do at least as well as when there is no coalition. Although network effects are not present in their model, the intuition seems to hold in our setting as well.

The network goods provided to different types of consumers were assumed to be compatible with each other so far. Let us briefly discuss the case of two types where the monopoly chooses the network good provided to low-type consumers to be incompatible with the high-types' goods, while the high-types's good has full compatibility. Now low-types benefit less from the network, so the monopoly cannot charge such a high tariff for them. However, a high-type consumer will now have less incentive to choose the menu devoted to low-type consumers, since then she excludes himself from using a part of the network, so information rent of high-type consumers should decrease as well, which is profitable for the monopoly. Therefore, it is a natural conjecture that if the monopoly chooses to make its good partially incompatible, then it will choose to do so with the good devoted to low-type consumers, since high-type consumers have a higher marginal utility for the network.

If the good devoted to low-type consumers is incompatible with the good devoted to high-type consumers, then in equilibrium low-type consumers' utilities depend only on low-type consumers' choices. This is exactly the same case as if low-type consumers had the pessimistic expectation that high-type consumers will stay out of the market, so the monopoly has to design the contract devoted to low-type consumers such that they would accept it 'without the high-types' as well. But if high-type consumers observe the contract devoted to low-type consumers, no matter how pessimistic prior expectations they had about low-types' behavior, they will realize that low-types will accept that contract in any case. Then they will make their choices by expecting low-type ones 'in the network', thus the monopoly can design the menu devoted to high-types accordingly. This "divide-and-conquer" strategy, presented also in Jullien (2002) and Segal (2003), may help to overcome the problem of multiple equilibria induced by different consumers' expectations and to end up with unique implementation. The latter two issues will be discussed in the next Chapter.

Chapter 3

Compatibility strategies and coordination problems in contracting with network effects

3.1 Introduction

This Chapter examines compatibility and coordination issues in a general model of second-degree discrimination in the presence of positive network effects. As we have seen in the previous Chapter, in the presence of network effects, consumers' expectations play a decisive role in which equilibrium the economy will end up with,[1] so it may be in the firms' best interest to influence the formation of these expectations, in order to avoid unfavorable outcomes.

Compatibility questions are strongly interlinked with network effects and were always the main focus of the literature on networks.[2] However, the use of compatibility strategies were mainly examined in oligopolistic markets and were strongly interlinked with the analysis of standards.[3] In the case of a monopolist supplier, the attention was turned to the use of compatibility strategies in intertemporal pricing. The main question of this dynamic approach was

[1] Farrell and Saloner (1985) and Katz and Shapiro (1985) were the first to raise this problem.

[2] See for example the articles in the Symposium on Compatibility, Journal of Industrial Economics, Spring 1992, and references therein.

[3] Gandal (2002) offers an overview and policy considerations.

whether different versions of the same product sold at different dates should be made compatible with each other or not, and whether the monopoly's optimal incentive for compatibility coincide with the social compatibility level.[4] In this Chapter we take a different perspective, and examine whether there exists incentives to use incompatibility strategies in a static screening setting.

Let us recite the example of the previous Chapter, which will be extended to demonstrate some results in this Chapter. There are only two types of consumers for a network good, let us call them sophisticated and normal, and sophisticated consumers benefit more both from individual consumption and network size. We have seen that in optimal contracts with asymmetric information the mechanism designer distorts the first-best optimal quantity devoted to normal consumers downwards in order to make switching less attractive to the sophisticated consumers. However, this results in a decrease in network size, and since complementarities are present due to positive network effects, the sophisticated consumers' utility from individual consumption is negatively affected. Thus, it is no more feasible to offer them the first-best optimal quantity, and their consumption should be distorted downwards as well.

A natural question is whether the monopoly can somehow decrease the costs to deter sophisticated consumers from switching. If the network good provided to normal consumers were made incompatible with the sophisticated ones' goods, then sophisticated consumers would have less incentive to switch, therefore the information rent could be decreased. There is a trade-off, however, since normal consumers would derive less network benefits as well, so the monopoly cannot charge such a high tariff for them. Incompatibility strategies therefore reshuffle the rents reaped from the consumers, but two questions remain: first, which good should be made incompatible, and second, whether it is optimal to use incompatibility strategies.

We show that if the monopoly chooses to make one of the goods partially incompatible, then it will choose to do so with the good devoted to normal consumers, since sophisticated ones have a higher marginal utility for the network. However, in our benchmark model, incompatibility strategies cannot be used profitably to improve the screening possibilities of the mechanism designer. The underlying reason is that consumers have a one-dimensional type, so it is suffi-

[4] See Choi (1992), Ellison and Fudenberg (2000) and Nahm (2005). Another important focus of these articles was to examine whether the Coase-conjecture holds with network effects.

cient to screen them only in dimension, which is the quantity variable. On the other hand, if consumers valuate compatibility differently, there is a screening motive for incompatibility, as we will see in the model developed in the next Chapter.

Nevertheless, incompatibility strategies may have a potential use in forming expectations that are a crucial factor in network models. As in most of the previous works on network effects, in the previous Chapter we also should have used an equilibrium selection criterion to overcome the multiple equilibrium problem due to different possible rational expectations. For example, another trivial equilibrium is when each agent expects the others not to purchase anything, so it is optimal for her not to choose either contract as well, and finally everybody is (weakly) worse off. In games with strategic complementarities, usually called coordination games, authors mostly use the Pareto-criterion, since positive network effects usually ensure the existence of Pareto-dominant equilibrium.[5] However, this equilibrium may be very risky, since consumers may fear ending up with a smaller or even negative utility if others fail to follow them on coordinating on this equilibrium, so players may be reluctant to play these strategies.[6] Therefore, the mechanism designer might prefer to bear some additional costs in order to exclude some of the equilibria due to pessimistic expectations.

Now suppose that in the previous example the two groups of consumers have pessimistic expectations that the other group will not join the network, but optimistic expectations about the others in their own group. Since they expect less benefits now, they would not choose the contract designed for them originally. The possibility of these pessimistic expectations clearly hurts the monopoly, and the question is how can it 'have all of them on board' and minimize the resulting loss in profit.

Let us introduce an artificial sequentiality in contracting with agents: the monopoly is able to contract first with all members of one group, and then members of the second group choose from the same menu of contracts. Now if the monopoly designs the contract devoted to the first group such that they can expect a non-negative utility if all first group members join but none of the second group, they will accept this contract. But since the second group also observes the same menu of contracts, no matter how pessimistic prior expectations they had,

[5] But this should not always be the case - see for example Ellison and Fudenberg (2000), where heterogenous consumers may prefer different equilibria.

[6] On experimental evidence demonstraing this result, see Cooper et al. (1990).

they realize that first group members should have accepted their contract in any case. So the second group members will make their choices by expecting everybody on board, since they have optimistic expectations about their own group. Therefore the monopoly is able to design the menu devoted to the second group to reap all network benefits (minus the possible costs of screening), while from the first group only the network benefits created by their own group. If the relative density of normal consumers is not too high, it is indeed optimal now to give away some profit for normal consumers in order to convince them to join the network, and target the full network benefits of sophisticated consumers, since the latter have a higher marginal utility for the network.

These so called divide-and-conquer strategies were first used by Innes and Sexton (1993), and later used in network effects contexts by Jullien (2002) and Segal (2003), although all for the case of perfect information. We should note here that with this kind of sequential contracting we are not working any more with rational expectations, since the pessimistic expectations of consumers contracted earlier about the behavior of consumers contracted later will not be fulfilled.

The use of divide-and-conquer strategies is technically possible by making the network good devoted to normal consumers partially incompatible with the network good devoted to sophisticated ones, since then the normal consumers' utilities depend only on the normal consumers' choices. Thus we see that compatibility decisions and coordination issues may be highly related, and divide-and-conquer strategies may help to overcome the problem of multiple equilibria. Software markets indeed give examples where normal consumers are supplied with a partially incompatible good, in order to ensure a higher network for sophisticated consumers.

However, we are not at unique implementation yet, since consumers of the same type may still have coordination problems among themselves, so the trivial "no purchases" equilibrium still remains. In some environments it might be desirable to examine the costs of unique implementation, as this provides the worst case scenario for the monopoly. By keeping the sequentially in contracting with the consumers, it should be ensured at every step that all decision-makers choose the menu devoted to them, no matter which type of expectations they might have about the final network size. This "better safe than sorry" approach can be applied for example if the mechanism designer is infinitely risk-averse.

In order to achieve unique implementation, we are using a continuous dividing of consumers, as it was done originally by Innes and Sexton (1993). The main advantage of this approach is that in this way the measure of consumers "entering" at each stage is non-measurable, so the choices of consumers entering at the same time do not have a significant effect on network size. However, we face additional problems when we combine divide-and-conquer strategies with screening techniques. The main difficulty is that if we want different consumer types to choose different quantities, then the different types' relevant incentive-compatibility constraints are for different expectations. Those who are to choose the higher amount to the lower one should be given proper incentives to do it for their most pessimistic possible expectations, since because of the complementarity between individual consumption and network size this is the hardest case to satisfy. However, for those who are to choose the lower amount to the higher one, the biggest temptation to deviate is when they have the most optimistic expectations about network size, so the relevant incentive-compatibility constraints should be formulated for this scenario.

We show that the screening motive is much stronger in designing the contracts for consumers at later stages, for two reasons. First, in the beginning the gap between the most optimistic and pessimistic expectations may be so big that it is too costly to separate the different types, and parallelly, for a consumer at later stages a higher network size is already assured, and the monopoly can exploit the differences in network valuation more effectively. So the monopoly might prefer to pool the different types in the early contracting stages, and concentrate only on building a large installed base. Although we are not working explicitly with a dynamic setup, this property seems to fit some evidence on network building, as for example in mobile telecommunications.

The remaining parts of this Chapter are as follows. First, Section 2 examines the profitability of incompatibility strategies, while Section 3 elaborates a detailed model combining divide-and-conquer and screening techniques that lead to unique implementation. In both Sections we build on the benchmark model of two-types presented in the first part of the previous Chapter. Finally, Section 4 concludes. The technical proofs are relegated to Appendix B at the end of the thesis.

3.2 Screening and compatibility

In the previous Chapter the network goods sold to different consumer groups were assumed to be perfectly compatible with each other. Now suppose the goods sold in the same menu are still perfectly compatible (it is not profitable to make it incompatible, since the monopoly does not want to discriminate consumers of the same type), but the monopoly may decide on the compatibility with the goods sold in other menus. Let us denote by s_1 the proportional degree of compatibility of good 1 (the good devoted to type 1 consumers) to good 2, and by s_2 the degree of compatibility of good 2 to good 1. Then the network sizes for the two types will differ:

$$\begin{aligned} \overline{q}_1 &= pq_1 + s_1(1-p)q_2, \\ \overline{q}_2 &= s_2 p q_1 + (1-p)q_2. \end{aligned}$$

3.2.1 The first-best case

As a benchmark case, suppose that the monopoly knows every consumer's type. Then the contracts (q_i, s_i, t_i) will be designed such that in equilibrium each type realizes non-negative utility:

$$\theta_1 V(q_1, \overline{q}_1) - t_1 \geq 0, \tag{3.1}$$

$$\theta_2 V(q_2, \overline{q}_2) - t_2 \geq 0. \tag{3.2}$$

In equilibrium these constraints should be binding, so substitution gives the following profit-maximization problem:

$$\begin{aligned} \max_{\{q_1,q_2,s_1,s_2\}} \Pi &= p\theta_1 V(q_1, \overline{q}_1) + (1-p)\theta_2 V(q_2, \overline{q}_2) - c\overline{q}, \\ \text{s.t. } s_1, s_2 &\in [0,1]. \end{aligned}$$

The first-order conditions in quantities are

$$\begin{aligned} \theta_1 V_1(q_1, \overline{q}_1) + p\theta_1 V_2(q_1, \overline{q}_1) + s_2(1-p)\theta_2 V_2(q_2, \overline{q}_2) &= c, \text{ and} \\ \theta_2 V_1(q_2, \overline{q}_2) + s_1 p\theta_1 V_2(q_1, \overline{q}_1) + (1-p)\theta_2 V_2(q_2, \overline{q}_2) &= c. \end{aligned}$$

Note that the network effects exerted by group i to group j are multiplied by the compatibility level s_j. The marginal profits of s_1 and s_2 are

$$\begin{aligned}\frac{\partial \Pi}{\partial s_1} &= p(1-p)\theta_1 q_2 V_2(q_1, \overline{q}_1),\\ \frac{\partial \Pi}{\partial s_2} &= p(1-p)\theta_2 q_1 V_2(q_2, \overline{q}_2).\end{aligned}$$

Since both of these marginal profits are always positive, $s_1^{FB} = s_2^{FB} = 1$, so $\overline{q}_1 = \overline{q}_2 = \overline{q}$. This result is not surprising, in the first-best (social) optimum we should have full compatibility.

3.2.2 The second-best case

In the second-best case the optimal menus, (q_1, s_1, t_1) and (q_2, s_2, t_2), should satisfy participation constraints (3.1) and (3.2) and the following incentive-compatibility constraints:

$$\theta_1 V(q_1, \overline{q}_1) - t_1 \geq \theta_1 V(q_2, \overline{q}_2) - t_2, \tag{3.3}$$

$$\theta_2 V(q_2, \overline{q}_2) - t_2 \geq \theta_2 V(q_1, \overline{q}_1) - t_1. \tag{3.4}$$

Adding the two incentive-compatibility constraints gives the following constraint:

$$V(q_2, \overline{q}_2) - V(q_1, \overline{q}_1) \geq 0. \tag{3.5}$$

Since now the network sizes for different consumer groups may differ, this condition is not a usual monotonicity constraint. For some values of s_1 and s_2, it may not be satisfied even if $q_2 \geq q_1$, and it also may be satisfied if $q_2 < q_1$. A possible sufficient condition is that $q_2 \geq q_1$ and $\overline{q}_2 \geq \overline{q}_1$. After some modifications the latter condition writes as

$$(1 - s_1)(1 - p)q_2 \geq (1 - s_2)pq_1,$$

so if $s_2 = 1$ (which is our conjecture for the optimum), the network size for high-type consumers is automatically higher.

The following Lemma characterizes the behavior of some constraints in optimum.

Lemma 11 *In optimum the participation constraint for high-type consumers (3.2) is slack, while low-types' participation constraint (3.1) and the high-types' incentive-compatibility constraint (3.4) are binding.*

Proof. *See Appendix B.1.* ■

Based on this Lemma, by rearranging constraints (3.1) and (3.4) we can write down the tariffs charged by the monopoly as functions of the quantities:

$$\begin{aligned} t_1 &= \theta_1 V(q_1, \overline{q}_1), \\ t_2 &= \theta_2 V(q_2, \overline{q}_2) - \Delta\theta V(q_1, \overline{q}_1). \end{aligned}$$

High-type consumers get an information rent of $\Delta\theta V(q_1, \overline{q}_1)$, which can be decreased not only by decreasing q_1, but also by decreasing s_1, since it decreases the network size for low-type consumers. So incompatibility for the low-type group may increase the monopoly's profit made on the high-type group.

By using the previous tariff functions, constraint (3.3) writes as condition (3.5). By attaching Kuhn-Tucker multiplier μ to this constraint, our profit-maximization problem writes as

$$\begin{aligned} \max_{\{q_1,q_2,s_1,s_2\}} \Pi &= p\theta_1' V(q_1, \overline{q}_1) + (1-p)\theta_2 V(q_2, \overline{q}_2) - c\overline{q} - \\ & -\mu\Delta\theta \left[V(q_1, \overline{q}_1) - V(q_2, \overline{q}_2)\right] \\ \text{s.t. } s_1, s_2 &\in [0,1], \end{aligned}$$

where $\theta_1' = \theta_1 - \frac{1-p}{p}\Delta\theta$.

The marginal profits of compatibility levels are

$$\begin{aligned} \frac{\partial \Pi}{\partial s_1} &= \left[p\theta_1' - \mu\right](1-p) q_2 V_2(q_1, \overline{q}_1), \\ \frac{\partial \Pi}{\partial s_2} &= \left[(1-p)\theta_2 + \mu\right] p q_1 V_2(q_2, \overline{q}_2). \end{aligned}$$

Since μ is non-negative, the marginal profit of the high-type group's compatibility level is always positive, so $s_2^{SB} = 1$, and therefore $\overline{q}_2 = \overline{q}$. This also follows from the simple observation that increasing s_2 is beneficial for profit without hurting the low-types' incentive constraint.

The marginal profit of the low-type group's compatibility level is of constant sign as well, so $s_1^{SB} \in \{0,1\}$, and $s_1^{SB} = 1$ only if $p\theta_1' - \mu > 0$.

