Unemployment Insurance Reform: the Origin of the German Labor Market Miracle

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Abstract

This paper develops a search and matching model with multi-worker firms and a two-tier unemployment insurance system to explore the role of the 2005 unemployment insurance reform (the Hartz IV reform) in reducing the cyclical volatility of German employment. Lower long-term unemployment benefits reduce firms' incentives to cut employment during downturns, and render adjustment along the intensive margin relatively more important. Calibrating my model to German pre-reform data, I find that the reform reduced the volatility of employment by 68% and was the main reason behind the mild response of the German labor market to the Great Recession. A short-time work policy, praised as the key to the German "miracle," played a minor role. I also find that the reform raised an average worker's welfare by 1.18%.

Keywords: German labor market reform, unemployment benefits, search and matching, employment volatility

JEL classification: E24, E32, J64, J65

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1 Introduction

Germany's jobs miracle hasn't received much attention in [the U.S.] — but it's real, it's striking. Krugman (2009)

In 2005, the German government restructured its unemployment insurance system by considerably reducing unemployment benefits for the long-term unemployed. This reform was the final step in a series of structural labor market reforms implemented in 2003-2005, known as the Hartz I-IV reforms, that aimed to reduce unemployment, increase job-search activities, and provide incentives for the unemployed to obtain jobs. Since 2005, the unemployment rate has fallen consistently, except for a slight increase during the Great Recession. The fact that the increase in the unemployment rate was the smallest among OECD countries — even though the fall in Germany's GDP was even deeper than that of the U.S. — made many observers speak of a German labor market "miracle."

There is ongoing debate regarding the origin of this German labor market miracle. Some emphasize the crucial role played by the mechanisms of labor adjustment along the intensive margin, such as a short-time work policy. As Figure 1 shows, the volatility of employment relative to the volatility of GDP in Germany fell after 2005 — four years before the Great Recession — compared to the U.S., where it stayed about the same. In this paper, I argue that the Hartz IV reform reduced firms' incentives to cut employment during downturns and was the main reason behind the mild response of the German labor market to the Great Recession.

Using aggregate German labor market data, I document that the volatility of employment relative to output fell by two-thirds after 2005, while the relative volatility of hours per worker increased by 10%. Prior to 2005, hours per worker were half as volatile as employment; after, they became almost twice as volatile.

To assess the quantitative implications of Germany's unemployment insurance reform on labor market volatility, I build a search and matching model with multi-worker firms and a two-tier unemployment insurance system. Firms face productivity shocks, search for new workers by posting vacancies, bargain with each of its workers over the hourly wage, and choose hours worked per worker. They can change hours per worker instantaneously, but must incur some adjustment costs in order to do so. The size of the match surplus plays a key role in generating a positive relationship between the level of unemployment benefits and employment volatility; lower long-term unemployment benefits imply a larger match surplus. Incentives to post vacancies depend on the size of percentage changes in the match surplus in response to changes in productivity. A larger surplus means that these percentage changes are smaller, as are the volatilities of vacancies and employment. Since the number of workers and the number of hours worked are substitutes in the production of goods, firms start relying more on the intensive margin of adjustment.

The introduction of multi-worker firms is motivated by the fact that in a standard one-worker one-firm setting the firm's choice of hours is independent of the search and matching frictions. As a result, changes in unemployment benefits have no effect on the volatility of hours per worker. As long as hours per worker do depend on the labor market tightness — which happens in a multi-worker firm environment with production function exhibiting decreasing returns to scale in the number of workers — the volatility of hours per worker is decreasing in the level of unemployment benefits.

I calibrate my model to German pre-reform data and find that the reform reduced the volatility of employment by 68%, while the volatility of hours per worker increased by 9%. After the reform, hours per worker became about 1.94 times as volatile as employment, which is close to what is observed in the data.

To examine the role of the short-time work policy in safeguarding jobs during the Great Recession, I introduce a government subsidy that lowers the cost of reducing working hours during times when the economy is in recession. In Germany, these costs include social security contributions on the worker's lost hours. Between 2009 and 2011, firms that participated in the short-time work program were required to pay 50% of those contributions and 0% after the first six months. I find that the subsidy reduces employment volatility further, by an additional 0.2 percentage points. This suggests that the short-time work policy played a minor role in protecting jobs during the Great Recession.

Using the calibrated model, I construct a sequence of shocks that match the observed dynamics of output during the Great Recession, and find that in the post-reform environment with the short-time work subsidy the unemployment rate increased by 0.5 percentage points, from 5.72% to 6.22%. I perform a counterfactual exercise and find that the unemployment rate would have increased by 1.6 percentage points, from 8.4% to 10%, if long-term unemployment benefits stayed at the pre-reform level. Without the reform, the increase in unemployment would have been three times higher in absolute terms and 9% higher in relative terms.

Finally, I analyze the impact of the unemployment insurance reform on workers' welfare and find that the reform made every type of workers better off. Welfare gains range from

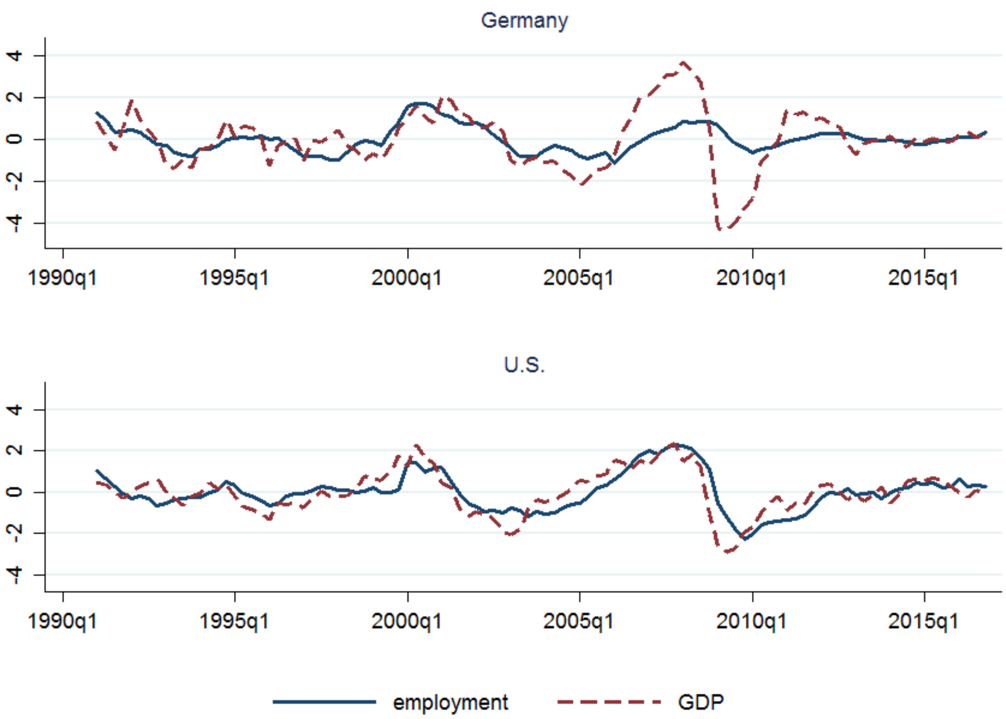

Figure 1: Employment and real GDP, quarterly data, seasonally adjusted, log deviation from an HP trend. Source: Eurostat and OECD.

0.47% for long-term unemployed workers to 1.21% for currently employed workers; the welfare of an average worker went up by 1.18%.

This paper is related to several strands of literature. From a theoretical perspective, I build on the search and matching model of Mortensen and Pissarides (1994). Following Andolfatto (1996), Krause and Lubik (2010), Kudoh et al. (2016), and Cacciatore et al. (2016), I introduce multi-worker firms and allow them to adjust labor along both extensive and intensive margins. My model environment is similar to Andolfatto (1996), but with a constant capital stock that does not depreciate. I follow Fredriksson and Holmlund (2001) and introduce a two-tier unemployment insurance system, in which short-term unemployment benefits expire with some exogenous probability.

My paper is related to the literature that studies the impact of labor market institutions on business cycle fluctuations. Veracierto (2008) analyzes the impact of firing costs on cyclical fluctuations in a real business cycle model and finds that lowering firing taxes increases the volatility of employment and output. Zanetti (2011) shows that in a model with labor market frictions and nominal rigidities, increasing firing costs and lowering unemployment benefits could lead to lower volatility of employment and job flows, but higher inflation volatility. Cacciatore and Fiori (2016) introduce endogenous product creation and labor market frictions into a real business cycle model and find that joint deregulation of product and labor markets, in terms of reducing entry costs, relaxing firing restrictions and lowering unemployment benefits, reduces aggregate volatility, which leads to a sizable reduction in the welfare costs of business cycles. However, none of these studies looks at the effects of actual labor market reforms. In contrast, I quantify the effects of the Hartz IV reform on labor market volatility based on pre- and post-reform data.