The first-order conditions in quantities are in q_1 and q_2 are respectively,

$$\underbrace{\theta_1 V_1(q_1,\overline{q}_1)}_{\text{FB ind. effect}} + \underbrace{p\theta_1 V_2(q_1,\overline{q}_1) + s_2(1-p)\theta_2 V_2(q_2,\overline{q}_2)}_{\text{FB network effect}} - \underbrace{\frac{1-p}{p}\Delta\theta V_1(q_1,\overline{q}_1)}_{\text{SB individual effect}} -$$

$$-\underbrace{(1-p)\Delta\theta V_2(q_1,\overline{q}_1)}_{\text{SB network effect}} + \mu\left[s_2 V_2(q_2,\overline{q}_2) - \frac{V_1(q_1,\overline{q}_1)}{p} - V_2(q_1,\overline{q}_1)\right] = c, \text{ and}$$

$$\theta_2 V_1(q_2,\overline{q}_2) + s_1 p\theta_1 V_2(q_1,\overline{q}_1) + (1-p)\theta_2 V_2(q_2,\overline{q}_2) -$$

$$-s_1(1-p)\Delta\theta V_2(q_1,\overline{q}_1) + \mu\left[\frac{V_1(q_2,\overline{q}_2)}{1-p} + V_2(q_2,\overline{q}_2) - s_1 V_2(q_1,\overline{q}_1)\right] = c.$$

Again, the network effects exerted by group i to group j are multiplied by the compatibility level s_j. If we combine these two equations for the case of $s_2 = 1$, we end up with the condition

$$\left[\theta_1' - \frac{\mu}{p}\right]\left[V_1(q_1,\overline{q}_1) + (1-s_1)pV_2(q_1,\overline{q}_1)\right] = \left[\theta_2 + \frac{\mu}{1-p}\right] V_1(q_2,\overline{q}_2).$$

As μ should be non-negative, all bracketed terms but the first are strictly positive, so $\theta_1' - \frac{\mu}{p} > 0$ should hold as well in order to get positive quantities. However, this means that we still need the no shutdown condition $\theta_1' > 0$, which implies that the optimal compatibility level is $s_1 = 1$.

This result shows that in our model incompatibility strategies cannot be used profitably to improve the screening possibilities of the mechanism designer. The underlying reason is that consumers have a one-dimensional type, so it is sufficient to screen them only in dimension, which is the quantity variable. However, if consumers valuate compatibility differently, there is a screening motive for incompatibility, as we will see in the model developed in the next Chapter.

Despite this negative result, the mechanism designer may still find it beneficial to use incompatibility strategies in order to influence expectations, as we have already demonstrated with an example in the Introduction.[7] In the next Section we present a more general model of

[7] However, it is not necessarily true that low-types should be contracted first. The optimal contracting order depends on the relative densities of the two groups.

these divide-and-conquer strategies.

3.3 Divide-and-conquer inside groups

In this Section we turn to the design of a mechanism which gives a stable and unique equilibrium, by using divide-and-conquer techniques. However, we have asymmetric information to deal with in our model as well. In what follows, we examine the case when the monopoly first divides the consumers and then screens them. We have chosen this timing for two reasons. First, if an effective dividing followed in the second stage, then in the screening stage consumers should sign a contingent contract depending on their later position, which does not seem to fit reality. Second, in this alternative way the monopoly's power does not seem to be reduced, since it can design the contracts such that the consumers' expected benefits from dividing will be zero.

Assume that the monopoly (or another randomizing device) first distributes a random number x among consumers, where x follows a uniform distribution on $[0,1]$, and in an increasing order it publicly offers the menu $\{(q_{1x}, t_{1x}), (q_{2x}, t_{2x})\}$ for each consumer of number x.[8] The whole mechanism will be denoted by $\{(q_i(x), t_i(x))\}$, where $i \in \{1,2\}$. In order to facilitate reading, we will use q_{ix} for realizations of $q_i(x)$, and leave the notation $q_i(x)$ to refer to the whole function itself. The same notation will apply to other variables as well. Although we do not use a dynamic approach here, this approach resembles the scenario that consumers arrive sequentially in a random order, and in the arriving stage they are offered a menu of screening contracts from which they choose one.

The publicly observable network size at any x is

$$\overline{q}_x = \int_0^x pq_{1s} + (1-p)q_{2s}ds.$$

If consumer of type θ_i and number x expects network size $\overline{q}_x^e$ and chooses menu (q_{jx}, t_{jx}), then her expected utility is

$$U(\theta_i, \overline{q}_x^e, q_{jx}, t_{jx}) = \theta_i V(q_{jx}, \overline{q}_x^e) - t_{jx}.$$

[8] This 'continuous dividing ' was done first by Innes and Sexton (1993), although only for a homogenous consumer group and without network effects.

A natural interpretation of x can be a time variable, and then one might ask why we do not include some appropriate discounting in the utility functions. However, as a consumer cannot choose the time of arrival of the market, the same discounting applies for all possible options, and therefore we might discard it without any loss of generality.

Consumers may form different expectations $\overline{q}_x^e$, as a function of two variables, x and $\overline{q}_x$.[9] Let us make the following weak assumptions about the different expectations consumers may form, where $\overline{q}_x^e$ is a function of x and $\overline{q}_x$:

1. For each x and $\overline{q}_x$ there exists a most pessimistic expectation $\overline{q}_x^p$, such that $\overline{q}_x^e \geq \overline{q}_x^p$ for all $\overline{q}_x^e$. Naturally, $\overline{q}_x^p \geq \overline{q}_x$.

2. For each x and $\overline{q}_x$ there exists a bounded most optimistic expectation $\overline{q}_x^o$, such that $\overline{q}_x^e \leq \overline{q}_x^o$ for all $\overline{q}_x^e$.

3. At the last contracting stage, all expectations should be the same, since everybody knows that their contribution to the network is negligible: $\overline{q}_1^p = \overline{q}_1^o = \overline{q}_1$.

4. For all $\overline{q}_x$, the most pessimistic expectations change more rapidly than the most optimistic ones: $\frac{d\overline{q}_x^p}{dx} \geq \frac{d\overline{q}_x^o}{dx}$.

One can also interpret the expectation about future network size $\overline{q}_x^e$ as an expectation about the forward compatibility of the currently sold product to the ones sold in the future. The condition $\overline{q}_x^e \geq \overline{q}_x$ expresses that the good is perfectly backward compatible, which property is widespread in network industries, however, firms' using forward incompatibility strategies to increase profit is well documented.[10]

If each consumer selects the menu devoted to her, then monopoly profit is

$$\Pi = \int_0^1 p[t_{1s} - cq_{1s}] + (1-p)[t_{2s} - cq_{2s}]ds.$$

[9] Note that these expectations are by definition no longer rational.

[10] See Ellison and Fudenberg (2000) and Nahm (2005).

3.3.1 Constraints and expectations

Now the monopoly faces participation and incentive-compatibility constraints for each possible expectation the consumers might have:

$$\begin{aligned} U(\theta_i, \overline{q}_x^e, q_{ix}, t_{ix}) &\geq 0, \text{ and} \\ U(\theta_i, \overline{q}_x^e, q_{ix}, t_{ix}) &\geq U(\theta_i, x, \overline{q}_x^e, q_{jx}, t_{jx}) \end{aligned}$$

should be satisfied for all $i \in \{1, 2\}, x \in [0, 1]$ and possible $\overline{q}_x^e$.

First, each consumer at x should expect at least zero utility in all scenarios. Since the network effects are always positive, utility is increasing in $\overline{q}_x^e$, so the participation constraints are the hardest to satisfy at the smallest expected network size $\overline{q}_x^p$:

$$\begin{aligned} [IR_{1x}] &: \theta_1 V(q_{1x}, \overline{q}_x^p) - t_{1x} \geq 0, \text{ and} \\ [IR_{2x}] &: \theta_2 V(q_{2x}, \overline{q}_x^p) - t_{2x} \geq 0. \end{aligned}$$

The formulation of the relevant incentive-compatibility constraints depends on the ordering of different types' consumption. Assume for the moment monotonicity of the output schedule in types, that is $q_{2x} \geq q_{1x}$ for all x, a property which will be proved shortly. Now the incentive constraints for high-type consumers at x writes as $\theta_2 \left[V(q_{2x}, \overline{q}_x^e) - V(q_{1x}, \overline{q}_x^e)\right] \geq t_{2x} - t_{1x}$. Due to the complementarity of individual consumption and network size, the left-hand side is increasing in $\overline{q}_x^e$, so the relevant constraint should be again for the most pessimistic expectation case, higher expected network sizes just strengthens the satisfaction of this constraint:

$$[IC_{2x}] : \theta_2 V(q_{2x}, \overline{q}_x^p) - t_{2x} \geq \theta_2 V(q_{1x}, \overline{q}_x^p) - t_{1x}.$$

We should be more careful with the incentive constraint of low-type consumers at x, since they should be incentivized to choose the smaller amount q_{1x}. Rearranging this inequality gives $\theta_1 \left[V(q_{1x}, \overline{q}_x^e) - V(q_{2x}, \overline{q}_x^e)\right] \geq t_{1x} - t_{2x}$, and since $V_{12} > 0$, the hardest case to satisfy this constraint is now when the expected network size is the largest, i.e. for the most optimistic scenario. Let us define the most optimistic expectation by $\overline{q}_x^o \geq \overline{q}_x^p$, and then the relevant incentive constraint is

$$[IC_{1x}] : \theta_1 V(q_{1x}, \overline{q}_x^o) - t_{1x} \geq \theta_1 V(q_{2x}, \overline{q}_x^o) - t_{2x}.$$

Now let us find the set of relevant constraints and express the tariffs charged by the monopoly as functions of the quantities.

Lemma 12 *In the optimal mechanism, high-type consumers' participation constraints are satisfied with a strict inequality, and in optimum low-types' participation constraints and the high-types' incentive-compatibility constraints are binding for all $x \in [0,1]$. Therefore the optimal tariffs*

$$\begin{aligned} t_{1x} &= \theta_1 V(q_{1x}, \overline{q}_x^p), \\ t_{2x} &= \theta_2 V(q_{2x}, \overline{q}_x^p) - \Delta\theta V(q_{1x}, \overline{q}_x^p). \end{aligned}$$

Proof. *See Appendix B.2.* ■

We still need to prove the conjectured monotonicity of output schedules in types, because if it does not hold, then we should reformulate the relevant incentive constraints and tariff functions for the appropriate expectations. Note that this proof relies on the complementarity property as well.

Proposition 13 *In the optimal incentive-compatible mechanism, $q_{2x} \geq q_{1x}$ should hold for all x.*

Proof. *Let us indirectly assume that for some x the monotonicity property does not hold, that is $q_{2x} < q_{1x}$. Then for this x the relevant incentive constraints are*

$$\begin{aligned} [IC'_{1x}] &: \theta_1 V(q_{1x}, \overline{q}_x^p) - t_{1x} \geq \theta_1 V(q_{2x}, \overline{q}_x^p) - t_{2x}, \text{ and} \\ [IC'_{2x}] &: \theta_2 V(q_{2x}, \overline{q}_x^p) - t_{2x} \geq \theta_2 V(q_{1x}, \overline{q}_x^p) - t_{1x}. \end{aligned}$$

However, as the high-types' incentive constraint is satisfied for the most optimistic expectations, it is satisfied for $\overline{q}_x^p$ as well, which is condition $[IC_{2x}]$. Now similarly as in Lemma 12, it is easy *to prove that $[IR_{1x}]$ and $[IC_{2x}]$ imply $[IR_{2x}]$, and therefore $[IR_{1x}]$ and $[IC'_{2x}]$ should be binding in the optimal mechanism. The tariffs are thus*

$$\begin{aligned} t'_{1x} &= \theta_1 V(q_{1x}, \overline{q}^p_x), \\ t'_{2x} &= \theta_2 V(q_{2x}, \overline{q}^p_x) - \theta_2 V(q_{1x}, \overline{q}^p_x) + \theta_1 V(q_{1x}, \overline{q}^p_x). \end{aligned}$$

Now if we substitute back these tariff functions into $[IC'_{1x}]$, we get

$$\theta_1 \left[V(q_{1x}, \overline{q}^p_x) - V(q_{2x}, \overline{q}^p_x)\right] - \theta_2 \left[V(q_{1x}, \overline{q}^o_x) - V(q_{2x}, \overline{q}^o_x)\right] \geq 0.$$

The bracketed terms are both positive, since $q_{2x} < q_{1x}$ by assumption. However, the first bracketed term cannot be bigger because of increasing differences and $\overline{q}^p_x \leq \overline{q}^o_x$, and since $\theta_1 < \theta_2$, we reached a contradiction. ■

So far we have seen that the equilibrium tariffs should be formulated for the most pessimistic expectations. However, since the low-types' relevant incentive constraint stands for the most optimistic expectation, the behavior of this constraint is uncertain. After substituting the optimal tariff functions, we have

$$[IC_{1x}] : \theta_2 \left[V(q_{2x}, \overline{q}^p_x) - V(q_{1x}, \overline{q}^p_x)\right] - \theta_1 \left[V(q_{2x}, \overline{q}^o_x) - V(q_{1x}, \overline{q}^o_x)\right] \geq 0.$$

Let us make three observations:

1. $[IC_{1x}]$ is always satisfied if we pool the two consumers groups at x, since for $q_{2x} = q_{1x}$ the left-hand side is trivially zero.

2. $[IC_{1x}]$ is always satisfied at $x = 1$, since for $\overline{q}^p_1 = \overline{q}^o_1$ the left-hand side is strictly positive.

3. If we do not change q_{2x} and q_{1x} between x and x', then the left-hand side is increasing, since $\overline{q}^o_x - \overline{q}^p_x$ is decreasing and $V_{12} > 0$.

Therefore we might have the following preliminary conjecture for the optimal mechanism: if pooling is present, then it should happen for earlier consumers, and later consumers are always differentiated.

3.3.2 The unconstrained optimum

For demonstration purposes, let us suppose now that the nontrivial constraint $[IC_{1x}]$ is satisfied everywhere along the mechanism, which we now call the unconstrained optimum. This is the case for example when $\overline{q}^o(x) = \overline{q}^p(x)$, where the constraint simplifies to the monotonicity condition.

The monopoly's unconstrained profit maximizing problem is now

$$\max_{\{q(x)\}} \Pi = \int_0^1 \{p[\theta_1' V(q_{1x}, \overline{q}_x^p) - cq_{1x}] + (1-p)\left[\theta_2 V(q_{2x}, \overline{q}_x^p) - cq_{2x}\right]\} dx,$$

where $\theta_1' = \theta_1 - \frac{1-p}{p}\Delta\theta$. The first-order condition in q_{1x} and q_{2x} are respectively

$$\theta_1' V_1(q_{1x}, \overline{q}_x^p) + \int_0^1 \left\{ \left[p\theta_1' V_2(q_{1s}, \overline{q}_s^p) + (1-p)\theta_2 V_2(q_{2s}, \overline{q}_s^p)\right] \frac{d\overline{q}_s^p / dq_{1x}}{p} \right\} ds = c,$$

$$\theta_2 V_1(q_{2x}, \overline{q}_x^p) + \int_0^1 \left\{ \left[p\theta_1' V_2(q_{1s}, \overline{q}_s^p) + (1-p)\theta_2 V_2(q_{2s}, \overline{q}_s^p)\right] \frac{d\overline{q}_s^p / dq_{2x}}{1-p} \right\} ds = c.$$

In order to simplify the analysis, let us make the following restriction on expectations.

Condition 14 *For all possible expectations $\overline{q}_x^e$ and $s, x \in [0,1]$, we have $\frac{d\overline{q}_s^e/dq_{1x}}{p} = \frac{d\overline{q}_s^e/dq_{2x}}{1-p}$.*

This condition simply says that the change of q_{1x} and the change of q_{2x} that have the same effect on the network size should have the same effect on expectation formation. Then the network effects are the same in both first-order conditions, so adding them simplifies to

$$\theta_1' V_1(q_{1x}, \overline{q}_x^p) = \theta_2 V_1(q_{2x}, \overline{q}_x^p). \tag{3.6}$$

We see again that θ_1' should be positive in order to get a solution. Additionally, $V_{11} \leq 0$ implies $q_{2x} > q_{1x}$ at all x, so we have separation along the whole mechanism.

3.3.3 The constrained optimum

We might follow a general approach to develop the first-order conditions of the constrained optimum case, but in order to prove our conjectured results, we should impose very technical assumptions - see the Appendix for some additional results. Here we are working further with a simple and frequently used utility function, which gives us easily interpretable results.

Let us have a utility function that is multiplicatively separable in the utilities derived from individual consumption and network size:

$$V(q_i, \overline{q}) = f(q_i)g(\overline{q})$$

In order to satisfy the requirements imposed so far, we should have f' and g' strictly positive and $f'' \leq 0$.

In the following Proposition, we provide conditions for our initial conjecture: pooling may be present for early consumers, but once we reach a critical point, the pessimistic and optimistic expectations are close enough such that it becomes possible to profitably separate all different types of consumers till the end of the mechanism.