I also contribute to the literature on the macroeconomic effects of the German labor market reforms of 2003-2005. Using calibrated macro models, Krause and Uhlig (2012) and Krebs and Scheffel (2013) find that the Hartz IV reform substantially reduced the long-run equilibrium unemployment rate, while Launov and Wälde (2013), using German microdata, estimate that the effect is close to zero. However, studies of the business cycle implications of Hartz reforms are rare. My paper is most closely related to Krebs and Scheffel (2017) and Gehrke et al. (2017).

Krebs and Scheffel (2017) study the effect of Hartz reforms on the output cost of recessions. They find that a reduction in unemployment benefits increases job-finding rates at all stages of the business cycle, which renders unemployment less volatile and reduces output losses during downturns. Although it is true that lower unemployment benefits lead to more vacancies being posted — which increases the level of job-finding rates — I show in this paper that the reason unemployment becomes less responsive to business cycle shocks is that the firm has fewer incentives to adjust employment over the business cycle. Thus, the volatility of vacancies, and consequently of job-finding rates and unemployment, goes down.

Gehrke et al. (2017) study Germany's labor market dynamics during the Great Recession and analyze the role of different shocks and institutions. They build a stochastic general equilibrium model with a search and matching labor market with endogenous separations and the possibility of firms' use of short-time work. They do not model unemployment insurance reform; instead, they introduce matching efficiency shocks, which they estimate from the data (together with other structural shocks), and find that positive matching efficiency shocks (likely caused by labor market reforms) were the underlying source of the unusual labor market dynamics. In this paper, I model the unemployment insurance reform explicitly. The fall in the level of unemployment and the volatility of unemployment are generated endogenously and result from firm's optimal behavior in response to the reduction in unemployment benefits.

In contrast to both Krebs and Scheffel (2017) and Gehrke et al. (2017), my model features a multi-worker firm that can adjust its labor input along both extensive and intensive margins. The data suggest that after the reform, German firms began to rely more on the intensive margin and my model can account for that.

This paper also contributes to the debate on the origin of the German labor market miracle. Short-time work subsidies are believed to have played an important role (Hijzen and Venn (2011), Brenke et al. (2011)). Cooper et al. (2017) study the employment effect of short-time work policy in a search model with heterogeneous multi-worker firms, and find that without short-time work. the miracle would disappear. They do not analyze the role played by the Hartz IV reform, arguing that it had no direct impact on firms' decisions to adjust hours. However, the data show that German firms began relying more on the intensive margin of adjustment after the reform but before the Great Recession hit and short-time work subsidies were extended. I show that quantitatively, the impact of short-time work subsidies is small.

The remainder of the paper is organized as follows. Section 2 describes the Hartz IV reform, presents evidence on changes in labor market dynamics after the reform, and provides an overview of the short-time work policy. Section 3 sets up the model and defines the equilibrium. Section 4 describes the calibration and discusses the findings. In Section 5, I conduct some sensitivity analysis to assess the robustness of my results. Section 6 concludes.

2 Empirical Evidence

In Section 2.1, I provide a brief overview of the Hartz IV reform. Section 2.2 describes the aggregate dynamics of the German labor market before and after the reform, and Section 2.3 gives an overview of the short-time work program.

2.1 The Hartz IV Reform

Between 2003 and 2005, the German government implemented extensive labor market reforms, known as the Hartz reforms. The first three (Hartz I-III) were aimed at improving job-search efficiency and employment flexibility. They included deregulation of the temporary work sector, improved job search assistance, and stronger incentives for the unemployed

Figure 2: Average net replacement rates, % of previous earnings of a single-person household earning the average income. Short-term refers to the initial phase of receiving benefits. Long-term refers to the 60th month of receiving benefits. Source: OECD.

to accept a job. The Hartz IV reform, implemented on January 1, 2005, constituted a major restructuring of the unemployment insurance system that significantly reduced the size and duration of unemployment benefits.

Before the reform, the German unemployment insurance system consisted of three layers[1]. Workers who had accumulated a sufficient number of working years prior to unemployment were eligible for unemployment benefits (UB) equal to 60% of previous net earnings (67% for parents with dependent children). For workers younger than 45, the benefit was limited to 12 months; older workers were eligible for up to 32 months. After UB were exhausted, and if the worker was still unemployed, she was eligible for unemployment assistance (UA) equal to 53% of previous net earnings (57% for parents with dependent children). UA could be claimed indefinitely, subject to a means test and an annual review. Those who did not qualify for UB or UA were eligible for social assistance (SA) — a means-tested lump-sum transfer that provided the least generous support.

The reform collapsed this system into two layers. The first layer, unemployment benefits I (UB I), was essentially UB relabeled. The main change was the introduction of unem-

[1] Source: Engbom et al. (2015)

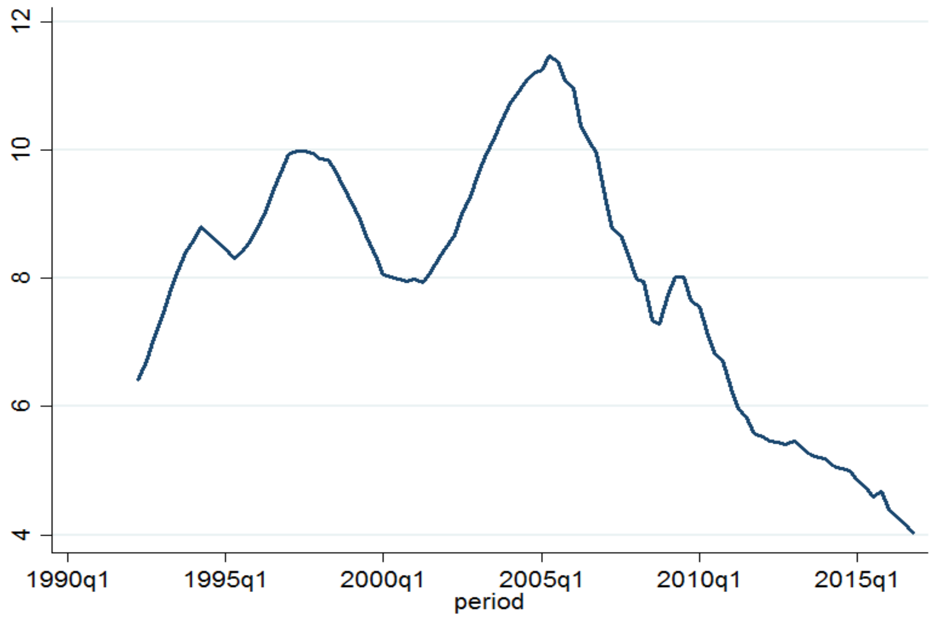

Figure 3: Unemployment rate, calculated as the ratio of total unemployment to active population, quarterly data, seasonally adjusted. Source: Eurostat.

ployment benefits II (UB II), which replaced UA and SA. Under the new system, workers who had exhausted their short-term benefit UB I were eligible for a means-tested lump-sum benefit that paid an amount similar to the old SA[2].

The effect of the Hartz IV reform can be seen in Figure 2, which shows the average net replacement rates for a single-person household (see Section 4.1 for details). The net replacement rate corresponds to the proportion of net income in work that is maintained after job loss. Clearly, the reform had almost no effect on short-term unemployed households, while the net replacement rate of long-term unemployed fell drastically.

2.2 Labor Market Dynamics

Figure 3 shows the unemployment rate in Germany after reunification. Prior to 2005, the unemployment rate had been on an upward trend, reaching 11% in 2005. Since 2005, the unemployment rate has fallen persistently (except for a slight increase during the Great Recession), and reached 4% by early 2017.

The Hartz IV reform has also changed the cyclical features of the German labor mar-

[2] As of 2013, UB II was equal to €345 a month plus rent allowance.

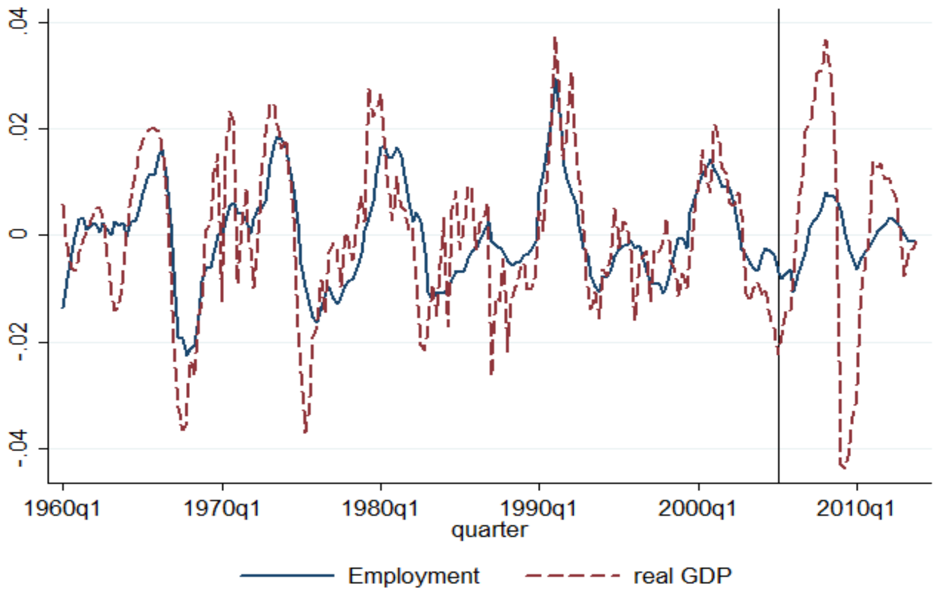

Figure 4: Employment and real GDP, quarterly data, seasonally adjusted, log deviation from an HP trend. Source: Eurostat and OECD.

ket. To document these changes, I use quarterly data on employment, hours per worker, total hours, and real GDP over the period 1960Q1-2013Q4 from the dataset constructed by Ohanian and Raffo (2012)[3]. They construct total hours series, H, as the product of hours worked per worker, h, and employment, N, normalized by the size of population aged 15-64 years and by the maximum number of hours per year to be shared between work and leisure (365 times 14). All variables are expressed in logs and detrended using an HP filter with smoothing parameter of 1600.

Figure 4 and Figure 5 show the cyclical fluctuations of employment and hours per worker relative to real GDP, respectively. It is clear from these graphs that the volatility of employment relative to output fell significantly after the reforms, while the relative volatility of hours per worker slightly increased. These findings are formalized in Table 1, which reports the relative volatility of employment, hours per worker, ratio between the two, and the relative volatility of total hours in the pre- and post-reform periods. The relative volatility of employment, measured as the ratio of the standard deviation of employment to the standard deviation of real GDP, fell by about two-thirds, from 0.671 to 0.224. At the same time,

[3]Using quarterly data on employment, hours per worker, and total hours from Eurostat, which is available from 1991Q1 until 2016Q4, yields similar results.

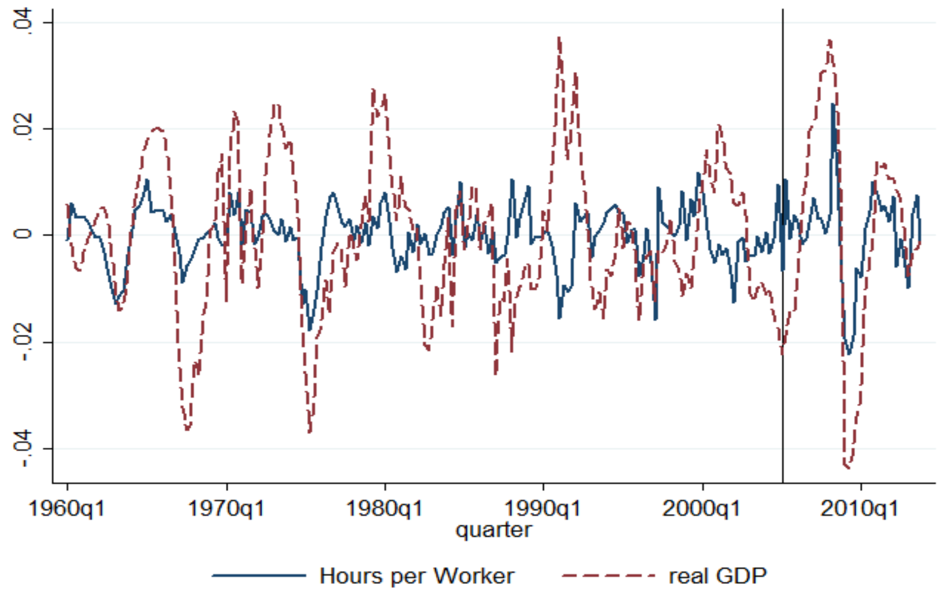

Figure 5: Hours per worker and real GDP, quarterly data, seasonally adjusted, log deviation from an HP trend. Source: Ohanian and Raffo (2012).

the relative volatility of hours per worker increased from 0.384 to 0.426. If we look at the volatility of hours per worker relative to the volatility of employment, we obtain the most striking result: Prior to the reform, hours per worker were half as volatile as employment; after, they became almost twice as volatile.

To conclude, not only did the Hartz IV reform reach its main objective of raising the employment rate, but it also significantly reduced firms' incentives to adjust employment in response to shocks.

2.3 Short-Time Work Policy

Of all the German government's anti-crisis measures, the extension of the short-time work program (*Kurzarbeit*) received the most attention. Short-time work (STW), which has existed in Germany for almost a century, allows firms facing temporary financial difficulties to cut workers' hours and reduce wages instead of laying them off. The firm applies to the Federal Employment Agency and, of the request is approved, workers receive between 60% and 67% of their lost income from the government. The firm still has to pay social security contributions based on the worker's full-time wage. Moreover, during short-time work the

	Pre-reform	Post-reform
σ^N/σ^Y	0.671	0.224
σ^h/σ^Y	0.384	0.426
σ^h/σ^N	0.571	1.900
σ^H/σ^Y	0.723	0.516

Table 1: Relative standard deviations. Data: quarterly, seasonally adjusted, log deviation from an HP trend. Pre-reform period: 1960Q1-2004Q4. Post-reform period: 2005Q1-2013Q4. Source: Ohanian and Raffo (2012). h – hours per worker, N – employment, H – total hours, Y – output.

firm is also responsible for the worker's share of social security contributions for the lost hours.

During the Great Recession, the maximum duration of short-time work was extended from 6 months to 24 months. Rules regarding social security contributions were loosened as well. From January 2009 to December 2011, firms only needed to pay 50% of social security contributions for the worker's lost hours during the first 6 months of short-time work (the remaining half was covered by the unemployment insurance fund). After the sixth month, the Federal Employment Agency covered 100% of those contributions. At the peak of the Great Recession in May 2009, the number of short-time workers reached 1.5 million (3.7% of total employment and 15% of employment in manufacturing sector). According to Brenke et al. (2011), the average reduction in working hours was just under 30% of the agreed working time.

3 Benchmark Model

In this section I present a benchmark model that is a discrete-time model of equilibrium unemployment with aggregate productivity shocks and multi-worker firms that adjust their labor input along extensive and intensive margins. The model features a two-tier unemployment insurance system with two types of unemployment benefits — short-term and long-term — and the former expire with an exogenous probability.

3.1 Technology and Preferences

The economy consists of a unit measure of infinitely lived workers, a unit measure of infinitely lived identical multi-worker firms, and a government. Workers and firms are risk-neutral and have a common discount factor $\beta \in (0, 1)$. There is one final good that can be used for consumption, production, and vacancy creation.

Worker's utility is given by

$$u(c_t, h_t) = c_t - \zeta \frac{h_t^{\mu+1}}{\mu + 1},$$

where c_t is consumption, h_t is hours worked, $\zeta > 0$ and μ is the inverse of Frisch elasticity.

Workers can either be employed, short-term unemployed, or long-term unemployed. Employed workers receive labor income and non-wage compensation from firms. Unemployed workers receive unemployment benefits, the level of which depends on their unemployment status. Workers own the firm (in equal shares) and receive all profits. Every worker, independent of her employment status, pays taxes that are used to finance unemployment benefits.

Firms use labor services to produce final goods according to the following technology:

$$Y_t = z_t(n_t h_t)^\alpha$$

where z_t is stochastic productivity, n_t is the number of workers, and h_t is hours worked per employee. The log of z_t follows the first-order autoregressive process

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_t^z, \text{ where } \varepsilon_t^z \sim N(0, \sigma_z^2).$$

3.2 Labor Market Structure

The labor market is frictional. Unemployed workers search for jobs, and firms with vacant positions search for workers. The search is undirected, so that firms have no ability to direct their search toward a particular type of unemployed worker. Because of the search frictions, only a fraction of job-seekers find jobs, and only a fraction of vacancies are filled each period. The number of worker-firm matches in period t is determined by the following matching function:

$$m(U_t, V_t) = mU_t^\xi V_t^{1-\xi},$$

where U_t is the measure of unemployed workers searching for jobs, V_t is the total number of job vacancies, m_0 is the parameter that governs matching efficiency, and ξ is the matching elasticity. Let $\theta_t \equiv V_t/U_t$ be labor market tightness. The probability that a vacancy is

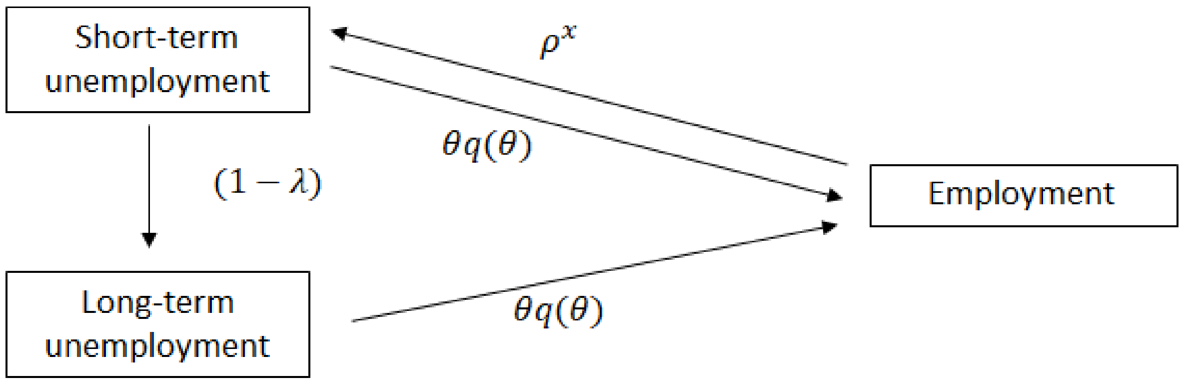

Figure 6: Labor market flows.

matched with a worker, *vacancy filling rate*, is

$$m(U_t, V_t)/V_t = m\theta_t^{-\xi} = q_t(\theta_t).$$

Similarly, the probability that a worker is matched with a vacancy, *job finding rate*, is

$$m(U_t, V_t)/U_t = m\theta_t^{1-\xi} = \theta_t q(\theta_t).$$

Figure 6 illustrates labor market flows. At the end of each period, a fraction ρ^x of employed workers are hit by the exogenous separation shock and become short-term unemployed. With probability $1-\lambda$, short-term unemployment benefits expire and the worker becomes long-term unemployed and remains in this status until she finds a job. The expected duration of short-term unemployment is $1/(1-\lambda)$. An unemployed worker of type i receives unemployment benefits b^i, where $i \in \{S, L\}$ and $b^S > b^L$.