Proposition 15 *In the case of a multiplicatively separable utility function, we have $q_{2x} = q_{1x}$ for $x \in [0, x_0]$ and $q_{2x} > q_{1x}$ for $(x_0, 1]$, where $0 \leq x_0 < 1$, if either one of the following conditions is satisfied:*

1. *$g'' \leq 0$, so the marginal utility of network size is decreasing, or*
2. *$\frac{d\overline{q}_x^o}{dx} = 0$, so the most optimistic expectations are constant.*

Proof. *In the case of a multiplicatively separable utility function the incentive constraint for low type consumers simplifies to*

$$[IC_{1x}] : [f(q_{2x}) - f(q_{1x})] \left[\theta_2 g(\overline{q}_x^p) - \theta_1 g(\overline{q}_x^o)\right] \geq 0.$$

The first term is always non-negative by the monotonicity condition and $f' > 0$. This means that if at a certain x we have $\theta_2 g(\overline{q}_x^p) - \theta_1 g(\overline{q}_x^o) < 0$ then we should have $q_{2x} = q_{1x}$. Now suppose that at a certain x we have $\theta_2 g(\overline{q}_x^p) - \theta_1 g(\overline{q}_x^o) \geq 0$, so $[IC_{1x}]$ is satisfied for all (q_{2x}, q_{1x}) that satisfy the required monotonicity condition. We have that

$$\frac{d}{dx}\left[\theta_2 g(\overline{q}_x^p) - \theta_1 g(\overline{q}_x^p)\right] = \theta_2 g'(\overline{q}_x^p)\frac{d\overline{q}_x^p}{dx} - \theta_1 g'(\overline{q}_x^o)\frac{d\overline{q}_x^o}{dx}$$

is always positive if any of the conditions given above hold, since $0 \leq \frac{d\overline{q}^o_x}{dx} \leq \frac{d\overline{q}^p_x}{dx}, \overline{q}^p_x \leq \overline{q}^o_x$ *and* $0 < \theta_1 < \theta_2$*. So if* $[IC_{1x}]$ *is once satisfied at* x_0*, then the left-hand side cannot be negative again, since the first-term is always non-negative and the second term is increasing starting from a non-negative value. Therefore in the interval* $[x_0, 1]$ *we have the unconstrained optimum case that leads to* $q_{2x} > q_{1x}$*. Finally, we know that* $x_0 < 1$*, since we necessarily have* $[IC_{1x}]$ *satisfied in a small environment of* $x = 1$*.* ■

Note that neither of these two sufficient conditions is that restrictive. First, if g'' has a constant sign, then it cannot be positive, since then the utility functions $U_i(\theta_1, q)$ have "increasing returns to scale" in q: for any $t > 1$, providing tq would generate a surplus more than $tU_i(\theta_1, q)$, and as the technology has constant returns to scale, the monopoly's optimal profit would not have a finite positive value. Second, if the most optimistic expectation is that everybody has rational expectations, then this would imply constant optimistic expectations.

3.3.4 The monotonicity of output schedules

We are now examining the behavior of the quantity schedules $q_2(x)$ and $q_1(x)$, and begin with the pooling regime. In order to simplify the argument, assume that the most pessimistic expectations at every x equal the actual network size, so $\overline{q}^p_x = \overline{q}_x$. If we know that there is pooling between 0 and x_0 (so the monopoly supplies q_x for both types at $t_x = \theta_1 V(q_x, \overline{q}^p_x)$), then the monopoly's the partial maximization program is

$$\max \int_0^{x_0} \theta_1 V(q_s, \int_0^s q_t dt) ds$$

$$\text{s.t.} \int_0^{x_0} q_t dt = \overline{q}_{x_0}$$

The first-order condition of the Lagrangian at x is

$$V_1(q_x, \overline{q}_x) + \int_x^{x_0} V_2(q_t, \overline{q}_t) dt + \frac{\lambda}{\theta_1} = 0.$$

Differentiating this first-order condition in x gives

$$V_{11}(q_x, \overline{q}_x)\frac{dq_x}{dx} + V_{12}(q_x, \overline{q}_x) q_x - V_2(q_x, \overline{q}_x),$$

so rearranging gives

$$\frac{dq_x}{dx} = \frac{V_{12}(q_x, \overline{q}_x)q_x - V_2(q_x, \overline{q}_x)}{-V_{11}(q_x, \overline{q}_x)}.$$

The denominator is always positive, so the sign of $\frac{dq_x}{dx}$ depends on the relative magnitude of the complementarity V_{12} (multiplied by the actual quantity supplied) and the network effect V_2, which are both positive. We see a trade-off between two effects:

1. Later consumers marginal utility is higher because of the increased network (captured by V_{12}), this positive individual effect is for the increasing of $q(x)$.

2. However, the quantity supplied to later consumers contributes to the (expected) network of less consumers and therefore decreases the rent to be transferred (captured by V_2), and this negative aggregated effect is for the decreasing of $q(x)$.

In the case of the multiplicatively separable utility function defined in the previous Section, the slope of the quantity schedule simplifies to

$$\frac{dq_x}{dx} = \frac{f'(q_x)g'(\overline{q}_x)q_x - f(q_x)g'(\overline{q}_x)}{-f''(q_x)g(\overline{q}_x)} = \frac{g'(\overline{q}_x)\left[f'(q_x)q_x - f(q_x)\right]}{-f''(q_x)g(\overline{q}_x)}.$$

Based on the assumptions made before, we can conclude that $q(x)$ is increasing (decreasing) in x if $f(x)$ is elastic (inelastic) at q_x, i.e. $\frac{f'(q_x)q_x}{f(q_x)}$ is larger (smaller) than 1.

In the screening regime differentiating the pair of first-order conditions

$$\begin{aligned}
\theta_1' V_1(q_{1x}, \overline{q}_x) + \int_x^1 p\theta_1' V_2(q_{1s}, \overline{q}_s) + (1-p)\theta_2 V_2(q_{2s}, \overline{q}_s)ds &= c, \text{ and} \\
\theta_2 V_1(q_{2x}, \overline{q}_x) + \int_x^1 p\theta_1' V_2(q_{1s}, \overline{q}_s) + (1-p)\theta_2 V_2(q_{2s}, \overline{q}_s)ds &= c
\end{aligned}$$

gives

$$\begin{aligned}
\frac{dq_{1x}}{dx} &= \frac{\left[pq_{1x} + (1-p)q_{2x})\right] V_{12}(q_{1x}, \overline{q}_x) - \frac{p\theta_1' V_2(q_{1x}, \overline{q}_x) + (1-p)\theta_2 V_2(q_{2x}, \overline{q}_x)}{\theta_1'}}{-V_{11}(q_{1x}, \overline{q}_x)}, \text{ and} \\
\frac{dq_{2x}}{dx} &= \frac{\left[pq_{1x} + (1-p)q_{2x})\right] V_{12}(q_{2x}, \overline{q}_x) - \frac{p\theta_1' V_2(q_{1x}, \overline{q}_x) + (1-p)\theta_2 V_2(q_{2x}, \overline{q}_x)}{\theta_2}}{-V_{11}(q_{2x,}, \overline{q}_x)}.
\end{aligned}$$

These slopes can be interpreted in the very same way as in the pooling regime, just here the type-specific network effects and quantities should be weighted by their respective densities. Note that the sign of $\frac{dq_{1x}}{dx}$ and $\frac{dq_{2x}}{dx}$ are not necessarily the same, so we can easily have cases where $q_2(x)$ is increasing in x and $q_1(x)$ is decreasing in x.

3.3.5 Comparison of equilibria

We have seen that different expectations provide additional difficulties for the mechanism designer to deal with, and the costs of unique implementation constrain the monopoly's power in practicing second-degree discrimination techniques. We now take a closer look at this distortion by comparing the allocations of the benchmark and divide-and-conquer regimes.

The monopoly's profit function in the benchmark second-best case is

$$\Pi^{SB} = p\theta_1' V(q_1, \overline{q}) + (1-p)\theta_2 V(q_2, \overline{q}) - c\overline{q},$$

which can be rewritten in order to compare it with the profit in the divide-and-conquer case as

$$\Pi^{SB} = \int_0^1 \left[p\theta_1' V(q_{1x}, \overline{q}_1) + (1-p)\theta_2 V(q_{2x}, \overline{q}_1)\right] dx - c\overline{q}_1.$$

In the benchmark model every consumer is contracted simultaneously, so $q_1(x)$ and $q_2(x)$ are constant functions, and all consumers rationally expect network size to be $\overline{q}_1$.

In the divide-and-conquer case let us make the simplifying assumption that the most pessimistic expectations at every x equal the actual network size, so $\overline{q}_x^p = \overline{q}_x$. Then

$$\Pi^{DC} = \int_0^1 \left[p\theta_1' V(q_{1x}, \overline{q}_x) + (1-p)\theta_2 V(q_{2x}, \overline{q}_x)\right] dx - c\overline{q}_1$$

Let us now take the decomposition $\overline{q}_1 = \overline{q}_x + \overline{q}_{-x}$, where $\overline{q}_{-x} = \int_x^1 pq_{1s} + (1-p)q_{2s} ds$. Then we can define the following parametrized profit function

$$\Pi(q, \beta) = \int_0^1 \left[p\theta_1' V(q_{1x}, \overline{q}_x + \beta\overline{q}_{-x}) + (1-p)\theta_2 V(q_{2x}, \overline{q}_x + \beta\overline{q}_{-x})\right] dx - c\overline{q}_1,$$

where q stands for $(q_1(x), q_2(x))$ and $\beta \in R_+$. This function encompasses both of the regimes enlisted above: $\beta = 1$ gives the standard case, while for $\beta = 0$ we have the divide-and-conquer case. This function is quasisupermodular in q, since it is a positive sum of quasisupermodular functions, and satisfies the strict single-crossing condition in (q, β) for the same reason.

A crucial difference from the previous monotone comparative statics discussion is, however, that now we face a constrained optimization problem characterized by the continuum of constraints $[IC_{1x}]$:

$$\theta_2 \left[V(q_{2x}, \overline{q}_x + \beta \overline{q}_{-x}) - V(q_{1x}, \overline{q}_x + \beta \overline{q}_{-x})\right] - \theta_1 \left[V(q_{2x}, \overline{q}_x^o) - V(q_{1x}, \overline{q}_x^o)\right] \geq 0.$$

As we have seen before, these constraint are always slack if $\beta = 1$, so the possibility set $S(1)$ is the set of continuous functions pair $(q_1(x), q_2(x))$ defined on $[0, 1]$ satisfying the monotonicity constraint $q_1(x) \leq q_2(x)$. However, $[IC_{1x}]$ may be binding in the case of $\beta = 0$, so the possibility set $S(0)$ cannot be larger than $S(1)$. If the possibility set is increasing in the parameter analyzed, then we may still use the Monotone Selection Theorem (see Milgrom-Roberts (1994), Theorem 4') that gives an easy proof of our conjecture.

Corollary 16 *The allocation scheme* $(q_1(x), q_2(x))$ *under the divide-and-conquer regime can never be larger than the allocation of the benchmark case of second-degree discrimination in the presence of network effects.*

Since the mechanism designer faces additional, potentially binding constraints in the divide-and-conquer regime, it is also trivial that Π^{DC} cannot be higher than Π^{SB}. On the other hand, there is no general direction of the change in consumers' utilites. Low types are better off, since in the benchmark mechanism they end up with zero surplus, while in the divide-and-conquer case they only expect zero utility, but the realized network size will be larger, so they end up with a positive surplus. Similarly, the tariff charged for high types reaps only the expected surplus which is smaller than the finally realized one, but since both individual consumption and network size is smaller in the divide-and-conquer regime, the aggregate effect is ambiguous. We can tell, however, that high-types consumers contracted at $x = 1$ are definitely worse off, since they are left only with the standard information rent, which is smaller as both $q_1(1)$ and $\overline{q}(1)$ decrease.

Second, we can also examine how the changes in expectations affect the equilibrium outcome. Let us assume that the most optimistic expectation is the established network size at the end of the mechanism, so $\overline{q}_x^o = \overline{q}_1$, and that the most pessimistic expectation at a given x is the weighted average of the actual network size and final network size, that is $\overline{q}_x^p = (1-\lambda)\,\overline{q}_x + \lambda\overline{q}_1$. Now the parameter $\lambda \in [0,1]$ measures the degree of optimism of the most pessimistic expectation, and the parametrized profit function is

$$\Pi(q,\lambda) = \int_0^1 \left[p\theta_1' V(q_{1x}, (1-\lambda)\,\overline{q}_x + \lambda\overline{q}_1) + (1-p)\theta_2 V(q_{2x}, (1-\lambda)\,\overline{q}_x + \lambda\overline{q}_1)\right] dx - c\overline{q}_1,$$

where q stands for $(q_1(x), q_2(x))$ and $\lambda \in R_+$. Again, it is easy to see that this function is quasisupermodular in q, since it is a positive sum of quasisupermodular functions, and satisfies the strict single-crossing condition in (q,λ). We should also check how the constraint set $S(\lambda)$, defined by the continuum of constraints $[IC_{1x}]$:

$$\theta_2 \left[V(q_{2x}, (1-\lambda)\,\overline{q}_x + \lambda\overline{q}_1) - V(q_{1x}, (1-\lambda)\,\overline{q}_x + \lambda\overline{q}_1\right] - \theta_1 \left[V(q_{2x}, \overline{q}_x^o) - V(q_{1x}, \overline{q}_x^o)\right] \geq 0,$$

changes with λ. The second square bracketed term is a constant, and as $(1-\lambda)\,\overline{q}_x + \lambda\overline{q}_1$ is increasing in λ, $q_{2x} \geq q_{1x}$ and $V_{12} > 0$, we have that $S(\lambda)$ is increasing in λ, Therefore, we can use the Monotone Selection Theorem to establish the following result.

Corollary 17 *If the most pessimistic expectations $\overline{q}_x^p$ get closer to the most optimistic expectation $\overline{q}_x^o$ (i.e. λ increases) then the allocation scheme $q(\lambda)$ increases as well.*

3.4 Conclusion

This Chapter examined compatibility and coordination issues in a general model of contracting with asymmetric information and positive network effects. We have shown that these two questions are widely interlinked. Although the first part demonstrated that incompatibility strategies are not an effective device to improve the screening possibilities of the mechanism designer, they might play a role in influencing the consumers' expectations about the possible equilibria.

In the second part of this Chapter we have shown how pessimistic expectations can decrease a monopoly's power in practicing second-degree discrimination techniques in the presence of positive network effects, and demonstrated how divide-and-conquer techniques may solve the consumers' coordination problem and uniquely implement a screening mechanism. We have seen that with sequential contracting in the presence of asymmetric information, different expectations about future network sizes were relevant in the incentive constraints of different consumer types, which provided additional problems to solve. However, we have demonstrated that screening is more profitable for consumers contracted in later periods, and it may be beneficial to pool different types of consumers in the early stages of contracting, and concentrate only on building a large installed base. We have also compared the equillibrium allocation of the divide-and-conquer regime to that of benchmark screening case, and concluded that the quantity scheme provided by the monopoly, and thereby its profit, decreases as well.

The "divide-and-conquer" approach, followed by this Chapter and other papers in the literature (Jullien (2002), Segal (2003)), builds on the property that consumers are contracted in an exogenous order, which does not seem to completely fit evidence in evolving industries. It would be therefore beneficial to develop a dynamic approach, where consumers' "timing" to join the network is also endogenous. In this case, however, we should not only formulate the expectation formation on future network sizes, but also on the menu of contracts offered by the mechanism designer in later stages, which provides additional modeling difficulties.

Chapter 4

Functional degradation and asymmetric network effects

4.1 Introduction

We often find firms removing some functions of their original product and selling the degraded version at a lower or zero price. This kind of product differentiation or versioning strategy is frequently observed in the markets for software or other information goods, where it is a common practice to initially develop a full-featured version and then introduce a read-only (or play-only) version by simply removing the writing (or producing) function from the original version. More interestingly, many of the downgraded versions are offered free of charge, mostly downloadable on the Internet. Several examples can be found. For example, this paper has been written in Scientific Workplace, and Mathematica has been used for some calculations (both software have a full and a read-only version), and finally it has been formatted into PDF by Adobe Acrobat so that readers can view the paper using Acrobat Reader, the archetypical example of this phenomenon. Adobe Acrobat, which is able to produce and read PDF files, remains quite expensive, while Acrobat Reader, having only the reading function, is legally downloadable from hundreds of webpages free of charge. Other examples include play-only versions of media software such as RealPlayer, Flash Viewer, and Shockwave Player. Even

Microsoft provides free read-only versions for its Office programs.[1]

We have a well-established body of literature on how a monopoly seller can benefit from differential pricing based on quality differentiation. The early works à la Mussa and Rosen (1978) and Maskin and Riley (1984) are largely focused on endogenizing the quality spectrum, assuming that goods of different quality are produced independently from each other, and moreover, the marginal production cost is increasing in quality. More closely related to our work is Deneckere and McAfee (1996) who analyze a situation where a monopolist first develops the good of the highest quality, and then degrades it in order to produce a good of lower quality. The basic idea in their model of damaged goods is that "by producing an inferior substitute, the manufacturer can sell to customers who do not value the superior product so much, without decreasing demand for the superior product very much". The authors also show that this degradation policy may lead to a Pareto-improvement, i.e. all the consumers and the monopoly seller benefit from the introduction of the damaged good.