Aggregate employment, N_t, evolves according to

$$N_{t+1} = (1-\rho^x)N_t + q(\theta_t)V_t.$$

The number of searching workers is equal to the currently unemployed, $U_t = 1 - N_t$. Unemployment consists of short-term and long-term unemployment

$$U_t = U_t^S + U_t^L.$$

Let $p_t^S = U_t^S/U_t$ denote the share of short-term unemployment. Short-term unemployment evolves according to

$$U_{t+1}^S = \rho^x N_t + \lambda(1 - q(\theta_t)\theta_t)U_t^S.$$

Short-term unemployment consists of workers who have just been separated from the firm and a fraction λ of previously short-term unemployed who could not find a job in this period. Long-term unemployment consists of previously long-term unemployed who could not find a job in this period and a fraction $1-\lambda$ of short-term unemployed who could not find a job and whose short-term unemployment benefits expired:

$$U_{t+1}^{L} = (1 - q(\theta_t)\theta_t)U_t^L + (1-\lambda)(1 - q(\theta_t)\theta_t)U_t^S.$$

3.3 Timing

The aggregate state of the economy is $X_t(z_t, N_t, p_t^S)$, and the individual state of a representative firm is (n_t^S, n_t^L), where n_t^S is the number of workers whose outside option is short-term unemployment benefits; this consists of previous period workers who weren't separated and newly hired short-term unemployed. n_t^L is the number of workers whose outside option is long-term unemployment benefits — i.e., long-term unemployed hired at the end of previous period.

At the beginning of the period, z_t is realized. The firm chooses hours of work h_t and vacancies v_t. Regardless of whether the match is new or the worker had been working for the firm in the previous period, the firm and each of its workers bargain over the corresponding hourly wage, $w_t^S = w(X_t, h_t)$ or $w_t^L = w(X_t, h_t)$[4]. After that, production and consumption take place, $\rho^x(n_t^S + n_t^L)$ of current workers leave the firm, and $p_t^S q(\theta_t)v_t$ short-term unemployed and $(1-p_t^S)q(\theta_t)v_t$ long-term unemployed are hired.

3.4 Workers

Each worker is characterized by a pair $\{e, b^i\}$, where $e \in \{0,1\}$ is the employment status of a worker and $b^i \in \{b^S, b^L\}$ is the level of unemployment benefits the worker is receiving if she is currently unemployed or would receive if she weren't currently employed. A worker chooses a sequence $\{c_t\}_{t=0}^{\infty}$ to maximize her expected lifetime utility

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(c_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} \right)$$

$$\text{s.t. } c_t = e(w_t^i h_t + \Gamma_t) + (1-e)b^i - T_t + \Pi_t,$$

[4] I assume that the firm ignores the fact that the marginal product of each worker depends on the total number of workers, n_t. As Krause and Lubik (2013) show, this barely affects the dynamics of the model, but makes it much easier to solve.

where w_t^i is the hourly wage, Γ_t is non-wage compensation, T_t is lump sum taxes, and Π_t is profits.

The value of being employed for a worker who was employed in the previous period is the same as the value of being employed for a short-term unemployed worker, because both have the same outside option, b^S, and receive w_t^S when employed. Given that the aggregate state of the economy is X_t, the value of being employed for a short-term unemployed worker is

$$V^{WS}(X_t) = w_t^S h_t + \Gamma_t - T_t + \Pi_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} + \beta \mathbb{E}_t[(1-\rho^x)V^{WS}(X_{t+1}) + \rho^x V^{US}(X_{t+1})]. \quad (1)$$

The flow value of being employed equals the sum of after-tax labor income, non-wage compensation, profits, and the disutility of working h_t hours. If the match is hit by the exogenous separation shock, which happens with probability ρ^x, the worker becomes short-term unemployed in the following period. Similarly, the value of being employed for a long-term unemployed worker is

$$V^{WL}(X_t) = w_t^L h_t + \Gamma_t - T_t + \Pi_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} + \beta \mathbb{E}_t[(1-\rho^x)V^{WS}(X_{t+1}) + \rho^x V^{US}(X_{t+1})]. \quad (2)$$

The value of being short-term unemployed is given by

$$\begin{aligned} V^{US}(X_t) = b^S - T_t + \Pi_t + \beta \mathbb{E}_t[\lambda[\theta_t q(\theta_t) V^{WS}(X_{t+1}) + (1-\theta_t q(\theta_t))V^{US}(X_{t+1})] \\ +(1-\lambda)[\theta_t q(\theta_t) V^{WL}(X_{t+1}) + (1-\theta_t q(\theta_t))V^{UL}(X_{t+1})]] \quad (3) \end{aligned}$$

The flow value of being short-term unemployed is equal to unemployment benefits minus taxes, $b^S - T_t$. With probability λ, the short-time unemployed worker does not change her unemployment status in the next period, and with probability $1-\lambda$, short-term unemployment benefits expire and she becomes long-term unemployed. In both cases, with probability $\theta_t q(\theta_t)$, the unemployed worker finds a job and starts working in the next period; otherwise, she remains unemployed. When the short-term unemployed worker becomes long-term unemployed, the worker stays in this status until she finds a job. The value of long-term unemployment is given by

$$V^{UL}(X_t) = b^L - T_t + \Pi_t + \beta \mathbb{E}_t[\theta_t q(\theta_t) V^{WL}(X_{t+1}) + (1-\theta_t q(\theta_t))V^{UL}(X_{t+1})]. \quad (4)$$

3.5 Representative Firm

The profits of the representative firm with n_t^S and n_t^L workers are

$$\Pi_t = z_t([n_t^S + n_t^L]h_t)^\alpha - [w_t^S n_t^S + w_t^L n_t^L]h_t - g(h_t)[n_t^S + n_t^L] - \kappa v_t,$$

where both wages are functions of hours worked, $w_t^S = w_t^S(X_t, h_t)$ and $w_t^L = w_t^L(X_t, h_t)$; κ is the vacancy posting cost; and $g(h_t)$ is non-wage compensation given by

$$g(h_t) = \phi|h_t - h|,$$

where h is the steady-state level of hours per worker.[5] I assume that non-wage compensation is increasing in the distance between h_t and h. It is costly for the firm to require workers to work more than a "normal" number of hours, e.g., overtime bonuses. If working hours are reduced below h, the firm incurs extra costs as well; e.g., social security contributions for the lost hours in Germany's case.

Apart from paying different wages, the firm cannot treat workers differently; therefore, workers of both types work the same number of hours. Following Kudoh et al. (2016), I assume that the firm chooses hours of work per employee to focus on the composition of labor demand.

Taking as given the labor market tightness, θ_t, share of short-time unemployment, p_t^S, and the law of motion of the aggregate state, Ω, the value of a firm is

$$\begin{aligned} J(X_t, n_t^S, n_t^L) &= \max_{v_t, h_t}\{\Pi_t + \beta\mathbb{E}_t\left[J(X_{t+1}, n_{t+1}^S, n_{t+1}^L)\right]\} \\ \text{s.t. } n_{t+1}^S &= (1-\rho^x)(n_t^S + n_t^L) + p_t^S q(\theta_t) v_t \\ n_{t+1}^L &= (1-p_t^S) q(\theta_t) v_t \\ X_{t+1} &= \Omega(X_t) \end{aligned}$$

The next-period number of workers of type S equals the fraction $1 - \rho^x$ of the current workforce plus newly hired short-term unemployed workers. The next-period number of workers of type L equals the number of long-term unemployed workers hired this period.

First-order conditions are given by

$$\kappa = q(\theta_t)\beta\mathbb{E}_t[p_t^S J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L) + (1-p_t^S) J_L(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \tag{5}$$

$$z_t \alpha h_t^{\alpha-1}[n_t^S + n_t^L]^\alpha - \frac{\partial g(h_t)}{\partial h_t}[n_t^S + n_t^L] = w_t^S n_t^S + w_t^L n_t^L + \frac{\partial w_t^S}{\partial h_t} n_t^S h_t + \frac{\partial w_t^L}{\partial h_t} n_t^L h_t. \tag{6}$$

[5] Non-wage compensation encompass a broad range of benefits, such as social security, health insurance, paid holidays, and overtime bonuses.

The value of having an additional short-time unemployed worker is given by

$$J_S(X_t, n_t^S, n_t^L) = \alpha z_t h_t^{\alpha}[n_t^S + n_t^L]^{\alpha-1} - w_t^S h_t - g(h_t) + (1-\rho^x)\beta\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]. \tag{7}$$

The value of hiring an additional long-time unemployed worker is

$$J_L(X_t, n_t^S, n_t^L) = \alpha z_t h_t^{\alpha}[n_t^L + n_t^L]^{\alpha-1} - w_t^L h_t - g(h_t) + (1-\rho^x)\beta\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]. \tag{8}$$

3.6 Bargaining

Every period, the firm and workers bargain over hourly wages, $w_t^S = w_t^S(X_t, h_t)$ or $w_t^L = w_t^L(X_t, h_t)$. Following Stole and Zwiebel (1996), I assume that the firm bargains with every individual worker and that each worker is treated as a marginal worker. Let $\omega \in (0,1)$ be the bargaining power of each worker. To derive the value of a marginal worker to the firm, suppose that the firm bargains with a group of type S workers of measure Δ. The threat point for the firm is $J(X_t, n_t^S - \Delta, n_t^L)$. The limit of the firm's surplus per worker as $\Delta \to 0$ gives us the value of a marginal worker:

$$\lim_{\Delta\to 0} \frac{J(X_t, n_t^S, n_t^L) - J(X_t, n_t^S - \Delta, n_t^L)}{\Delta} = J_S(X_t, n_t^S, n_t^L),$$

where $J_S(\cdot)$ is the derivative of $J(\cdot)$ with respect to n_t^S. Therefore, $w_t^S(X_t, h_t)$ is the solution to the following Nash bargaining problem:

$$\max_{w_t^X} [V^{WS}(X_t) - V^{US}(X_t)]^{\omega}[J_S(X_t, n_t^S, n_t^L)]^{1-\omega}.$$

Similarly, $w_t^L(X_t, h_t)$ is the solution to the following Nash bargaining problem:

$$\max_{w_t^L} [V^{WL}(X_t) - V^{UL}(S_t)]^{\omega}[J_L(X_t, n_t^S, n_t^L)]^{1-\omega}.$$

Equilibrium hourly wages are derived in Appendix A. The resulting expressions for equilibrium wages are quite cumbersome, but it is possible to rewrite them in a compact manner to understand the underlying logic. For example, w_t^S can be written as

$$w_t^S = \frac{\omega}{h_t} w_t^{FS} + \frac{1-\omega}{h_t} w_t^{WS},$$

where

$$w_t^{FS} = \alpha z_t h_t^{\alpha}[n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]$$

$$w_t^{WS} = \zeta\frac{h_t^{\mu+1}}{\mu+1} + b^S - \Gamma_t - \beta\mathbb{E}_t[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}$$

$$-\rho^x\frac{\omega}{1-\omega}J_S(X_{t+1}) + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_L(X_{t+1}) - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}]\ .$$

Essentially, w_t^S is a weighted average of the marginal value of a worker to the firm per hour worked, w_t^{FS}/h_t, and the the minimum hourly wage the worker is willing to accept, w_t^{WS}/h_t. w_t^{FS} is equal to the sum of the marginal product of the worker, minus hours adjustment costs, and plus the value of keeping the worker until the next period. w_t^{WS} corresponds to the opportunity costs of not working, which includes unemployment benefits, an increase in utility from not working minus the value of entering the next period employed. The hourly wage is decreasing in the total number of employees, increasing in the level of unemployment benefits, and nonlinear in hours worked per worker.

3.7 Equilibrium

Given the sequence of productivity shocks $\{z_t\}_{t=0}^{\infty}$, the initial level of aggregate employment, N_0, the initial share of short-term unemployment, p_0^S, and the initial levels of employment at the representative firm, n_0^S and n_0^L, an equilibrium is a sequence of wages $\{w_t^S, w_t^L\}_{t=0}^{\infty}$, the firm's choices of hours and vacancies $\{h_t, v_t\}_{t=0}^{\infty}$, the firm's employment levels $\{n_t^S, n_t^L\}_{t=1}^{\infty}$, aggregate labor market outcomes $\{\theta_t, N_{t+1}, p_{t+1}^S, U_t, V_t\}_{t=0}^{\infty}$, taxes $\{T_t\}_{t=0}^{\infty}$, profits $\{\Pi_t\}_{t=0}^{\infty}$, non-wage compensation $\{\Gamma_t\}_{t=0}^{\infty}$, and the law of motion of the aggregate state $\{\Omega_t\}_{t=0}^{\infty}$, such that

- *w_t^S and w_t^L are solutions to the corresponding Nash bargaining problems*
- *h_t and v_t satisfy the firm's optimality conditions*
- *θ_t satisfies*

$$\kappa = q(\theta_t)\beta\mathbb{E}_t[p_t^S(w_{t+1}^L h_{t+1} - w_{t+1}^S h_{t+1}) + J_L(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]$$

- *The aggregate number of vacancies satisfies*

$$V_t = \theta_t U_t$$

- *Aggregate employment is equal to the employment at the representative firm*

$$N_t = n_t^S + n_t^L$$

- *The aggregate number of vacancies is equal to the number of vacancies created by the representative firm*

$$V_t = v_t$$

- *Aggregate employment evolves according to*

$$N_{t+1} = (1 - \rho^x)N_t + q(\theta_t)V_t$$

- *The number of unemployed workers satisfies*

$$U_t = 1 - N_t$$

- n_t^S *and* n_t^L *evolve according to*

$$n_{t+1}^S = (1 - \rho^x)(n_t^S + n_t^L) + p_t^S q(\theta_t)v_t$$
$$n_{t+1}^L = (1 - p_t^S)q(\theta_t)v_t$$

- *Short-time unemployment evolves according to*

$$p_{t+1}^S U_{t+1} = \rho^x N_t + \lambda(1 - q(\theta_t)\theta_t)p_t^S U_t$$

- *Non-wage compensation is given by*

$$\Gamma_t = g(h_t)$$

- *The government budget is balanced*

$$T_t = b_t^S p_t^S U_t + b_t^L(1 - p_t^S)U_t$$

- *Profits are given by*

$$\Pi_t = z_t([n_t^S + n_t^L]h_t)^\alpha - [w_t^S n_t^S + w_t^L n_t^L]h_t - g(h_t)[n_t^S + n_t^L] - \kappa v_t$$

- Ω_t *is consistent with the law of motion of aggregate state variables*

$$(z_{t+1}, N_{t+1}, p_{t+1}^S) = \Omega_t(z_t, N_t, p_t^S) \text{ for all } t, z_t, z_{t+1}.$$

The model is solved in Dynare. The system of equations that characterizes the equilibrium can be found in Appendix B.

4 Quantitative Analysis

4.1 Calibration

Parameter	Description	Value	Target/Source
β	discount factor	0.99	4% annual interest rate
α	labor share	0.64	Karabarbounis and Nieman (2014)
μ	inverse of Frisch elasticity	2.00	Chetty (2012)
ξ	matching elasticity	0.50	Pissarides (2009)
ω	worker's bargaining power	0.50	($\xi = \omega$) Hosios condition
ρ	separation rate, %	3.50	Literature
λ	probability of staying ST unemployed	0.75	duration of ST unemployment
	Steady State Targets		
m	matching efficiency	0.50	job finding rate 0.35
κ	vacancy posting costs	0.10	vacancy filling rate 0.7
ζ	disutility parameter	0.69	unemployment rate 9.1%
b_0^S	pre-reform ST benefits	0.42	ST replacement rate 60%
b_0^L	pre-reform LT benefits	0.28	LT replacement rate 54%
b_1^L	post-reform LT benefits	0.05	LT replacement rate 17%
	Business Cycle Targets		
ϕ	non-wage compensation parameter	0.0072	pre-reform $\sigma^h/\sigma^N = 0.571$
	Stochastic Process Targets		
ρ_z	persistence of productivity	0.94	persistence of real GDP
σ_z	volatility of productivity, %	0.93	st.dev. of real GDP

Table 2: Calibration.

The model is calibrated at a quarterly frequency to the German data. Table 2 summarizes parameters and calibration targets. The discount factor β is 0.99, which corresponds to an annual interest rate of 4.1%. Using the data from Karabarbounis and Nieman (2014), I set the labor share $\alpha = 0.64$, which corresponds to the average labor share over the period 1980-2011. Empirical estimates of Frisch elasticity are in the range of 0.5, implying $\mu = 2$ (Chetty (2012)). Following Pissarides (2009), I set the matching elasticity ξ to be equal to 0.5. I further assume that a Hosios condition holds, so that $\omega = 0.5$. Estimates for the separation rate for Germany range from 3% (Christoffel et al. (2009)) to 4% (Gartner et al. (2009)), so I select the midpoint, setting $\rho^x = 3.5\%$. The probability of staying short-term unemployed, λ, is set to 0.75 so that the expected duration of short-term unemployment in the model equals 4 quarters.