In all of these previous models, however, quality is measured by an universal number, to which the consumers' preference for quality is attached, usually in a multiplicative functional form. This unidimensional approach is ill-suited for analyzing the above-mentioned practice of functional degradation. First, the functional degradation strategy can be only artificially interpreted as if the relative valuation of one function to another is identical across all consumers. Second and more importantly, it neglects how firms can exploit differences in consumers' preferences regarding different quality dimensions. Third, these models completely ignore network effects (consumers' valuation of a particular product increases as the number of people who use the product increases), which are pervasive in markets for software and are becoming more significant nowadays due to the advance of the Internet.[2]

We build a functional degradation model, aiming to explain when and why firms have incentives to introduce a functionally-degraded good in markets subject to significant network effects. In our model it is assumed that consumers differ only in their valuation of network

[1] A similar phenomenon is observed in telecommunications as well, where two specific features of the service are calling and receiving, just like writing and reading (or playing) in the software examples above. In fact, there are communications devices designed to receive calls only, not being able to make calls, such as pagers and some mobile phones in the US. Hahn (2003) provides some discussion on this two-way communication issue in the context of monopoly pricing.

[2] See Katz and Shapiro (1994) and Economides (1996) for a survey on network effects. Gandal (1994) provides some empirical evidence on the existence of network effects in software markets.

effects, which allows us to focus on the pure impact network effects have on the profitability of the degradation strategy, abstracting from quality differentiation based motives.[3] Moreover, in order to capture the nature of functional degradation more accurately, we model network effects to be specific to individual functions imbedded in the good (e.g. writing and reading for word-processors), and also allow the intensity of network effects to be asymmetric across different functions. Our analysis shows that introducing a functionally-degraded good can be profitable to the firm if the consumer preference structure in terms of the valuations of networks is biased towards the function removed in the process of creating the degraded version (e.g. the writing function in the read-only version), and that the firm may wish to offer the degraded version free of charge if the bias in network valuations is sufficiently large. This provides a theoretical foundation to the main argument made by a recent case study on Acrobat, "The key was to separate Acrobat Reader from the full version of Acrobat and to give it away."[4]

Our findings differ from Deneckere and McAfee (1996) in several aspects. We provide a pure network effects based theory of damaged goods, not relying on the price discrimination motivation based on consumer heterogeneity in preferences for quality. Also, we provide a detailed analysis on economic logic behind functional degradation, which we believe is a more frequently observed versioning technique in real world markets, in particular in markets for software. Furthermore, there is a striking difference between the two models in the way the firm realizes the increase in its profit via degradation. In our model the firm's main purpose of introducing the degraded good is to extract more surpluses from full version buyers who benefit from extra network effects created by the degraded version, while in Deneckere and McAfee (1996) the main motivation for degradation is to increase profits by selling damaged goods without sacrificing demands for the original goods.[5] The practical importance of functional degradation is especially noticeable, given that the condition under which the proportional degradation is profitable is rather stringent and fails to hold in many specifications, as pointed out by Deneckere and McAfee (1996). The two approaches, however, should be considered as

[3] So our model differs from previous work on quality differentiation with network effects where the extent of network effects is assumed to be identical across all products regardless of the quality of the good, as in the papers by Haruvy and Prasad (1998, 2001), for example.

[4] "Trapeze artists", *The Economist* (December 12, 2002).

[5] These results are also comparable to the standard bundling literature (e.g. Adams and Yellen, (1976), McAfee et al. (1989)), which will be discussed later.

complements rather than substitutes.

Last, we examine the welfare consequences of versioning (taking the case of selling only the full version as a benchmark), and establish sufficient conditions under which the functional degradation strategy leads to a Pareto-improvement. We find that the firm's private incentive for introducing a degraded good is very much aligned with a social planner's objective, i.e. the introduction of damaged network goods tends to improve social welfare and therefore should not prohibited by public policy. We show that whether the introduction of the read-only version enlarges market sizes plays a key role in this analysis, which shows some similarities to the results established by Schmalensee (1981) and Varian (1985) in the context of the third-degree price discrimination.[6] In our case, however, we need to check how the installed bases of both functions change, not the amount of software sold, since we are implicitly dealing with two markets.

Closely related to our work is the literature on illegal piracy in the presence of network effects (Conner and Rumelt, 1991; Takeyama, 1994; King and Lampe, 2003). The main argument in this line of research is that allowing illegal copying of intellectual property may induce greater firm profits in the presence of network effects because it increases the user base of the product. This argument, however, crucially depends on a set of assumptions, in particular those allowing consumers to differ in some dimension(s) other than their valuation of network effects (e.g. different preferences for quality as in Takeyama and exogenously separated groups of consumers in terms of the ability to pirate as in King and Lampe). So, in their models the contribution of network effects to the firm's incentive for allowing illegal copies is entangled with the standard screening incentive based on some differentiation factors, such as quality and the ability to pirate. In fact, most of their results fail to hold if consumers are homogeneous, suggesting that network effects may be important for the firm's incentive to deliberately ignore illegal copies, but is effective only when consumers are sufficiently heterogeneous in some dimensions. Also, their models assume network effects are the same across different versions of the good. In contrast, assuming that consumers are identical (other than their valuation of network effects) and network effects are realized asymmetrically depending on specific functions imbedded in different versions, our analysis focuses on pure impacts the network effect brings on the firm's

[6] Note, however, that these authors looked at conditions for total welfare increase, not for Pareto-improvement.

incentives for introducing a functionally damaged good.

There exists a recent literature analyzing software versioning strategy from a two-sided markets viewpoint, where two exogenously separated groups of consumers interact via an intermediary (see Parker and Van Alstyne (2002a,b), Armstrong (2004), and Rochet and Tirole (2003) among others). Their main research questions are very similar to ours, namely what determines whether one side of the market is subsidized in order to reap more surplus form the other side, and whether the resulting outcome is socially efficient. However, we deal with a single market where all consumers ex ante belong to a single group, and individual consumers' group membership is determined endogenously in equilibrium, which property seems to have a better fit for software or telecommunications markets. The asymmetric network effects created by functional degradation also allows us to analyze the situation where some consumers are present on both sides of the market, while some choose to enter only one market.

The rest of the Chapter is organized as follows. Section 2 sets up the basic model and introduces its main assumptions. In Section 3, we use a simple two-type model to present the monopolist's incentives for implementing the functional degradation strategy and its relations to the traditional one-dimensional approach as well as to the bundling model. In Section 4, we develop a general continuous-type model, and derive the necessary and sufficient conditions characterizing the cases where the firm finds it profitable to introduce the read-only version and where it is provided for free. The welfare effects are analyzed for each case, and in the last part of Section 4 we work out a simplified model, where welfare results can be easily interpreted in terms of model primitives. Finally, we summarize and discuss the possible extensions of our model in Section 5. The proofs of more technical parts and Lemmas are found in Appendix C at the end of the thesis.

4.2 General model assumptions

Consider a monopolist who has developed a product which is consisted of two basic functions or components. The first function will be called writing, and it allows users to produce information goods (text, music, etc.) which can be consumed by users owning a product with the second function, say reading. Naturally, the software with the writing function includes the reading function as well, so the monopolist faces two options: it can sell the original full version alone or introduce a functionally-degraded version by eliminating one particular function or component from the original good and sell the two differentiated versions of the product. We assume that the unit production cost of the goods is the same for both goods and further normalize it to zero. For simplicity, any fixed costs including the cost of degradation are ignored.

Due to the separation of these two components the product is subject to a specific type of asymmetric network effects. Let w_i and r_i measure type-i consumers' preferences for the two functions, where w stands for writing and r for reading in the analogy of word processors. Following the Lancasterian characteristic approach, consumers' gross utility is assumed to be additively separable in the two valuations. Specifically, a type-i consumer's willingness-to-pay is given by

$$U_i = \begin{cases} (n_f + n_r)w_i + n_f r_i, & \text{when buying the original full version} \\ n_f r_i, & \text{when buying the read-only version} \end{cases},$$

where n_f and n_r respectively denotes the fraction of consumers who purchase the full and the read-only version. In order to concentrate on the pure impact of asymmetric network effects we normalize the stand-alone value of the good to zero for all consumers, so we consider a pure network good.[7]

This utility specification is inspired by the following stylized fact in today's software industry. For the writing function the size of the network benefit depends on the total number of consumers who can read the output produced by the particular piece of software (i.e. the owners of either the full or the read-only version), whereas for the reading function it depends only on the number of other consumers who are capable of producing the outputs (i.e. the owners of the original version). This implicitly implies that the extent of network effects is stronger for the original full version than for the degraded read-only version.

The main distinctive feature of this utility specification is that network effects are differ-

[7] The model can be extended to a utility structure incorporating the so-called stand-alone value of the good, but adding one more dimension would heavily complicate the analysis, mainly because of multiple equilibria. Our main results, however, do not change provided the value of the network is sufficiently large relative to the stand-alone value. For more on multi-dimensional screening problem, see Armstrong and Rochet (1999) and the references therein.

entiated depending on different functions as well as different versions, which to our knowledge has not been analyzed in the previous literature. For analytical simplicity, we implicitly assume that owners of the full version produce the same amount of information good, so we can use the number of writers as a measure of the total network benefit conferred to the reading function. Similarly, all users are assumed to read each outputs produced only once - or if they derive any additional utility from reading the same output repeatedly, then this is included in their valuation for the writing function. We do not examine the explicit incentives for writing either - if writers derive any direct or monetary utility for producing information goods, it is included in their valuation for the writing function.

Consumers are heterogeneous, and their type cannot be observed by the firm (or even if it is, it is not possible to directly discriminate among different consumers). The timing in the model is as follows.

1. The monopolist decides whether to introduce the degraded version or not, and sets the price(s) of the product(s).

2. Consumers observe the price(s), and form expectations about the sizes of the networks relevant to each function, n_f^e and n_r^e.

3. Each consumer decides which version to purchase or buys nothing.

We require that consumers' expectation must be fulfilled in equilibrium (rational expectation). With network effects, however, multiple equilibria may arise given prices, corresponding to various scenarios of consumer expectations and coordination.[8] To deal with this problem, we assume that if at a certain pair of prices there exist more than one pair of (n_f, n_r) that satisfies our equilibrium condition, and they are Pareto-ranked (i.e. one of them makes every consumer better off), then consumers expect the Pareto-optimal allocation to prevail in equilibrium. Milgrom and Roberts (1996) show that if there exists a Pareto-superior strategy in pure strategies (as in our case), then it is the only coalition-proof (correlated) equilibrium under any admissible coalition communication structure.[9]

[8] For more details on the equiliblium concept and the multiplicity of equilibria, see Rohlfs (1974), Farrell and Saloner (1985), and Katz and Shapiro (1985) among others.

[9] However, it may be that the Pareto-optimal equilibrium is more risky, and the risk-dominant equilibrium is the one with a lower participation, but here we do not examine this issue.

4.3 A simple two-type model

Let us begin with a discrete-type model to illustrate the basic intuitions of our results. We assume there are two types of consumers indexed by $i = h, l$, and for each type there are a continuum of homogenous consumers with measure λ_h and λ_l respectively. The total population of consumers is normalized to unity (i.e. $\lambda_h + \lambda_l = 1$). We make the following assumption on the distribution of consumers' valuations for the two functions:

A1: $\lambda_h(w_h + r_h) > w_l + r_l$.

This assumption says that preferences for the full version are sufficiently differentiated between the two types: the utility high-type consumers derive from using the full version when they are the only group using it is larger than the utility low-type consumers get when both groups use it. This condition ensures that high-type consumers always decide to buy the full-version when low-type consumers buy it, and it will be also a necessary condition for a partial participation equilibrium to exist, as will be shown later. Note that, however, this assumption does not exclude the cases where type-l consumers have stronger preferences for the degraded good than type-h consumers (i.e. $r_h \leq r_l$).

4.3.1 The full version only case

First, we examine the benchmark case when the monopolist sells only the full version at price p. Since in this case the expected number of writers and readers should be the same, a type-i consumer gets utility of $n_f(w_i + r_i) - p$ if she buys the good. Under the Pareto criterion, one of the following two equilibria can occur.[10]

Partial participation equilibrium: Under assumption A1 this case occurs only if the firm sells the full version to type-h consumers only, and is characterized by $\lambda_h(w_h + r_h) \geq p > w_l + r_l$. Assumption A1 also ensures that there exists at least one feasible p satisfying the above condition. The firm will fully extract the type-h consumers' surplus by charging $p = \lambda_h(w_h + r_h)$, and the resulting profit is $\pi^1 = (\lambda_h)^2(w_h + r_h)$.

[10] Note that if assumption A1 does not hold, the only possible equilibrium is the one with the firm serving both types.

Full participation equilibrium: Since under assumption A1 type-h consumers always buy the product if type-l consumers do, this equilibrium is characterized by $w_l + r_l \geq p$. The optimal price is $p = w_l + r_l$, and the resulting profit is $\pi^2 = w_l + r_l$. Serving type-l consumers forces the firm to give away informational rents of $(w_h + r_h) - (w_l + r_l)$ to type-h consumers.

Hence, the firm will serve type-h consumers alone if

$$(\lambda_h)^2(w_h + r_h) - (w_l + r_l) > 0 \tag{4.1}$$

and serves both types of consumers otherwise.

4.3.2 The option of introducing the read-only version

Notice that if the monopolist considers to introduce a differentiated version that contains both the writing and reading functions to differentiate between the two groups of consumers (e.g. a downgraded second or student version which is fully compatible with the original good), then this policy cannot be effective. It is because the degraded version involves no losses of network effects, therefore this type of quality differentiation does not affect consumers' willingness to pay for the goods, so all consumers would buy the good with the lower price. The only possible way to differentiate is a specific type of versioning strategy called *functional degradation*, in which the firm introduces a read-only version by eliminating the writing function from the original full version and sell the two differentiated versions of the product. What is particularly interesting in this kind of versioning is that the degraded good not only creates but also receives smaller network effects relative to the original version.

Suppose that the monopolist sets prices p_f and p_r for the full and read-only versions, respectively. The firm's problem is to choose the profit-maximizing pair of the prices subject to the following incentive and participation constraints:

$$\begin{aligned} w_i + \lambda_i r_i - p_f &\geq \lambda_i r_i - p_r, & (4.2)\\ w_i + \lambda_i r_i - p_f &\geq 0, & (4.3)\\ \lambda_i r_j - p_r &\geq w_j + \lambda_i r_j - p_f, & (4.4)\\ \lambda_i r_j - p_r &\geq 0, & (4.5) \end{aligned}$$

where type-i consumers are induced to buy the full version and type-j consumers to buy the read-only version ($i, j = h, l$). Given only two types of consumers, the incentive and participation constraints imply full participation in equilibrium ($n_f + n_r = 1$). Note that given a continuum of consumers in each type, an individual consumer's contribution to the network is ignored in both incentive constraints. In other words, conditions (4.2) and (4.4) imply that each consumer has no incentive to switch *individually* from the targeted version to the other version.

Not surprisingly, under assumption A1 selling the full version to type-l consumers and the read-only version to type-h consumers never results in higher profit than by selling the full version alone (π^2). If type-l consumers buy the full version, type-h consumers would obtain a strictly positive net surplus from purchasing the full version under assumption A1. Then, given that the price of the read-only version cannot exceed the price of the full version, the maximum profit the firm can get from this screening strategy is always smaller than π^2. Therefore, we focus on the case when the firm induces type-h consumers to buy the full version and type-l consumers to buy the read-only version. The joint satisfaction of the two incentive constraints (4.2) and (4.4) with $i = h$ and $j = l$ requires that $w_l \leq p_f - p_r \leq w_h$. So, for the separating equilibrium to be implementable it must be that $w_h \geq w_l$ (i.e. the preference for the writing function is greater for type-h consumers than type-l consumers).

Consider first the pair of prices that make the two types' participation constraints binding, i.e. $\overline{p}_f = w_h + \lambda_h r_h$ and $\overline{p}_r = \lambda_h r_l$.[11] Plugging in those prices into the two incentive constraints, it is easily observed that under assumption A1 the type-l consumers' incentive constraint is always satisfied, but the incentive constraint of type-h consumers is satisfied only if $r_h \leq r_l$. In this case, the pair of prices $(\overline{p}_f, \overline{p}_r)$ is feasible and in fact optimal, so the firm can exercise perfect discrimination even with incomplete information about consumer types. With a negative correlation in the preferences for the two functions ($w_h \geq w_l$ and $r_h \leq r_l$) the two types of consumers are clearly divided into two groups (one interested in writing and the other interested in reading) even though the valuation for the full version is still larger for type-h consumers than type-l consumers, thus the firm can take the full advantage of functional degradation,

[11] This is *not* as if the firm has complete information about consumer type. With complete information, the firm would sell the full version to both types of consumers with prices equaling to their respective reservation values.

extracting both types' surpluses. The full-surplus extraction story is not generally true when there exist more than two types (see next Section), but it sharply contrasts the functional degradation with the traditional proportional quality differentiation where a firm must give away some informational rent to high-valuation consumers when selling quality-differentiated goods (even with only two types of consumers).

For $r_h > r_l$, the valuations for the two functions are more or less aligned between the two types (a positive correlation), and the firm is forced to reduce the price of the full version to the extent that type-h consumers are indifferent between buying the full version and switching to the read-only version.

To summarize, the optimal prices are given by

$$\begin{aligned} p_f &= \begin{cases} w_h + \lambda_h r_h & \text{for } r_h \leq r_l \\ w_h + \lambda_h r_l & \text{for } r_h > r_l \end{cases}, \\ p_r &= \lambda_h r_l \text{ for all cases,} \end{aligned}$$

and the resulting profit is

$$\pi^{FD} = \begin{cases} \lambda_h(w_h + \lambda_h r_h + \lambda_l r_l) & \text{for } r_h \leq r_l \\ \lambda_h(w_h + r_l) & \text{for } r_h > r_l \end{cases}.$$

Proposition 18 *Suppose that the firm sells the full version to type-h consumers and the read-only version to type-l consumers. Then, for $r_h \leq r_l$ the firm always introduces the read-only version, and for $r_h > r_l$ the firm has the incentive the read-only version if and only if*

$$\lambda_l w_h - \lambda_h r_h + r_l > 0 \textit{ for } \pi^1 > \pi^2,$$

and

$$\lambda_h w_h - w_l - \lambda_l r_l > 0 \textit{ for } \pi^1 \leq \pi^2.$$

Proof. *First, for the case of $r_h \leq r_l$ the incremental profit from introducing the read-only* *version is given by*

$$\pi^{FD} - \max\left\{ \pi^1, \pi^2 \right\} = \begin{cases} \lambda_h(w_h + r_l) & \text{for } \pi^1 > \pi^2 \\ \lambda_h(w_h + \lambda_h r_h r_l + \lambda_l r_l) - (w_l + r_l) & \text{for } \pi^1 \leq \pi^2 \end{cases},$$

which is always positive under assumption A1 and $r_h \leq r_l$. For the case of $r_h > r_l$, the above two conditions are immediate from $\pi^{FD} > \pi^1$ and $\pi^{FD} > \pi^2$. ■

The above result clearly shows that introducing the read-only version is more likely to be profitable when the preferences of high-valuation consumers (who value the full version more) are more biased toward the writing function than the reading function, relative to the low-valuations consumers. This is the case where the firm can extract greater network benefits from the full-version buyers without giving them too much information rent compared to the benchmark partial participation case, and also can gain a greater screening benefit without losing too much network benefits (of the read-only version buyers) from taking away the writing function for the full participation case in the benchmark.

The underlying logic is quite similar to the case of commodity bundling. It is well known that bundling is optimal when consumers' willingness-to-pay for the bundle is less dispersed than the willingness-to-pay for the components (e.g. Adams and Yellen (1976)). A similar principle applies to functional degradation: it is more effective when consumers' valuations are more dispersed across functions. In our case, however, the firm's decision problem is the exact opposite of the bundling model - whether or not to disaggregate the already-bundled original good. Indeed, bundling increases profits by reducing the dispersion in consumers' valuation, while functional degradation allows the firm to directly exploit the valuation dispersion. Furthermore, the functional degradation is basically a consumer screening device, whereas in bundling the firm gets more profits by bunching different types of consumers.

Another important implication of the presence of network effects is that if we allow for a positive marginal cost the firm may wish to sell the degraded version under cost.[12] In this case the firm loses money on every sale of the read-only version, but the enlarged reading base and its network effects allow the firm to charge a higher price for the full version, increasing

[12] See Hahn (2001) for a formalization of the idea.

thereby its profit. In the next Section we examine a special case of cross-subsidization pricing policy (namely, offering the read-only version free of charge) more rigorously using a general continuous type model.

4.3.3 Welfare effects

The welfare effect of the functional degradation is paralleled with the conventional wisdom established in standard price discrimination models: it depends on whether the introduction of the read-only version expands the market or not, relative to the benchmark case of the full version only.

Proposition 19 *The profitable introduction of the read-only version is (weakly) Pareto-improving if it increases the total demand, and is welfare-reducing otherwise.*

This result can be easily seen to hold in the present setting. If we have partial participation in the benchmark case, then the new product enlarges the market size and creates new network effects. High-type consumers' net surplus either remains the same (in the negative correlation case) or increases by the informational rent (in the positive correlation case), while low-type consumers end up with zero net surplus in all the cases. The firm's profit naturally increases if it chooses versioning. However, in the positive correlation case there exist some parameter values under which the firm does not introduce the read-only version even if it would increase total surplus (consumer surplus plus profit).

When the monopolist covers all the market by selling the full version only, consumer surplus cannot be increased by introducing the read-only version. Low-type consumers end up with zero net surplus in all the cases, and high-type consumers' net surplus decreases (or even disappears in the negative correlation case). The decrease in consumer surplus is a natural consequence of screening, but on the top of that, total surplus decreases as well, since introducing the read-only version destroys some network effects by taking away the writing function from low-type consumers.

4.4 The continuous type model

Now index consumers by their valuation for the writing function, denoted by v (so $v = w$). We assume that v has a continuous cumulative distribution function $F(v)$, with density $f(v) > 0$ on its support normalized to $[0, 1]$. As usual in the standard adverse selection model, we assume the monotone hazard rate property: $\frac{d}{dv}\frac{1-F(v)}{f(v)} < 0$, which will be guaranteed by the convexity of the distribution function ($f'(v) \geq 0$).[13]

In this continuous type model we focus on the case of perfect and positive correlation between the two valuations, since we have seen in the discrete-type case that with a negative correlation the monopoly always has the incentive to introduce the read-only version. So, we assume that there exists a continuous function $g(v)$ with $g'(v) > 0$, which gives the type-v consumer's valuation for the reading function (so $g(v) = r$). We assume that $g(0) = 0$, i.e. the consumers having zero valuation for the writing function have zero valuation for the reading function as well.[14] Our key assumption will be that $g''(v) \leq 0$, i.e. the valuation for the reading function changes more slowly compared to the valuation for the writing function. This property ensures that the two functions describing the valuation for writing and reading cross at most once for all types, and as will be seen later, it allows the monopoly to implement her optimal screening policy in case of versioning.

4.4.1 The full version only case

Suppose the monopolist sells only the full version. Given a price p, let v_f be the type of consumers who are indifferent between buying and not buying the product, i.e.

$$n_f^e[v_f + g(v_f)] - p = 0,$$

where n_f^e is consumers' expectation on the fraction of consumers who end up buying the full version. Since $v + g(v)$ is increasing in v, consumers with a type higher than v_f will purchase

[13] The convexity assumption is needed only for ensuring that the second-order conditions of the problem are always satisfied. In fact, it would be sufficient to state a condition ensuring that the distribution function is "not too concave", but it would entail more technical difficulties.

[14] If $g(0) > 0$, the monopolist would never offer the read-only version free of charge, the case we do not want to exclude from the beginning.

the good, and those with a type lower than v_f will not buy it. In order for the expectation to be fulfilled it must be that $n_f^e \equiv 1 - F(v_f)$, leading to the following equilibrium condition:

$$p(v_f) = [1 - F(v_f)][v_f + g(v_f)], \tag{4.6}$$

which immediately shows that for a given price there exist more than one expectation-fulfilling equilibrium demands, as usual in a model with network effects.[15] However, a higher demand makes the good more valuable, hence everyone would be better off by coordinating on the smallest v_f, which gives the (unique) fulfilled-expectation equilibrium under the Pareto criterion.

Having derived a demand function, we can now write the monopolist's profit as a function of the marginal type v_f instead of the price p, as usual in the mechanism design literature. The monopolist will then choose the price p that satisfies equilibrium condition (4.6), and at this price the expectation-fulfilling demand will be the profit-maximizing one. The firm's profit is then

$$\Pi(v_f) = p(v_f)[1 - F(v_f)] = [1 - F(v_f)]^2[v_f + g(v_f)].$$

Maximizing the profit function with respect to v_f gives the following first-order condition:[16]

$$\frac{1 - F(v_f)}{f(v_f)} = \frac{2[v_f + g(v_f)]}{1 + g'(v_f)} \tag{4.7}$$

which gives a unique solution, given that the left-hand side is decreasing in v_f under the monotone hazard rate property and its value is 0 at $v_f = 1$, while the right-hand side is increasing in v_f and its value is 0 at $v_f = 0$. We call this equilibrium the full version only optimum.

4.4.2 The option of versioning

Now we examine the case where the monopolist introduces the functionally-degraded good and chooses prices p_f and p_r for the full and the read-only version respectively. Let v_w denote

[15] Since $p(0) = p(1) = 0$ in (4.6), for any p there exist more than one v_f (so more than one quantity sold) that satisfy the equilibrium condition (4.6).

[16] We postpone the discussion of second-order conditions for the moment. In the next subsection we show that this maximization program is part of a more general one, for which the first-order conditions are necessary and sufficient.

the type of consumers who are indifferent between buying the full and the read-only version (the marginal writer), and v_r the type of consumers who are indifferent between buying the read-only version and not buying at all (the marginal reader), i.e.

$$(n_f^e + n_r^e)v_w + n_f^e g(v_w) - p_f = n_f^e g(v_w) - p_r$$

and

$$n_f^e g(v_r) - p_r = 0.$$

Rearranging the above two equations gives the following expressions for the prices:

$$p_f - p_r = (n_f^e + n_r^e)v_w, \tag{4.8}$$

$$p_r = n_f^e g(v_r), \tag{4.9}$$

and therefore

$$p_f = (n_f^e + n_r^e)v_w + n_f^e g(v_r). \tag{4.10}$$

Equations (4.8) and (4.9) imply the standard relations between prices in the second-degree price discrimination: the price of the read-only version is equal to the marginal readers' expected valuation for the reading function, and the price differential between the full and the read-only version is equal to the marginal writers' expected valuation for the additional writing function (and therefore the marginal writers realize information rent of $n_f^e[g(v_w) - g(v_r)]$).

As in standard models of vertical differentiation, the marginal consumers' types "cut" the mass of consumers into different purchasing groups.

Lemma 20 *Every consumer having a type at least v_w purchase the full version, consumers of type $v_r \leq v < v_w$ buy the read-only version, and consumers with a type less than v_r buy nothing.*

Proof. *See Appendix C.1.* ■

Under fulfilled-expectation equilibria it must be that $n_f^e = 1 - F(v_w)$ and $n_r^e = F(v_w) - F(v_r)$, which reduce equations (4.9) and (4.10) to

$$p_f = [1 - F(v_r)]v_w + [1 - F(v_w)]g(v_r) \tag{4.11}$$

and

$$p_r = [1 - F(v_w)]g(v_r). \tag{4.12}$$

As before, for given prices (p_w, p_r) there can be multiple pairs of (v_w, v_r) that satisfy conditions (4.11) and (4.12). Nevertheless, any two different equilibrium pairs of (v_w, v_r) can be ordered. It is because for any (v_w', v_r') and (v_w'', v_r'') satisfying equation (4.12) it must be that $[1 - F(v_w')]g(v_r') = [1 - F(v_w'')]g(v_r'')$. So, if $v_w' < v_w''$ then $g(v_r') < g(v_r'')$ should hold, which implies $v_r' < v_r''$. Since the smaller pair means a larger writing base as well as a larger reading base, all consumers have the same ordering of the possible equilibria. Therefore, under the Pareto criterion consumers expect the smallest pair (v_w, v_r) among those satisfying equations (4.11) and (4.12) to be realized in equilibrium.

Note that the positively correlated valuations of the two functions played a crucial role in ordering the different equilibria. However, suppose that the read-only version is provided for free, that is $v_r = 0$. Then, equation (4.11) is simplified to $p_f = v_w$, and therefore there is only one equilibrium. Thus the free read-only version can be seen as an effective coordination-facilitating device to overcome the problem of multiple equilibria, even in the case of positive correlation.

Using equations (4.11) and (4.12), the firm's profit as a function of the marginal types is

$$\Pi(v_w, v_r) = [1 - F(v_w)]p_f + [F(v_w) - F(v_r)]p_r = [1 - F(v_w)][1 - F(v_r)][v_w + g(v_r)].$$

The function $\Pi(v_w, v_r)$ is strictly quasi-concave (see Appendix C.2 for the proof), which property will be very useful in the following analysis.

The monopolist's profit-maximization problem is

$$\begin{aligned} &\max_{\{v_w, v_r\}} [1 - F(v_w)][1 - F(v_r)][v_w + g(v_r)] \\ &\text{s.t.} \quad 0 \le v_r \\ &\qquad\;\; v_r \le v_w \\ &\qquad\;\; v_w \le 1. \end{aligned}$$

The second constraint states that the number of writers cannot exceed the number of readers.

The third constraint is always slack in optimum, since setting $v_w = 1$ would result in no sales of the full version, making the read-only version totally useless and therefore generating zero profit. Let λ and μ be the Kuhn-Tucker multipliers of the first and the second constraint respectively.

The first-order conditions for the profit-maximization are

$$\frac{1 - F(v_w)}{f(v_w)} + \frac{\mu}{[1 - F(v_r)]f(v_w)} = v_w + g(v_r) \tag{4.13}$$

and

$$\frac{1 - F(v_r)}{f(v_r)} g'(v_r) + \frac{\lambda - \mu}{[1 - F(v_w)]f(v_r)} = v_w + g(v_r). \tag{4.14}$$

Due to the strict quasi-concavity of the profit function, the solution of the problem is unique, and the first-order conditions are sufficient if one of the constraints is binding. So we need to check the second-order conditions only when all the constraints are slack (Appendix C.3 shows that they are indeed satisfied in this case). There can be three different cases, depending on which of the first two constraints is binding in the above profit-maximization problem.[17] We examine each case separately in the following Subsections.

The non-versioning case

Suppose the second constraint is binding ($v_w = v_r$), i.e. the firm chooses not to introduce the read-only version. Then the first constraint must be slack at the optimum, i.e. $\lambda = 0$. Plugging $v_r = v_w$ into the first-order conditions (4.13) and (4.14) and adding these two equations leads to equation (4.7), which was the first-order condition of selling the full version only. Therefore, in the non-versioning case we have $v_w = v_r = v_f$.

The following Proposition shows when the firm has the incentive to introduce the read-only version.

Proposition 21 *The firm chooses not to introduce the read-only version if and only if $g'(v_f) \geq 1$, where v_f is the full version only optimum defined by the equation $\frac{1-F(v_f)}{f(v_f)} = \frac{2[v_f+g(v_f)]}{1+g'(v_f)}$. Therefore, if $g'(v_f) < 1$, the firm always introduces the read-only version.*

[17] The two constraints cannot be binding simultaneously in optimum, because if they are, then $v_w = v_r = 0$ and $\Pi(v_w, v_r) = 0$.

Proof. *First, assume that $v_w = v_r = v_f$. Based on the two first-order conditions, for μ to be non-negative the following two conditions have to hold simultaneously:*

$$\begin{aligned} \frac{1-F(v_f)}{f(v_f)} &\leq v_f + g(v_f), \text{ and} \\ \frac{1-F(v_f)}{f(v_f)} g'(v_f) &\geq v_f + g(v_f). \end{aligned}$$

These equations can be satisfied only if $g'(v_f) \geq 1$.

Second, if $g'(v_f) \geq 1$ then for non-negative Kuhn-Tucker multipliers the two first-order conditions are satisfied only if μ is positive, so $v_w = v_r$. Finally, the second claim is immediate from the first one. ■

The condition for the introduction of the read-only version to be profitable is in fact similar to the sorting (or single-crossing) condition in standard adverse selection models: in order to successfully screen the different types, the consumer valuation for the writing (high-type) function should change more rapidly than the valuation for the reading (low-type) function for the marginal type in the no discrimination regime (v_f), and for all consumers of higher types given that $g(\cdot)$ is concave.[18] The underlying reasoning is as follows. Versioning decreases the writing base,[19] reducing the network effects created by writing consumers. If $g'(v_f) \geq 1$, the corresponding profit loss is larger than the profit gain coming from the newly created network effects by the enlarged reading base and from the sales of the read-only version, since the consumers around the marginal type v_f are more sensitive to the reading function than the writing function. Hence, in this case any attempt to screen consumers by introducing the degraded version would reduce the firm's profit.