I set the matching efficiency, m_0, vacancy posting costs, κ, disutility parameter, ζ, pre-reform short-term and long-term unemployment benefits, b_0^S and b_0^L, and post-reform long-term unemployment benefits, b_1^L, to jointly match the following steady-state targets: pre-reform short-term and long-term replacement rates, post-reform long-term replacement rate, pre-reform steady-state level of unemployment, and pre-reform job-finding and job-filling rates.

Pre- and post-reform replacement rates are taken from OECD data, which report long-term and short-term average net replacement rates for different subgroups of households over the period 2001-2014. "Short term" refers to the initial phase of unemployment, and "long term" refers to the 60th month of receiving benefits. I focus on a single-person household without children. To obtain target values for pre-reform short-term and long-term replacement rates, I compute the averages of the corresponding time series over the period 2001-2004, and get 60% and 54%. The target value for the post-reform long-term replacement rate is calculated as the average over the period 2005-2014 and is equal to 17%. In the model, the net replacement rate is defined as the ratio of unemployment benefits to labor income.

The target for the pre-reform steady-state value of the unemployment rate is set to 9.1%, which corresponds to the average employment rate over the period 1993Q1-2004Q4. Using data from Gartner et al. (2009), I set the pre-reform job-finding rate to 0.35, which corresponds to the average over the same period, 1993Q1-2004Q4. The vacancy-filling rate is set to 0.7, which is in line with estimates for European countries that have low job turnover (Amaral and Tasci (2016)).

The remaining three parameters, ϕ, ρ_z, and σ_z, are calibrated to match the ratio of volatility of hours per worker to the volatility of employment, persistence, and volatility of output in pre-reform data. The estimated persistence and volatility of the HP-filtered logarithm of real GDP are 0.79 and 1.4%, respectively.

4.2 Findings

Steady State

Table 3 shows the effect of the reform on the model's steady state. A reduction in long-term unemployment benefits reduces the outside option of long-term unemployed, so that the match surplus of hiring an additional long-term unemployed worker increases, while her hourly wage falls by half. The hourly wages of currently employed workers (and of newly hired short-time unemployed) decline slightly. As a result, the firm posts more vacancies, and hires more workers. The share of short-time unemployment increases, the aggregate unemployment goes down to 5.9%, and labor market tightness goes up from 0.5 to 1.26. The unemployment rate in the model does not fall as much as in the data. By the end of 2016, the unemployment rate in Germany reached 4%, suggesting that the model can explain about 62% of the observed fall. Since the number of workers and the number of hours worked are substitutes in the production of final goods, the firm cuts back on hours worked by each worker.

Variable	Pre-reform	Post-reform
Unemployment rate	0.091	0.059
Hours per worker	1.120	1.114
Tightness	0.500	1.256
Share of ST unemployment	0.683	0.833
Vacancies	0.045	0.075
ST hourly wage	0.632	0.623
LT hourly wage	0.471	0.259
Output	1.012	1.031

Table 3: Steady state values of endogenous variables before and after the reform.

Elasticities

Following Hagedorn and Manovskii (2008), I calculate the productivity elasticities of labor market tightness, employment, hours per worker, and output assuming that the economy is at the steady state and, for simplicity, that $b^S = b^L = b$ (see Appendix C). The elasticity of labor market tightness, employment, hours per worker, and output are given by

$$\varepsilon_{\theta,z} = \frac{zM}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)}$$

$$\varepsilon_{n,z} = \varepsilon_{\theta,z}\frac{\rho^x(1-\xi)}{(\rho^x + \theta q(\theta))}$$

$$\varepsilon_{h,z} = \frac{1}{\mu+1-\alpha}\left[1-(1-\alpha)\varepsilon_{n,\theta}\varepsilon_{\theta,z}\right]$$

$$\varepsilon_{y,z} = 1 + \alpha(\varepsilon_{n,z} + \varepsilon_{h,z})$$

where $K > 0$ and $M > 0$.

Elasticities of market tightness and employment are increasing in the level of unemployment benefits, while the elasticity of hours per worker is decreasing in b. As the level of unemployment benefits goes down, the match surplus increases. Incentives to post vacancies depend on the size of percentage changes of the match surplus in response to changes in productivity. A larger surplus means that these percentage changes are smaller, as are the volatilities of vacancies, tightness, and employment. Hours per worker become more responsive to fluctuations in productivity, since the firm now has more incentives to adjust labor input along the intensive margin[6]. The impact of reducing b on the elasticity of output depends on the behavior of the sum of employment and hours per worker elasticities.

Business Cycle Statistics

Figure 7 displays the pre- and post-reform impulse responses to a one-percentage-point negative productivity shock. On impact, vacancies, hours per worker, and output decline. The initial fall in hours per worker and output is almost the same in both calibrations. However, in the post-reform calibration, vacancies decline by about 3 percentage points on impact compared to 7.6 percentage points in the pre-reform calibration. Given that unemployment is unchanged in the period when the shock occurs, labor market tightness goes down. Fewer vacancies lead to an increase in unemployment one period after the shock. In the pre-reform calibration, unemployment keeps rising for 3 periods when it is 2.26 percentage points above the steady-state level, while in the post-reform calibration it starts to decline 2 periods earlier when it is just 1.0 percentage points above steady state. In the post-reform calibration, output recovers faster than in the pre-reform model, while hours per

[6]As it is shown in Appendix D, in a one-worker firm environment, the elasticity of hours per worker is a function of model parameters only.

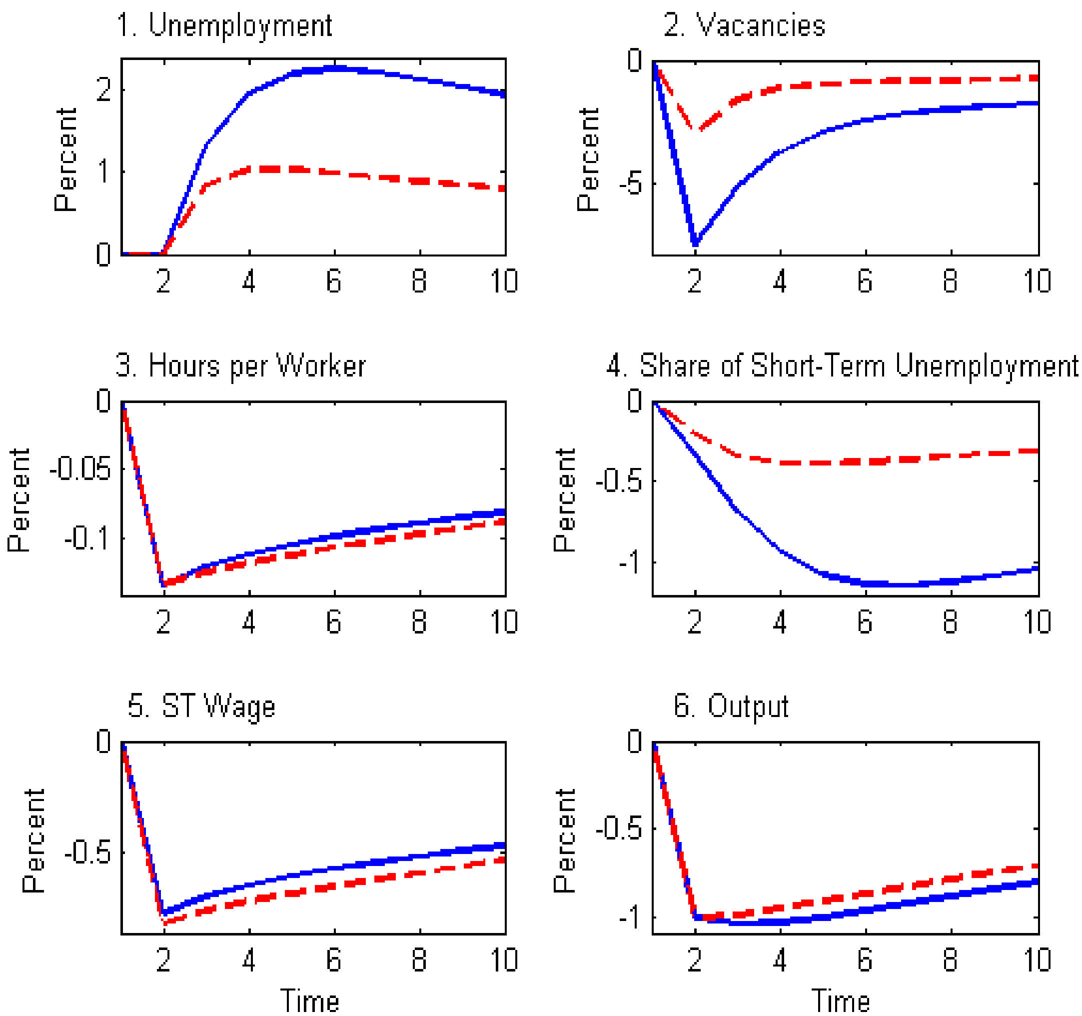

Figure 7: Impulse response functions to a negative one-percentage-point productivity shock. Each panel shows the percentage-point deviations from the steady state. The horizontal axes measure time, expressed in quarters. Pre-reform (solid line). Post-reform (dashed line).

	data	(1)	(2)	(3)	(4)	(5)
Pre-reform						
σ^N/σ^Y	0.671	0.201	0.199	0.189	0.280	0.293
σ^h/σ^Y	0.384	0.114	0.117	0.163	0.159	0.170
σ^h/σ^N	0.571	0.567	0.585	0.859	0.569	0.581
σ^H/σ^Y	0.723	0.306	0.310	0.334	0.432	0.459
Post-reform						
σ^N/σ^Y	0.224	0.064	0.063	0.060	0.075	0.076
σ^h/σ^Y	0.426	0.124	0.127	0.177	0.186	0.203
σ^h/σ^N	1.900	1.937	2.000	2.971	2.472	2.655
σ^H/σ^Y	0.516	0.187	0.190	0.224	0.261	0.279

Table 4: Observed and simulated relative standard deviations. Data: quarterly, seasonally adjusted, log deviation from an HP trend. Pre-reform Period: 1960Q1-2004Q4. Post-reform period: 2005Q1-2013Q4. Source: Ohanian and Raffo (2012). h – hours per worker, N – employment, H – total hours, Y – output. The simulated business cycle statistics are based on 5000 simulations of 10000 quarter horizon and are HP-filtered for comparison. Simulated figures are averages across simulations. (1) – benchmark model; (2) – benchmark model with $s = 0.08$; (3) – benchmark model with $s = 0.67$; (4) – benchmark model with $\gamma = 0.1$; (5) – benchmark model with $\gamma = 0.1$ and $s = 0.08$

worker recover more slowly.

The third column of Table 4 reports the relative standard deviations of the simulated benchmark model. The relative volatility of employment falls by 68%, from 0.201 to 0.064, while the relative volatility of hours per worker increases by 9%, from 0.114 to 0.124. The post-reform ratio of volatility of hours per worker to the volatility of employment is 1.94, which is close to the one observed in German post-reform data. In line with Shimer (2005) critique, my model does not generate enough fluctuations in vacancies to match the level of employment volatility in the data[7]. However, it does well in terms of capturing the relative changes in volatility: In the data, the relative volatility of employment fell by two-thirds, while the volatility of hours per worker went up by 10% (see Table 5).

Role of the Short-Time Work Policy

The effect of the short-time work policy on the labor market is twofold. Workers receive a short-time working allowance (a fraction of their lost full-time wages from the government, where full time represents some notion of "normal" — i.e., steady state — hours per worker).

[7]The robustness of my results to the Shimer's critique is discussed in Section 5.2.

	data	(1)	(2)	(3)	(4)	(5)
$\sigma^N_{post}/\sigma^N_{pre}$	0.328	0.319	0.317	0.294	0.270	0.259
$\sigma^h_{post}/\sigma^h_{pre}$	1.105	1.091	1.121	1.522	1.170	1.209
$\sigma^H_{post}/\sigma^H_{pre}$	0.708	0.611	0.621	0.752	0.604	0.610

Table 5: Observed and simulated ratios of pre- and post-reform standard deviations. h - hours per worker, N - employment, H - total hours. The simulated business cycle statistics are based on 5000 simulations of 10000 quarter horizon and are HP-filtered for comparison. Simulated figures are averages across simulations. Ratios are calculated using pre-reform standard deviations of the benchmark model. (1) – benchmark model; (2) – benchmark model with $s = 0.08$; (3) – benchmark model with $s = 0.67$; (4) – benchmark model with $\gamma = 0.1$; (5) – benchmark model with $\gamma = 0.1$ and $s = 0.08$.

At the same time, it is less costly for the firm to reduce hours worked per worker below the "normal" level, because part of the social security contribution is paid by the government. In my model, I abstract from the short-time working allowance[8].

To analyze the contribution of short-time work policy to the unemployment dynamics during the Great Recession, I solve and simulate my model assuming that the firm faces lower costs of reducing hours. In particular, I assume that the government introduces a subsidy s such that new non-wage compensation is

$$\tilde{g}(h_t) = \begin{cases} \phi(h_t - h),\ h_t \geq h \\ (1-s)\phi(h - h_t),\ h_t < h. \end{cases}$$

The subsidy is financed by higher taxes that are now equal to

$$T_t = b^S_t p^S_t U_t + b^L_t (1 - p^S_t) U_t + s\phi \max\{h - h_t, 0\}.$$

To pin down s, I use data on the duration of short-time work from the Federal Employment Agency (see Figure 8). The average subsidy is calculated as a weighted average of the share of social contributions covered by the government during the first 6 months of short time work (50%) and the share of social contribution covered by the government after the sixth month (100%). Weights are equal to the average shares of short-time workers of corresponding duration. The resulting subsidy level is 67%. To account for the fact that

[8]As Cooper et al. (2017) show, if firms were able to appropriate part of the workers' surplus that comes from the short-time working allowance, workers would be willing to accept lower wages, and it would be optimal for firms to increase employment during recessions, which is contrary to the data. In a risk-neutral environment, the model in which wages are not renegotiated to account for the short-time working allowance is essentially equivalent to the one without it.

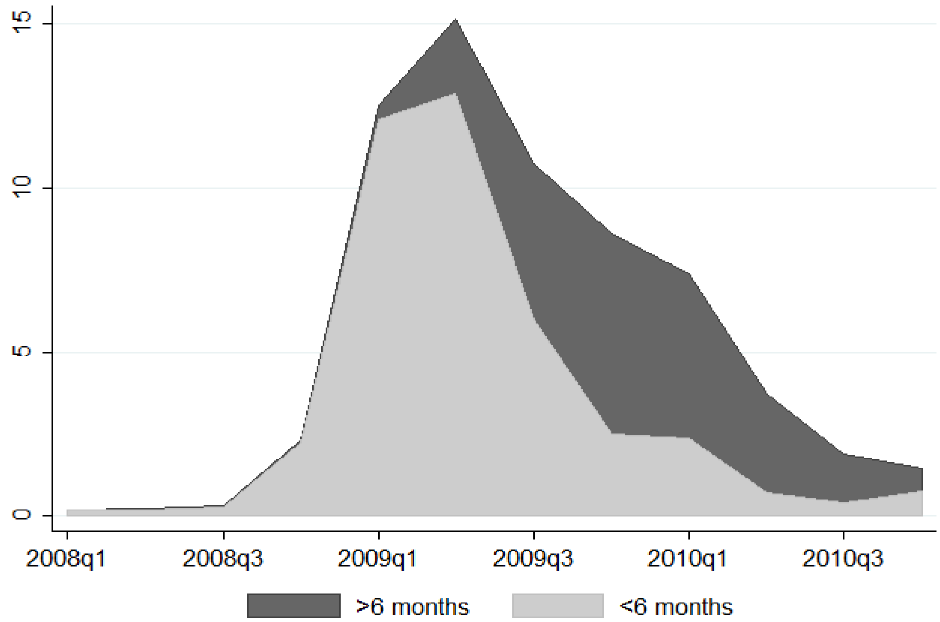

Figure 8: Short-time manufacturing workers by duration of short-time work (expressed as a share of total employment of manufacturing sector, %). Source: Federal Employment Agency.

only a fraction of workers participated in short-time work — in 2009 the average share of manufacturing sector workers on a short-time work was 12% — I assume that only 12% of the firm's workers are eligible for this subsidy, so the resulting value of s is 0.08.

To solve the model, I use the OccBin toolkit developed by Guerrieri and Iacoviello (2015). OccBin is a library of numerical routines designed as an add-on to Dynare and is used to solve models with occasionally binding constraints. The main idea is that the model with asymmetric compensation can be represented as a model with two regimes. Under the "reference" regime, hours per worker are above the steady state level and $\tilde{\phi} = \phi$. Under the "alternative" regime, hours per worker are below the steady-state level and $\tilde{\phi} = (1 - s)\phi$. The model under each regime is log-linearized around the deterministic steady state. The algorithm employs a guess-and-verify approach: Guess the periods in which each regime applies, verify the guess, and update it if necessary.

The fourth column of Table 4 shows the relative standard deviations of the pre- and post-reform simulated models with the short-time work subsidy. Since changing both hours per worker and employment is costly, the subsidy makes it cheaper for the firm to reduce hours per worker, instead of posting fewer vacancies, when it is hit by a negative productivity shock.