Standard versioning

Suppose now that neither of the constraints is binding, i.e. the monopolist introduces the read-only version and sells it at a positive price. Plugging in $\lambda = \mu = 0$ into the first-order conditions (4.13) and (4.14) leads to

$$\frac{1-F(v_w)}{f(v_w)} = v_w + g(v_r) \tag{4.15}$$

[18] Note that the marginal change of the valuation for the writing function is 1.
[19] It will be shown in Proposition 23 below.

and

$$\frac{1 - F(v_r)}{f(v_r)} g'(v_r) = v_w + g(v_r). \tag{4.16}$$

Proposition 22 *If $0 < v_r < v_w < 1$, then $g'(v_r) < 1$.*

Proof. *Given that $v_r < v_w$, monotone hazard rate property implies $\frac{1-F(v_w)}{f(v_w)} < \frac{1-F(v_r)}{f(v_r)}$. From conditions (4.15) and (4.16) we have $\frac{1-F(v_w)}{f(v_w)} = \frac{1-F(v_r)}{f(v_r)} g'(v_r)$, which holds only if $g'(v_r) < 1$.* ■

This Proposition is in line with our previous argument concerning the sorting condition: if $g'(v_r) < 1$, then $g'(v) < 1$ for all $v \in [v_r, 1]$ by the concavity of $g(v)$, and so the versioning policy can be successfully implemented for the whole range of consumers.

We now show that the introduction of the read-only version enlarges the reading base but reduces the writing base, compared to the benchmark case of selling the full version only.

Proposition 23 *If $0 < v_r < v_w < 1$, then $v_r < v_f < v_w$.*

Proof. *The proof is by contradiction. First let us suppose that $v_r < v_w \leq v_f$. Monotone hazard rate property implies that $\frac{1-F(v_f)}{f(v_f)} \leq \frac{1-F(v_w)}{f(v_w)}$. Rearranging this inequality by using optimum conditions (4.7) and (4.15), we have that*

$$\frac{2}{1 + g'(v_f)} [v_f + g(v_f)] \leq v_w + g(v_r).$$

But this is a contradiction, because $1 + g'(v_f) \leq 1 + g'(v_r) < 2$ and $v_f + g(v_f) > v_w + g(v_r)$.given the concavity of $g(v)$ and Proposition 22.

Suppose now that $v_w > v_r \geq v_f$, so by the monotone hazard rate property $\frac{1-F(v_r)}{f(v_r)} \leq \frac{1-F(v_f)}{f(v_f)}$. Rearranging this inequality by using optimum conditions (4.7) and (4.16), we have

$$v_w + g(v_r) \leq \frac{2g'(v_r)}{1 + g'(v_f)} [v_f + g(v_f)].$$

This is a contradiction again, since $v_w + g(v_r) > v_f + g(v_f)$, and $1 + g'(v_f) \geq 1 + g'(v_r) > 2g'(v_r)$ by the concavity of $g(v)$ and Proposition 22. ■

The comparison of the equilibrium prices is ambiguous. We can easily see that the price of the read-only version, $p_r = [1 - F(v_w)]g(v_r)$, is smaller than the price of the full version when it is the only version sold, $p = [1 - F(v_f)][v_f + g(v_f)]$), because $1 - F(v_w) < 1 - F(v_f)$ and $g(v_r) < v_f + g(v_f)$ by Proposition 23. However, the ordering is not clear between p and the price of the full version in the versioning case, $p_f = [1 - F(v_r)]v_w + [1 - F(v_w)]g(v_r)$.[20]

The free read-only version

Finally, suppose that the first constraint is binding ($v_r = 0$), i.e. the monopolist covers all the market by selling the read-only version at zero price. Then the second constraint must be slack (i.e. $\mu = 0$), and first-order conditions (4.13) and (4.14) are simplified to

$$\frac{1 - F(v_w)}{f(v_w)} = v_w \tag{4.17}$$

and

$$\frac{1 - F(0)}{f(0)} g'(0) + \frac{\lambda}{[1 - F(v_w)]f(0)} = v_w. \tag{4.18}$$

Condition (4.17) yields a unique solution, which we denote by $\widetilde{v}_w$.

First, we show that our sorting condition holds for the whole range of consumer types.

Proposition 24 *If* $0 = v_r < \widetilde{v}_w$*, then* $g'(0) < 1$*.*

Proof. *By using conditions (4.17) and (4.18) and the fact that* $\frac{\lambda}{[1-F(\widetilde{v}_w)]f(0)}$ *is positive, we can see that* $\frac{1-F(\widetilde{v}_w)}{f(\widetilde{v}_w)} > \frac{1-F(0)}{f(0)} g'(0)$*. Since* $0 < \widetilde{v}_w$*, then* $\frac{1-F(\widetilde{v}_w)}{f(\widetilde{v}_w)} < \frac{1-F(0)}{f(0)}$ *by the monotone hazard rate property, so* $g'(0) < 1$*.* ■

Similar to the standard versioning case, the monopolist sells a smaller amount of the full version compared to the case of selling the full version only, and it naturally enlarges the reading base ($v_r = 0$). Moreover, this case results in the smallest demand of the full version (defined by condition (4.17)), which is exactly the same as the optimal quantity in the standard single-product monopoly problem without network effects.

Proposition 25 *If* $0 = v_r < \widetilde{v}_w$*, then* $0 < v_f < \widetilde{v}_w$*, and* $\widetilde{v}_w$ *is bigger than any* v_w *of the standard versioning case.*

[20] Even if this ordering were clear, we could not derive a clear welfare implication from this fact alone, since versioning changes the writing and reading bases of the product as well as the extent of network effects. In Subsection 4.4.3 we examine these welfare issues in more details.

Proof. *First, suppose that* $0 < \widetilde{v}_w \leq v_f$*, so by the monotone hazard rate property* $\frac{1-F(v_f)}{f(v_f)} \leq \frac{1-F(\widetilde{v}_w)}{f(\widetilde{v}_w)}$*. Rearranging this inequality by using optimum conditions (4.7) and (4.17) we have*

$$\frac{2}{1+g'(v_f)}[v_f + g(v_f)] \leq \widetilde{v}_w.$$

But this is a contradiction, since by the concavity of $g(v)$ *and Proposition 24,* $1 + g'(v_f) < 1 + g'(0) < 2$*, and* $v_f + g(v_f) \geq \widetilde{v}_w$*.*

For the second part suppose that $\widetilde{v}_w \leq v_w$*, where* v_w *stands for any optimal marginal type in the standard versioning case. Then* $\frac{1-F(v_w)}{f(v_w)} \leq \frac{1-F(\widetilde{v}_w)}{f(\widetilde{v}_w)}$ *by the monotone hazard rate condition. Replacing both sides of the inequality by using optimum conditions (4.15) and (4.17) yields*

$$v_w + g(v_r) \leq \widetilde{v}_w,$$

which is a contradiction, since in the standard versioning case $g(v_r) > 0$*.* ■

Contrary to the standard versioning case, the comparison of prices of the full version in the two regimes is unambiguous. The firm's profit is $p[1 - F(v_f)]$ in the full version only case, and $p_f[1 - F(v_w)]$ in the free read-only version case. Given that $1 - F(v_f) > 1 - F(v_w)$ by Proposition 25 and the firm has optimally chosen to introduce the free read-only version, p_f must be larger than p. So if the read-only version is provided for free, it increases the price of the full version.

Since one of the constraints is slack and λ should be non-negative, we can state the following necessary and sufficient condition for this case to happen.

Corollary 26 *The firm introduces the read-only version for free if and only if* $\widetilde{v}_w f(0) \geq g'(0)$*, where* $\widetilde{v}_w$ *is implicitly defined by the equation* $\widetilde{v}_w = \frac{1-F(\widetilde{v}_w)}{f(\widetilde{v}_w)}$*.*

If the monopolist raises v_r to a slightly positive level, it decreases the reading base, which greatly reduces consumers' willingness-to-pay for the writing function. If $\widetilde{v}_w f(0) < g'(0)$, the profit loss of $\widetilde{v}_w f(0)$ on each writer (of mass $1 - F(\widetilde{v}_w)$) would not be compensated by the small rent of $[1 - F(\widetilde{v}_w)]\, g'(0)$ collected from each reader (of mass 1).

4.4.3 Welfare effects

We now briefly examine the welfare effect of introducing the read-only version. We have already observed from Propositions 23 and 25 that the versioning always increases the reading base but decreases the writing base, hence not only creating but also destroying network effects. Therefore, consumers are affected by the changes of network effects (both positively and negatively) as well as the price changes. In the discrete-type example in Section 3, we have seen that the versioning may lead to a (weak) Pareto-improvement. Our discussion is focused on whether there exist some conditions under which the same result holds in this continuous-type model.

Consider the following condition:

$$[1 - F(v_w)] + [1 - F(v_r)] > 2[1 - F(v_f)] \tag{4.19}$$

The first term of the left-hand side is the amount of the full version sold (i.e. the installed base for writing) under the versioning, and the second term is the total amount of sales including the read-only version (i.e. the installed base for reading) under versioning. The right-hand side is twice the amount of the full version sold without versioning (i.e. the sum of the installed writing and reading bases). This condition is very similar to the Pareto-improving condition derived in the discrete-type case: the introduction of the read-only version increases the market size. In our case, however, what is important is not the amount of firm's output (which is always increasing since $v_r < v_f$) but the amount of the two functions imbedded in the products.[21] The following Proposition shows that once this condition holds, the welfare effect hinges on how the marginal writers' utility is affected by the versioning policy.[22]

Proposition 27 *Suppose condition 4.19 holds. Then versioning by functional degradation leads to a Pareto-improvement if and only if the utility of consumers of type v_w increases.*

Proof. *See Appendix C.4.* ■

An intuitive sufficient condition for the increase of the marginal writers' utility would be

[21] This point has been blurred in the previous two-type model where the versioning changed only the writing or the reading base, and so it was sufficient to check whether the overall participation of consumers increased or not.

[22] Note that this condition is very easy to check since we need to look at only the marginal writers. Although it does not hold in general, it is satisfied for many specifications, as in the illustrative example presented below.

a decrease in the price of the full version. In this case condition (4.19) immediately implies a welfare improvement, since the changes of network effects due to versioning provide the marginal writers with a higher writing and reading base in total at a smaller price. This conjecture proves to be true provided the highest type's valuation for the reading function does not exceed her valuation for the writing function, as shown in the following Lemma.

Proposition 28 ***Lemma 29*** *Suppose condition 4.19 holds and $g(1) \leq 1$. Then the utility of consumers of type v_w increases if $p_w \leq p_f$.*

Proof. *See Appendix C.5.* ■

Summing up, the above arguments lead to the following sufficient condition for a Pareto-improving versioning.

Corollary 30 *If the versioning increases the sum of the installed writing and reading bases and the price of the full version falls, it leads to a Pareto-improvement.*

The price fall for the high-quality product (the full version in our model) was a key condition for a Pareto-improvement in Deneckere-McAfee (1996) as well. As in their model without network effects, however, this criterion fails to hold for many specifications. A simple explanation for this is that in order to make at least as much profit with versioning as in the full version only case, the firm is likely to charge a higher price for the smaller writing base since the price of read-only version falls considerably. We have observed this logic in the free read-only case, in which the introduction of the read-only version always results in a higher price for the full version.

4.4.4 An illustrative example

In order to be able to have more explicit results based on model primitives, we now present a simplified model. Let the valuation for the reading function be proportional to the valuation for the writing function, i.e. $g(v) = kv$, where $k > 0$. On top of the two-dimensional approach we have applied so far, this setting allows us an alternative interpretation. Now consumers can be viewed heterogenous in their general (one-dimensional) valuation for network effects, and k reflects an intrinsic feature of the product market. The market can be called writing-oriented if $k < 1$, reading-oriented if $k > 1$, and balanced between the two if $k = 1$.[23] Furthermore, let v be uniformly distributed on $[0, 1]$.

Let us briefly summarize the possible cases.

The non-versioning case: This case occurs if and only if $g'(v_f) \geq 1$, i.e. $k \geq 1$ in this example. From condition (4.7), the optimal marginal type is $v_f = \frac{1}{3}$, and the equilibrium price is $p = \frac{2}{9}(1+k)$. The firm' profit is $\frac{4}{27}(1+k)$.

The free-read only case: The necessary and sufficient condition for this case to occur is that $\widetilde{v}_w f(0) - g'(0) \geq 0$, where $\widetilde{v}_w$ is implicitly defined by the equation $\widetilde{v}_w = \frac{1-F(\widetilde{v}_w)}{f(\widetilde{v}_w)}$. Solving the latter equation gives $\widetilde{v}_w = \frac{1}{2}$, and then the inequality holds if and only if $k \leq \frac{1}{2}$. From condition (4.11), the equilibrium price of the full version is $p_f = \frac{1}{2}$, and obviously $p_r = 0$. The firm' profit is $\frac{1}{4}$.

The standard versioning: The results of the above two cases immediately imply that this case occurs if and only if $\frac{1}{2} < k < 1$. Solving the system of equations (4.15) and (4.16) leads to $v_w = \frac{2}{3}\left(1 - \frac{k}{2}\right)$ and $v_r = \frac{2}{3}\left(1 - \frac{1}{2k}\right)$. Finally, from equation (4.11) and (4.12) the equilibrium prices are given by $p_f = \frac{2}{9k}(1+k)\left(k^2 - k + 1\right)$ and $p_r = \frac{1}{9}(1+k)(2k-1)$. Equilibrium profit is $\frac{1}{27k}(1+k)^3$.

Corollary 31 *The firm introduces the read-only version if and only if the market is writing-oriented* $(k < 1)$*, and*

i) sells it at a strictly positive price if the market is moderately writing-oriented $(1/2 < k < 1)$*, and*

ii) offers it free of charge if the market is highly writing-orientated $(k < 1/2)$*.*

Note that versioning always increases total installed base, since $(1 - v_w) + (1 - v_r) = \frac{1}{3k}(k+1)^2$, which always exceeds $2(1 - v_f) = \frac{4}{3}$ for $k \in (0, 1)$, so condition (4.19) is always satisfied.

The fact that the increase in the reading base is larger then the decrease in the writing base clearly shows the basic intuitions behind the functional degradation strategy. If the market is

[23] By writing-oriented we mean that consumer preferences are biased towards network benefits stemming from the writing function (i.e. they value network effects coming from the reading base more than those coming from the writing base).

reading-oriented, then all consumers derive a higher surplus from the presence of the writing consumers, and since this surplus is partly reaped by the monopolist, it does not want to decrease the installed base that brings more profit. If the market becomes more and more writing-oriented (i.e. it decreases from $k = 1$ towards $k = \frac{1}{2}$), then it becomes more profitable to increase the reading base, even though it decreases the writing base.[24] Finally, if the market reaches a sufficient degree of writing-orientation, then the monopolist decides to sacrifice any possible profits made on readers and builds the maximum possible reading base by giving away the read-only version free of charge, and targets the increased network benefits of full version buyers.

Welfare effects

Our first result implies that the monopolist's incentives for functional degradation perfectly coincide with those of a social welfare maximizer.

Proposition 32 *The monopolist introduces the read-only version if and only if this policy increases social welfare.*

Proof. The total welfare in the singe (full) version equilibrium is given by

$$W^F = \int_{v_f}^{1} (1 - v_f)(1 + k)vdv = \frac{8}{27}(1 + k)$$

and the welfare in the functional degradation equilibrium is

$$\begin{aligned} W^{FD} &= \int_{v_w}^{1} (1 - v_d)vdv + \int_{v_r}^{1} (1 - v_w)kvdv \\ &= \begin{cases} \frac{8}{27}(1+k) & \text{if } k \geq 1 \\ \frac{2}{27k}(1+k)^3 & \text{if } \frac{1}{2} < k < 1 \\ \frac{3+2k}{8} & \text{if } k \leq \frac{1}{2} \end{cases} . \end{aligned}$$

If we compare W^F and W^{FD}, we see that $W^{FD} = W^F$ for $k \geq 1$ and $W^{FD} > W^F$ for $k < 1$, which is exactly the necessary and sufficient condition for the profitable introduction of the

[24] It can be easily checked that for the values of $k = 1$ and $k = \frac{1}{2}$, the marginal types and prices converges to those of the non-versioning and of the free-read-only case, respectively.

read-only version. ■

However, consumers do not necessarily benefit from the introduction of the read-only version, since part of their increased surplus is reaped by the monopolist. We have argued in the previous Section that the more the market is writing-oriented, the larger the surplus extraction effect is, so this observation might hint that for low values of k consumers might be worse off.

Proposition 33 *The profitable introduction of the read-only version*

(i) makes consumers collectively better off only if $k > \frac{5}{22}$*, and*

(ii) is never Pareto-improving.

Proof. (i) The sum of consumer net surpluses in the single-version equilibrium is

$$CS^F = W^F - \Pi^F = \tfrac{4}{27}(1+k)$$

and the total consumer net surplus in the functional degradation equilibrium is

$$CS^{FD} = W^{FD} - \Pi^{FD} = \begin{cases} \frac{1}{27k}(1+k)^3 & \text{if } \frac{1}{2} < k < 1 \\ \frac{1+2k}{8} & \text{if } k \leq \frac{1}{2} \end{cases}.$$

By inspection it is immediate that $CS^{FD} > CS^F$ only if $k > \frac{5}{22}$.

(ii) Since total installed base increased, so Condition 4.19 is satisfied, we need to check only how the net surplus of type-v_f consumers (marginal writers) changes. By substituting equilibrium values, we have that

$$\begin{aligned} \Delta U(v_f) &= [k(1-v_w)v_w - p_r] - [(1+k)(1-v_f)v_f - p] \\ &= k(1-v_w)(v_w - v_r) - (1+k)(1-v_f)(v_w - v_f) \\ &= \begin{cases} -(1-k)^2(1+k) & \text{if } \frac{1}{2} < k < 1 \\ (5k-4) & \text{if } k \leq \frac{1}{2} \end{cases}, \end{aligned}$$

which is always strictly negative for these parameter values. Hence, the (profitable) introduction of the read-only version cannot be Pareto-improving by Proposition 27. ■

For the first sight, it might seem counterintuitive that the introduction of the read-only version decreases welfare in the regime with sufficiently low k where the monopoly gives away the degraded good for free and covers the whole market. However, notice that since consumers' relative valuation for the reading function is very small, the reading-based network effects created by versioning are not large enough to compensate for the loss of the writing-based network effects, although the expansion of the reading base is bigger then the shrinking of the writing base.

The result that in this simplified model versioning can never lead to a Pareto-improvement should not be interpreted as it can never happen. It is a well-known fact that with linear demand curves implied by uniform distribution of types third-degree discrimination cannot be welfare improving (see Schmalensee (1981) and references therein), but this negative result does not rule out that it might happen in more general environments.

4.5 Conclusion

This Chapter has examined a monopolist's incentives for introducing a functionally degraded version by removing some functions from its original product in a pure network effects based framework. The main focus of the analysis is on the interaction of function-specific asymmetric network effects and how it affects the profitability of the functional degradation strategy. Therefore, we offer a pure network effects based explanation for the functional degradation practice. It has been shown that the functional degradation strategy can be profitable to the firm if consumers value more the network effect associated with the function removed in the process of degradation (e.g. the writing function in the Acrobat example) than the network effect associated with the function contained only in the degraded version, and furthermore the firm may wish to offer the degraded version free of charge if the valuation difference is sufficiently large. The main contribution of our analysis to the existing literature is that we have shown that network effects alone can provide firms with sufficient incentives for introducing degraded goods. On the technical side, it has been shown that offering the read-only version for free is an effective way of overcoming the problem of multiple equilibria with network effects. The welfare effects of this type of product versioning turns out to be ambiguous, but conditions have been derived under which the introduction of the degraded version leads to a Pareto-improvement.

Extending the current framework in the following directions seems fruitful. First, introducing dynamics in the model would give some additional insights for the phenomenon. It will be interesting to incorporate the durable good nature of the product in the model with asymmetric network effects, in which case we need to take into account the timing of introducing degraded versions and even consumers' intertemporal upgrading behaviors (i.e. switching from the degraded version to the full version). Takeyama (1997, 2002), Inderst (2003) and Hahn (2005) have considered some of those issues in the standard quality differentiation or degradation framework, but they ignore any network effects. Also, firms constantly introduce new products, upgrading their original products, and this naturally raises the issues of forward and backward compatibility between old and new versions as well as price discrimination based on purchase history (e.g. upgrade discounts). Some recent studies on related topics shed light on how to tackle those issue in our setup (see, for example, Waldman (1993), Choi (1994), Fudenberg and Tirole (1998) and Ellison and Fudenberg (2000)).[25] Second, competitive aspects of the practice have not been discussed in our analysis. In particular, analyzing the damaged good phenomenon as an incumbent's preemptive action to deter potential entries seems warranted. Fudenberg and Tirole (2000) examine an incumbent's pricing behavior in a market with network effects, but they do not consider the possibility of using a second version as an entry-deterring device. Also, Jullien (2001) examines competition between a dominant and a challenging network in the presence of network effects.

[25] An interesting point in this line of research is that firms often make the newer version only backward compatible, degrading the network value of the old version and therefore forcing consumers to buy the new version, as formalized by Waldman (1993) and Choi (1994). Here the mechanism by which the firm copes with network effects, contracting the product line (i.e. making old versions obsolescent), is quite contrasted with the one considered in our analysis, extending the product line (i.e. introducing degraded goods).

Appendix A

Appendix for Chapter 2

A.1 Proof of Lemma 1

Since $f(x)$ has increasing marginal returns for all choice variables, it trivially satisfies the strict single-crossing property in (x, t). Therefore we can apply the Monotone Selection Theorem, so $t'' > t'$ implies $x^*(t'') \geq x^*(t')$ for every selection of maximizers. Since the maxima are interior, we have

$$\frac{\partial f(x^*(t'), t')}{\partial x_i} = 0$$

for all $i = 1, ..., n$, and since we have increasing marginal returns in any x_i and $t'' > t'$,

$$\frac{\partial f(x^*(t'), t'')}{\partial x_i} > \frac{\partial f(x^*(t'), t')}{\partial x_i} = 0.$$

Therefore $x^*(t')$ is not optimal given t'', so there exists at least one choice variable such that $x_i^*(t'') > x_i^*(t')$.

Now let us indirectly assume that $x^*(t'')$ is identical to $x^*(t')$ in $0 < k < n$ coordinates, and they differ in the remaining $l = n - k$ coordinates. We may assume without any loss of generality that the matching coordinates are the first k ones, so $x^*(t'') = (x_k'', x_l'')$ and $x^*(t') = (x_k', x_l')$, where $x_k'' = x_k'$, and $x_l'' > x_l'$. Now pin down the last l coordinates to x_l' and find $x_k^*(x_l, t) \in \underset{x_k \in R^k}{\arg\max} f(x_k, x_l, t)$. By definition $x_k^*(x_l', t'') = x_k'$, and let $x_k^*(x_l', t'')$ be denoted by $\overline{x}_k$. Since we have increasing marginal returns in all variables, we can replicate the arguments given in the first part of the proof, so $\overline{x}_k$ cannot be smaller than x_k', and differs from $\overline{x}_k$ in at least one coordinate. Based on these points, we can define $\overline{x} = (\overline{x}_k, x'_l)$ and $\overline{x}' = (\overline{x}_k, x''_l)$, and then $x^*(t'') \vee \overline{x} = \overline{x}'$ and $x^*(t'') \wedge \overline{x} = x^*(t'')$. Basically we are constructing a rectangle characterized by the points $x^*(t'), x^*(t''), \overline{x}_k$ and $\overline{x}'_k$.

By the definition of optimum, $f(\overline{x}, t'') > f(x^*(t'), t'')$, so $f(\overline{x}, t'') > f(x^*(t'') \wedge \overline{x}, t'')$. Since $f(x,t)$ is quasisupermodular in x on X for all $t \in T$, the former inequality implies

$$f(x^*(t''), t'') < f(x^*(t'') \vee \overline{x}, t'') = f(\overline{x}', t''),$$

so given t'', $\overline{x}'$ gives a higher value then $x^*(t'')$, which is a contradiction. This method can be applied for any positive k, so after at most $n-1$ steps we can conclude that $x^*(t'')$ should differ from $x^*(t')$ in all coordinates. *Q.E.D.*

A.2 Proof of Lemma 5

First, the participation constraint (P_i) will be automatically satisfied for all types $i \geq 2$ if the constraints (P_1) and (IC_{i1}) are satisfied, since

$$\theta_i V(q_i, \overline{q}) - t_i \geq \theta_i V(q_1, \overline{q}) - t_1 > \theta_1 V(q_1, \overline{q}) - t_1 \geq 0.$$

Second, by adding any two incentive constraints (IC_{ij}) and (IC_{jk}) such that $i > j > k$, we have

$$\theta_i V(q_i, \overline{q}) - t_i \geq (\theta_i - \theta_j) V(q_j, \overline{q}) + \theta_j V(q_k, \overline{q}) - t_k.$$

Rearranging the monotonicity constraint (2.20) gives $(\theta_i - \theta_j) V(q_j, \overline{q}) + \theta_j V(q_k, \overline{q}) \geq \theta_i V(q_k, \overline{q})$. Therefore

$$\theta_i V(q_i, \overline{q}) - t_i \geq \theta_i V(q_k, \overline{q}) - t_k,$$

so the incentive constraint (IC_{ik}) is satisfied. The same reasoning can be done for the case of $i < j < k$, thus the local incentive constraints (the ones involving adjacent types) imply the global incentive constraints.

Now suppose that (IC_{ij}) for $i < j$ will not be binding in equilibrium, so we ignore them for the moment and check later whether they will be satisfied. Then the remaining n constraints

constraints should be binding in equilibrium, implying the optimal tariff functions $(t_1) - (t_n)$. Finally, by substituting the corresponding tariff functions into the ignored upward local incentive constraints, we see that they are indeed satisfied if

$$\Delta\theta_{i-1}[V(q_i, \overline{q}) - V(q_{i-1}, \overline{q})] \geq 0, \text{ for } i = 2, ..., n,$$

which is fulfilled by the monotonicity constraint (2.20). *Q.E.D.*

A.3 Proof of Lemma 6

First, suppose that in a rational expectations equilibrium with expected network size $\overline{q}'$, θ_i-types choose c_k and θ_j-types choose c_l. Then the expected utilities to be realized are $U_i = \theta_i V(q_k, \overline{q}') - t_k$, and $U_j = \theta_j V(q_l, \overline{q}') - t_l$. However, if a θ_i-type considers to individually switch to c_l, she expects to realize $U_i' = \theta_i V(q_l, \overline{q}') - t_l$,while if a a θ_j-type considers to individually switch to c_k, she expects to realize $U_j' = \theta_j V(q_k, \overline{q}') - t_k$.If we sum the expected difference in the two consumers' utilities, we have

$$\left(U_i' + U_j'\right) - (U_i + U_j) = (\theta_j - \theta_i)\left[V(q_k, \overline{q}') - V(q_l, \overline{q}')\right].$$

This difference is positive by the monotonicity constraint and $\theta_i < \theta_j$, therefore at least one of the consumers expect a higher utility by individually switching and this outcome cannot be a self-fulfilling equilibrium.

Second, Lemma 5 proved that the downward local incentive constraints $(IC_{i(i-1)})$ are binding for all $i \in N$ (for $i = 1$ this is equivalent to constraint P_1):

$$\theta_i V(q_i, \overline{q}) - t_i = \theta_i V(q_{i-1}, \overline{q}) - t_{i-1}.$$

As $V_{12} > 0$ and $\overline{q}' < \overline{q}$, we have

$$\theta_i V(q_i, \overline{q}') - t_i < \theta_i V(q_j, \overline{q}') - t_j$$

for all $j < i$ (since the monotonicity constraint implies $q_j \leq q_i$ for for $j < i$), so all types will have an incentive to "jump down".

Third, (IC_{ij}) writes as

$$\theta_i V(q_i, \overline{q}) - t_i \geq \theta_i V(q_j, \overline{q}) - t_j.$$

If $\overline{q}' > \overline{q}$, then by $V_{12} > 0$ we should have

$$\theta_i V(q_i, \overline{q}') - t_i > \theta_i V(q_j, \overline{q}') - t_j$$

for all $j < i$, as in this interval the monotonicity constraint implies $q_i \geq q_j$, so no type has an incentive to "jump down". *Q.E.D.*

Appendix B

Appendix for Chapter 3

B.1 Proof of Lemma 11

The first result is immediate by

$$\theta_2 V(q_2, \overline{q}) - t_2 \geq \theta_2 V(q_1, \overline{q}_1) - t_1 > \theta_1 V(q_1, \overline{q}_1) - t_1 \geq 0.$$

Second, if we write up the constrained maximization problem (ignoring for the moment the constraints $s_1, s_2 \in [0,1]$) by attaching multipliers λ_1, μ_1 and μ_2 for the constraints (3.1), (3.3) and (3.4), respectively, the Lagrangian is

$$\begin{aligned} L &= pt_1 + (1-p)t_2 - c\overline{q} - \lambda_1 \left[t_1 - \theta_1 V(q_1, \overline{q})\right] - \\ &\quad -\mu_1 \left[t_1 - t_2 + \theta_1 V(q_2, \overline{q}_2) - \theta_1 V(q_1, \overline{q}_1)\right] - \\ &\quad -\mu_2 \left[t_2 - t_1 + \theta_2 V(q_1, \overline{q}_1) - \theta_2 V(q_2, \overline{q}_2)\right]. \end{aligned}$$

The first-order conditions in respect of tariffs are

$$\begin{aligned} [dt_1] &: \quad p - \lambda_1 - \mu_1 + \mu_2 = 0, \\ [dt_2] &: \quad (1-p) + \mu_1 - \mu_2 = 0. \end{aligned}$$

Since all the multipliers are non-negative and $p \in (0,1)$, the second condition implies $\mu_2 > 0$, so (3.4) should be binding. Adding the two equations yields $\lambda_1 = 1 > 0$, so (3.1) is binding as well. *Q.E.D.*

B.2 Proof of Lemma 12

By using $[IR_{1x}]$ and $[IC_{2x}]$, we immediately got that $[IR_{2x}]$ is slack for positive q_{1x}-s:

$$\theta_2 V(q_{2x}, \overline{q}_x) - t_{2x} \geq \theta_2 V(q_{1x}, \overline{q}_{1x}) - t_{1x} > \theta_1 V(q_{1x}, \overline{q}_{1x}) - t_{1x} \geq 0.$$

Second, we can maximize Π pointwise in respect of tariffs. By attaching multipliers λ_{1x}, μ_{1x} and μ_{2x} for the constraints $[IR_{1x}]$, $[IC_{1x}]$ and $[IC_{2x}]$, respectively, the Lagrangian for x is

$$\begin{aligned} L_x &= p[t_{1x} - cq_{1x}] + (1-p)[t_{2x} - cq_{2x}] - \lambda_{1x}\left[t_1 - \theta_1 V(q_1, \overline{q})\right] - \\ &\quad - \mu_{1x}\left[t_{1x} - t_{2x} + \theta_2 V(q_{2x}, \overline{q}_x) - \theta_1 V(q_{1x}, \overline{q}_x)\right] - \\ &\quad - \mu_{2x}\left[t_2 - t_1 + \theta_2 V(q_{1x}, \overline{q}_x) - \theta_2 V(q_{2x}, \overline{q}_x\right]. \end{aligned}$$

The first-order conditions in respect of tariffs are

$$\begin{aligned} [dt_{1x}] &: \; p - \lambda_{1x} - \mu_{1x} + \mu_{2x} = 0, \\ [dt_{2x}] &: \; (1-p) + \mu_{1x} - \mu_{2x} = 0. \end{aligned}$$

Since all the multipliers are non-negative and $p \in (0,1)$, the second condition implies $\mu_{2x} > 0$, so $[IC_{2x}]$ should be binding. Adding the two equations yields $\lambda_{1x} = 1 > 0$, so $[IR_{1x}]$ is binding as well. *Q.E.D.*

B.3 The constrained optimum case in general

Now let us develop the full mechanism. By attaching Kuhn-Tucker multiplier μ_x to the $[IC_{1x}]$ constraints, the Lagrangian writes as

$$
\begin{aligned}
L &= \int_0^1 p[\theta_1' V(q_{1s},\overline{q}_s^p) - cq_{1s}] + (1-p)[\theta_2 V(q_{2s},\overline{q}_s^p) - cq_{2s}]ds + \\
&+ \int_0^1 \mu_s \left\{\theta_2[V(q_{2s},\overline{q}_s^p) - V(q_{1s},\overline{q}_s^p) - \theta_1[V(q_{2s},\overline{q}_s^o) - V(q_{1s},\overline{q}_s^o)]\right\} ds.
\end{aligned}
$$

The first-order condition in respect of q_{1x} is the following:

$$
\begin{gathered}
\theta_1' V_1(q_{1x},\overline{q}_x^p) + \int_0^1 \left\{ \left[p\theta_1' V_2(q_{1s},\overline{q}_s^p) + (1-p)\theta_2 V_2(q_{2s},\overline{q}_s^p)\right] \frac{d\overline{q}_s^p/dq_{1x}}{p} \right\} ds+ \\
+\frac{\mu_x}{p}\left[-\theta_2 V_1(q_{1x},\overline{q}_x^p) + \theta_1 V_1(q_{1x},\overline{q}_x^o)\right] + \\
+\int_0^1 \left\{ \mu_s\theta_2 \left[V_2(q_{2s},\overline{q}_s^p) - V(q_{1s},\overline{q}_s^p)\right] \frac{d\overline{q}_s^p/dq_{1x}}{p} \right\} ds- \\
-\int_0^1 \left\{ \mu_s\theta_1 \left[V_2(q_{2s},\overline{q}_s^o) - V_2(q_{1s},\overline{q}_s^o)\right] \frac{d\overline{q}_s^o/dq_{1x}}{p} \right\} ds = c,
\end{gathered}
$$

while in respect of q_{2x} it is

$$
\begin{gathered}
\theta_2 V_1(q_{2x},\overline{q}_x^p) + \int_0^1 \left\{ \left[p\theta_1' V_2(q_{1s},\overline{q}_s^p) + (1-p)\theta_2 V_2(q_{2s},\overline{q}_s^p)\right] \frac{d\overline{q}_s^p/dq_{2x}}{1-p} \right\} ds+ \\
+\frac{\mu_x}{1-p}\left[\theta_2 V_1(q_{2x},\overline{q}_x^p) - \theta_1 V_1(q_{2x},\overline{q}_x^o)\right] + \\
+\int_0^1 \mu_s\theta_2 \left[V_2(q_{2s},\overline{q}_s^p) - V(q_{1s},\overline{q}_s^p)\right] \frac{d\overline{q}_s^p/dq_{2x}}{1-p} ds- \\
-\int_0^1 \mu_s\theta_1 \left[V_2(q_{2s},\overline{q}_s^o) - V_2(q_{1s},\overline{q}_s^o)\right] \frac{d\overline{q}_s^o/dq_{2x}}{1-p} ds = c.
\end{gathered}
$$

By Condition 14 the network effects are the same in both first-order conditions, so combining them gives

$$
\begin{gathered}
\theta_1' V_1(q_{1x},\overline{q}_x^p) = \theta_2 V_1(q_{2x},\overline{q}_x^p) + \\
+\mu_x \left\{ \theta_2 \left[\frac{V_1(q_{2x},\overline{q}_x^p)}{1-p} + \frac{V_1(q_{1x},\overline{q}_x^p)}{p} \right] - \theta_1 \left[\frac{V_1(q_{2x},\overline{q}_x^o)}{1-p} + \frac{V_1(q_{1x},\overline{q}_x^o)}{p} \right] \right\}.
\end{gathered} \tag{B.1}
$$

We can rearrange for the Kuhn-Tucker multiplier:

$$\mu_x = \frac{-\left[\theta_2 V_1(q_{2x}, \overline{q}_x^p) - \theta_1' V_1(q_{1x}, \overline{q}_x^p)\right]}{\theta_2\left[\frac{V_1(q_{2x},\overline{q}_x^p)}{1-p} + \frac{V_1(q_{1x},\overline{q}_x^p)}{p}\right] - \theta_1\left[\frac{V_1(q_{2x},\overline{q}_x^o)}{1-p} + \frac{V_1(q_{1x},\overline{q}_x^o)}{p}\right]}.$$

We can now better formalize our preliminary observations:

1. If $[IC_{1x}]$ is slack then we have separating, since $\mu_x = 0$ implies $q_{2x} > q_{1x}$.

2. Pooling implies a binding $[IC_{1x}]$ with $\mu_x > 0$, since $q_{2x} = q_{1x}$ and $\mu_x = 0$ is not possible by the previous point.

3. A screening regime necessarily exists, because $[IC_{1x}]$ holds with a strict inequality at $x = 1$ and therefore $q_{21} > q_{11}$. Additionally, we have separating in a small environment of $x = 1$, if the quantity schemes are continuous functions of x.

We now provide conditions for the monotonicity of the constraint $[IC_{1x}]$, which is a sufficient condition for our main result in Proposition 15 to hold.[1] If we differentiate

$$[IC_{1x}] : \theta_2\left[V(q_{2x}, \overline{q}_x^p) - V(q_{1x}, \overline{q}_x^p)\right] - \theta_1\left[V(q_{2x}, \overline{q}_x^o) - V(q_{1x}, \overline{q}_x^o)\right] \geq 0.$$

in x, we get

$$\begin{aligned}&\theta_2\left[V_1(q_{2x}, \overline{q}_x^p) - V_1(q_{1x}, \overline{q}_x^p)\right] - \theta_1\left[V_1(q_{2x}, \overline{q}_x^o) - V_1(q_{1x}, \overline{q}_x^o)\right] + \\ &\theta_2\left[V_2(q_{2x}, \overline{q}_x^p) - V_2(q_{1x}, \overline{q}_x^p)\right]\frac{d\overline{q}_x^p}{dx} - \theta_1\left[V_2(q_{2x}, \overline{q}_x^o) - V_2(q_{1x}, \overline{q}_x^o)\right]\frac{d\overline{q}_x^o}{dx}\end{aligned}$$

A possible set of sufficient conditions for the positivity of this expression are $V_{112} \leq 0$, that is strategic complementarity is decreasing in individual consumption, and $\frac{d\overline{q}_x^o}{dx} = 0$. Then the difference of differences in the first line is positive by the first assumption and, the first term in the second line is positive by $V_{12} > 0$ and $\frac{d\overline{q}_x^p}{dx} \geq 0$, while the last term is zero.

[1] An alternative way could be to look for conditions for the monotonicity of the Kuhn-Tucker multiplier $\mu(x)$, but this approach leads to more technical diffulties.

Appendix C

Appendix for Chapter 4

C.1 Proof of Lemma 20

Consider first consumers with a type $v > v_w$. If they prefer to purchase the full version, then the following inequalities should be satisfied:

$$(n_f^e + n_r^e)v + n_f^e g(v) - p_w \geq n_f^e g(v) - p_r, \text{ and} \tag{C.1}$$

$$(n_f^e + n_r^e)v + n_f^e g(v) - p_w \geq 0. \tag{C.2}$$

Substituting equation (4.8) in condition (C.1) and equation (4.10) in condition (C.4) give

$$\begin{aligned}(n_f^e + n_r^e)(v - v_w) &\geq 0, \text{ and}\\ (n_f^e + n_r^e)(v - v_w) + n_f^e(g(v) - g(v_w)) &\geq 0,\end{aligned}$$

which are clearly satisfied by the monotonicity of $g(\cdot)$, so they all purchase the full version.

Consider now consumers with a type $v \in (v_r, v_w)$. They prefer to buy the read-only version if

$$n_f^e g(v) - p_r \geq (n_f^e + n_r^e)v + n_f^e g(v) - p_w, \text{ and} \tag{C.3}$$

$$n_f^e g(v) - p_r \geq 0. \tag{C.4}$$

Substituting equation (4.8) in condition (C.3) and equation (4.9) in condition (C.4) give

$$\begin{aligned}(n_f^e + n_r^e)(v_w - v) &> 0, \text{ and} \\ n_f^e(g(v) - g(v_r)) &\geq 0,\end{aligned}$$

which are both satisfied, so they all purchase the read-only version.

Last, consider consumers with a type $v < v_r$. By purchasing the full version they would derive a utility of $(n_f^e + n_r^e)(v - v_w) + n_f^e(g(v) - g(v_r))$, or by buying the read-only version their utility would be $n_f^e(g(v) - g(v_r))$, which are both negative, so they do not purchase at all. *Q.E.D.*

C.2 The strict quasi-concavity of $\Pi(v_w, v_r)$

We will prove the strict quasi-concavity of the function for all the pairs $(v_w, v_r) \in [0,1) \times [0,1)$, where $\nabla\Pi(v_w, v_r) \neq 0$. Condition $\nabla\Pi(v_w, v_r) = 0$ is satisfied only in the standard versioning optimum, and for this case we will check the second-order condition in the next Appendix. Since it will be satisfied, the function is strictly concave in that point, which implies strict quasi-concavity.

The function $\Pi(v_w, v_r) = [1 - F(v_w)][1 - F(v_r)][v_w + g(v_r)]$ is strictly quasi-concave if the bordered Hessian has a positive determinant, that is

$$D = 2\frac{\partial\Pi(v_w, v_r)}{\partial v_w}\frac{\partial\Pi(v_w, v_r)}{\partial v_r}\frac{\partial^2\Pi(v_w, v_r)}{\partial v_w \partial v_r} - [\frac{\partial\Pi(v_w, v_r)}{\partial v_w}]^2\frac{\partial^2\Pi(v_w, v_r)}{\partial v_r^2} - [\frac{\partial\Pi(v_w, v_r)}{\partial v_r}]^2\frac{\partial^2\Pi(v_w, v_r)}{\partial v_w^2} > 0.$$

In order to simplify notations, let us introduce two new variables, D_{v_w} and D_{v_r}, in the following way:

$$\frac{\partial\Pi(v_w, v_r)}{\partial v_w} = [1 - F(v_r)][1 - F(v_w) - f(v_w)(v_w + g(v_r))] = [1 - F(v_r)]D_{v_w}$$

and

$$\begin{aligned}
\frac{\partial \Pi(v_w, v_r)}{\partial v_r} &= [1 - F(v_w)][(1 - F(v_r))g'(v_r) - f(v_r)(v_w + g(v_r))] \\
&= [1 - F(v_w)]D_{v_r}.
\end{aligned}$$

Note that $\nabla \Pi(v_w, v_r) = 0$ only if $D_{v_w} = D_{v_r} = 0$, since the other term is always positive. Using these new variables, we can write the second derivatives as follows:

$$\begin{aligned}
\frac{\partial^2 \Pi(v_w, v_r)}{\partial v_w \partial v_r} &= -f(v_r)[1 - F(v_w)] - f(v_w)D_{v_r} \\
&= -f(v_w)[1 - F(v_r)]g'(v_r) - f(v_r)D_{v_w}, \\
\frac{\partial^2 \Pi(v_w, v_r)}{\partial v_w^2} &= [1 - F(v_r)][-2f(v_w) - f'(v_w)(v_w + g(v_r))], \\
\frac{\partial^2 \Pi(v_w, v_r)}{\partial v_r^2} &= [1 - F(v_w)][(1 - F(v_r))g''(v_r) - 2f(v_r)g'(v_r) - \\
&\quad - f'(v_r)(v_w + g(v_r))].
\end{aligned}$$

Expanding D leads to

$$\begin{aligned}
D &= [1 - F(v_w)][1 - F(v_r)]D_{v_w}D_{v_r}[-f(v_r)D_{v_w} - f(v_w)D_{v_r} - \\
&\quad -2f(v_w)f(v_r)(v_w + g(v_r))] - \\
&\quad -[1 - F(v_w)]^2[1 - F(v_r)]D_{v_r}^2[-2f(v_w) - \underbrace{f'(v_w)(v_w + g(v_r))}] - \\
&\quad -[1 - F(v_r)]^2[1 - F(v_w)]D_{v_w}^2[\underbrace{(1 - F(v_r))g''(v_r)} - \\
&\quad -2f(v_r)g'(v_r) - \underbrace{f'(v_r)(v_w + g(v_r))}].
\end{aligned}$$

Since $2[1 - F(v_w)][1 - F(v_r)]$ is positive, simplifying using this term does not change the sign of D. Additionally, the highlighted terms (with their respective signs) are all positive since $f'(v) > 0$ and $g''(v) < 0$). So, omitting these terms simplifies the expression. Then, we are left with

$$\begin{aligned}
D' &= -D_{v_w}D_{v_r}f(v_w)f(v_r)(v_w+g(v_r)) + D_{v_r}^2 f(v_w)[(1-F(v_w)) - \frac{D_{v_w}}{2})] + \\
&\quad + D_{v_w}^2 f(v_r)[(1-F(v_r))g'(v_r) - \frac{D_{v_r}}{2})] \\
&\geq -D_{v_w}D_{v_r}f(v_w)f(v_r)(v_w+g(v_r)) + D_{v_r}^2[f(v_w)]^2(v_w+g(v_r)) + \\
&\quad + D_{v_w}^2[f(v_r)]^2(v_w+g(v_r)) \\
&\geq (v_w+g(v_r))[D_{v_w}f(v_r) - D_{v_r}f(v_w)]^2 \geq 0.
\end{aligned}$$

Since $D > D'$, D must be positive. *Q.E.D.*

C.3 Second order conditions for the standard versioning case

We need to check whether the Hessian of the objective function $\Pi(v_w, v_r)$ is negative definite under the first-order conditions (4.15) and (4.16). Since $\frac{\partial^2\Pi(v_w,v_r)}{\partial v_w^2} = [1-F(v_r)][-2f(v_w) - f'(v_w)(v_w+g(v_r))]$ is always negative, all we have to do is to check if the determinant of the Hessian is positive. i.e.

$$D = \frac{\partial^2\Pi(v_w,v_r)}{\partial v_w^2}\frac{\partial^2\Pi(v_w,v_r)}{\partial v_r^2} - [\frac{\partial^2\Pi(v_w,v_r)}{\partial v_w \partial v_r}]^2 > 0.$$

Using the results of the previous proof, and the fact that in this case $D_{v_w} = D_{v_r} = 0$, we have

$$[\frac{\partial^2\Pi(v_w,v_r)}{\partial v_w \partial v_r}]^2 = [1-F(v_w)][1-F(v_r)]f(v_w)f(v_r)g'(v_r)$$

and

$$\begin{aligned}
\frac{\partial^2\Pi(v_w,v_r)}{\partial v_w^2}\frac{\partial^2\Pi(v_w,v_r)}{\partial v_r^2} &= [1-F(v_w)][1-F(v_r)] * [-2f(v_w) - \\
&\quad - \underbrace{f'(v_w)(v_w+g(v_r))}][\underbrace{(1-F(v_r))g''(v_r)} - \\
&\quad -2f(v_r)g'(v_r) - \underbrace{f'(v_r)(v_w+g(v_r))}] \\
&> 4[1-F(v_w)][1-F(v_r)]f(v_w)f(v_r)g'(v_r)
\end{aligned}$$

since all the highlighted terms are negative. So, we have $D > 0$. *Q.E.D.*

C.4 Proof of Proposition 27

The firm introduces the read-only if and only if it gets strictly better off. If $v_r \neq 0$, consumers of type in $[0, v_r)$ do not purchase at all in either case, so their utility is not affected by the versioning. Consumers of type in $[v_r, v_f)$, who would not purchase without versioning, now buy the read-only version and derive a non-negative utility. Consumers of type in $[v_f, v_w)$, who would purchase the full version without versioning, now buy the read-only version under versioning. The change in their utility can be expressed by

$$\Delta U_r(v) = [(1 - F(v_w))g(v) - p_r] - [(1 - F(v_f))(v + g(v)) - p],$$

and it is decreasing in v:

$$\begin{aligned}
\frac{d\Delta U_r(v)}{dv} &= (1 - F(v_w))g'(v) - (1 - F(v_f))(1 + g'(v)) \\
&= -(1 - F(v_f)) + (F(v_f) - F(v_w))g'(v) < 0
\end{aligned}$$

given that the first term is negative, $F(v_f) - F(v_w)$ is negative, and $g'(v) > 0$. So, if $\Delta U_r(v_w) \geq 0$, then it must be that $\Delta U_r(v) > 0$ for all $v \in [v_f, v_w)$.

Consumers of type $[v_w, 1]$ end up buying the full version in both cases. The change in their utility is

$$\begin{aligned}
\Delta U_w(v) &= [(1 - F(v_r))v + (1 - F(v_w))g(v) - p_f] - \\
&\quad -[(1 - F(v_f))(v + g(v)) - p].
\end{aligned}$$

Note that since v_w is the type of consumers who are indifferent between the full and the read-only version, it must be that $\Delta U_w(v_w) = \Delta U_r(v_w)$. We now prove that $\Delta U_w(v)$ increases in v:

$$\begin{aligned}
\frac{d\Delta U_w(v)}{dv} &= 1 - F(v_r) + (1 - F(v_w))g'(v) - (1 - F(v_f))(1 + g'(v)) \\
&= (1 - F(v_r)) - (1 - F(v_f)) + [(1 - F(v_w)) - (1 - F(v_f))]g'(v) \\
&> (1 - F(v_w)) + (1 - F(v_r)) - 2(1 - F(v_f)) > 0,
\end{aligned}$$

where the first inequality follows from the fact that $(1-F(v_w))-(1-F(v_f))<0$ and $g'(v)<1$ for all $v \in [v_w, 1]$, and the second inequality from condition (4.19). So, if $\Delta U_w(v_w) > 0$, then it must be that $\Delta U_w(v) > 0$ for all $v \in [v_w, 1]$. *Q.E.D.*

C.5 Proof of Lemma 29

Expanding $\Delta U_w(v_w)$ leads to

$$\begin{aligned}
\Delta U_w(v_w) &= [(1-F(v_r))v_w + (1-F(v_w))g(v_w) - p_f] - \\
&\quad -[(1-F(v_f))(v_w + g(v_w)) - p] \\
&\geq [(1-F(v_r)) - (1-F(v_f))]v_w - \\
&\quad -[(1-F(v_f)) - (1-F(v_w)]g(v_w) \\
&> (1-F(v_w)) + (1-F(v_r)) - 2(1-F(v_f)) > 0,
\end{aligned}$$

where the first inequality follows from $p_f \leq p$, the second from $g(v_w) < 1$ and $(1-F(v_f)) - (1-F(v_w) > 0$, and the third from condition (4.19). *Q.E.D.*

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