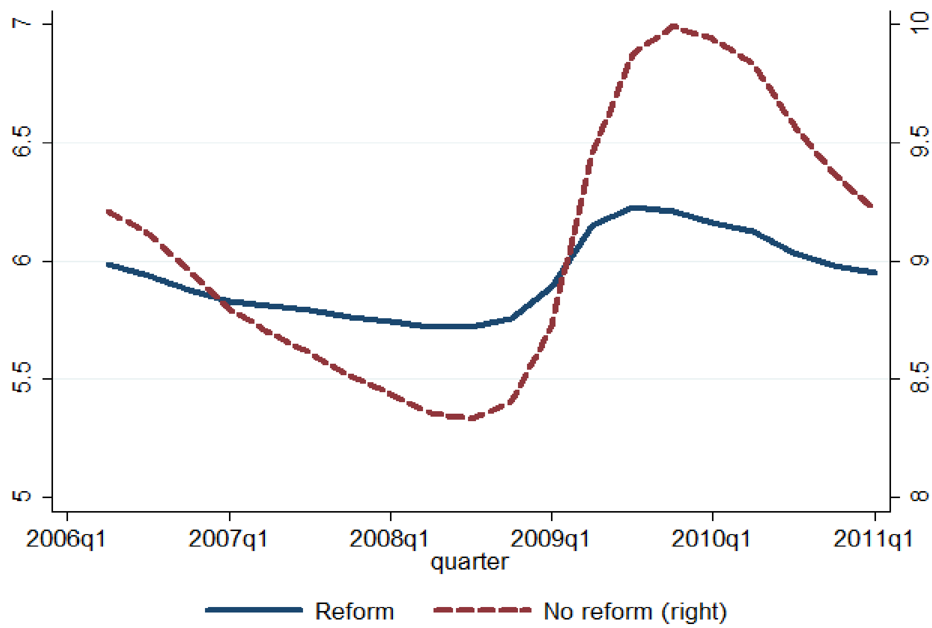

Figure 9: Counterfactual for the unemployment rate: reform vs no reform.

The volatility of employment goes down, while hours per worker become more volatile. The effect, however, is very small: 0.2 and 3 percentage points, respectively (see fourth column of Table 5). Comparing this to the 68% decline in employment volatility brought about by the reform suggests that short-time work played only a minor role in generating the German labor market miracle.

Unemployment Dynamics during the Great Recession

In this section I conduct a counterfactual exercise to analyze what would have happened with the unemployment rate during the Great Recession if there had been no unemployment insurance reform. First, I compute a sequence of productivity shocks such that the post-reform calibration with the short-time work subsidy replicates the dynamics of the cyclical component of real GDP over the period 2006Q2-2011Q1. I then feed this sequence of shocks into the pre-reform benchmark calibration and compare the dynamics of unemployment to those of the post-reform model with the short-time work subsidy[9].

[9]Due to the fact that the effect of the short-time work subsidy is small, the dynamics of the unemployment rate in the post-reform benchmark model without the subsidy is almost the same as those of the post-reform benchmark model with the short-time work subsidy

Figure 9 shows that in the post-reform calibration, the unemployment rate increases by 0.5 percentage points, from 5.72% to 6.22%, while without reform it would have increased by 1.6 percentage points, from 8.4% to 10%. In relative terms, without the reform the unemployment rate would have gone up by a factor of 1.19, compared to the increase by a factor of 1.09 in the reform case.

Welfare

To analyze the impact of the unemployment insurance reform on workers' welfare, I compare the welfare of workers in the pre-reform steady state to welfare along the deterministic transition path in the post-reform steady state. Pre- and post-reform steady state values of workers' value functions are derived in Appendix E, while the transition paths of all endogenous variables are calculated using policy functions obtained from solving the deterministic version of the model.

Suppose that the economy reaches the post-reform steady state in T periods, and consider a long-term unemployed worker. In period T, her value is $V_T^{UL} = V_{\text{post,ss}}^{UL}$, where $V_{\text{post,ss}}^{UL}$ is the post-reform steady-state value of V^{UL}. Using the deterministic version of equation (4) and $V_{\text{post,ss}}^{WL}$, I calculate V_{T-1}^{UL}. Similarly, I calculate the values of V_{T-1}^{US}, V_{T-1}^{WL}, and V_{T-1}^{WS}. I keep iterating backward until I reach period 0, when the reform was implemented and the transition started.

Comparing the welfare of each worker type, I find that the reform makes everyone better off, with welfare gains ranging from 0.5% for long-term unemployed workers to 1.2% for currently employed workers. It turns out that welfare gains stemming from lower taxes, higher profits, higher chances of finding a job, and higher employment values outweigh welfare losses from lower long-term unemployment benefits. In particular, even though the long-term unemployed suffer the most because of lower benefits, their continuation value rises a lot because hiring a long-term unemployed worker is very profitable for the firm and some of this surplus is shared with the worker in the form of higher value of employment.

To assess the impact of the reform on an average worker, I calculate the following,

$$W_{\text{no reform}} = U^S V_{\text{pre,ss}}^{US} + U^L V_{\text{pre,ss}}^{UL} + n^S V_{\text{pre,ss}}^{WS} + n^L V_{\text{pre,ss}}^{WL}$$
$$W_{\text{reform}} = U^S V_0^{US} + U^L V_0^{UL} + n^S V_0^{WS} + n^L V_0^{WL},$$

where weights are equal to the measure of each worker type in the pre-reform steady state. I find that the unemployment insurance reform improved the welfare of an average worker by 1.17%.

To calculate the impact of the reform on welfare in a stochastic setting, I generate 5,000 sequences of productivity shocks, each 10,000 quarters long, calculate the change in welfare along each, and take the average. The resulting increase in well-being ranges from 0.47% for a long-term unemployed worker to 1.21% for a currently employed worker and 1.18% for the average worker.

5 Robustness

5.1 Short-Time Work Subsidy

To assess how robust my findings are to the level of the short-time work subsidy, I solve and simulate the model assuming that all firm's workers are eligible for the subsidy, i.e., $s = 0.67$. As the fifth column of Table 5 shows, higher subsidy significantly increases the volatility of hours per worker, by additional 42.8 percentage points, on the top of a 9.4% increase due to the reform. It also reduces employment volatility by extra 2.3 percentage points. This suggests that even if all workers were involved in the short-time work program, its contribution to the miracle would still be small.

5.2 Fixed Matching Costs

Shimer (2005) argues that the standard search and matching model of the labor market fails to account for the observed high volatility of unemployment and vacancy rates. To increase the volatility, I follow Pissarides (2009) and introduce fixed matching costs that include the costs of negotiating with the successful job applicant, putting her on the firm's payroll, and training her. I assume that when a worker arrives, the firm pays a fixed fee γ before the Nash bargaining takes place. Since these costs are sunk at the time of the Nash bargaining, they do not enter Nash bargaining equations. The constant posting cost κ is now replaced by the cost $\tilde{\kappa} + \gamma q(\theta_t)$, which falls in tightness.

To see why the introduction of fixed matching costs increases the volatility of vacancies, consider first the case when $\gamma = 0$. After a positive productivity shock, the firm posts more vacancies at cost κ each. The entry of new vacancies reduces the vacancy filling rate, $q(\theta_t)$, so that the expected cost of hiring a worker, $\kappa/q(\theta_t)$, goes up. The increase in the hiring costs reduces the response of tightness to the productivity shock. When $\gamma > 0$, the hiring costs rise less than in proportion to $1/q(\theta_t)$ so that the firm's incentives to post vacancies remain high.

For any κ, there exist different combinations of $\tilde{\kappa}$ and γ such that the steady state of the model is unaffected, while the volatility of vacancies differ. Specifically, as we increase γ, the value of $\tilde{\kappa}$ falls and the volatility of vacancies goes up. Since the vacancy posting cost is assumed to be positive, the set of possible γ's is limited from above.

To assess whether my results hold in a more volatile environment, I set γ to 0.1 and recalibrate the benchmark model. First, I find that for the pre-reform steady state to remain unchanged, $\tilde{\kappa}$ should be equal to 0.03. After that, I recalibrate ϕ, ρ_z and σ_z[10]. As the sixth column of Table 4 shows, in the pre-reform calibration the volatility of employment goes up by about 40%, so does the volatility of hours per worker. Although the impact of the reform on employment volatility is bigger, — it falls by 73% compared to 68% in the benchmark model — the effect of the short-time work subsidy is small as before (see Table 5).

6 Conclusion

In this paper, I study the effects of the Hartz IV reform on the cyclical volatility of the German labor market. I build a search and matching model with multi-worker firms and a two-tier unemployment insurance system. The fall in long-term unemployment benefits leads to a larger and less volatile match surplus, and reduces firms' incentives to change vacancies and employment over the business cycle. Firms start relying more on the intensive margin of adjustment. I calibrate my model to German pre-reform data and find that the effect of the reform on volatility of employment and hours per worker is consistent with the data. I show that most of the German labor market miracle can be attributed to the unemployment insurance reform of 2005, and that without it the unemployment rate would have gone up much more, both in absolute and relative terms. I also find that the short-time work program played a minor role in safeguarding jobs during the Great Recession.

From a policy perspective, my paper has two main implications. The first is that unemployment insurance reforms are more effective in reducing the cyclical volatility of employment and safeguarding jobs during downturns than temporary policies that promote labor hoarding, such as the short-time work program. However, for reforms to be welfare-improving, policymakers need to enhance efforts to ensure that gains from reforms are not

[10]The new values are as follows: $\phi = 0.0032$, $\rho_z = 0.954$ and $\sigma_z = 0.94\%$.

appropriated solely by employers, and that newly created jobs are not inferior to existing ones in terms of duration and job security.

There are several areas for future research. Job separations in my model are exogenous, while according to recent empirical studies, in Germany changes in unemployment inflows (the job-separation rate) are more important than changes in outflows (the job-finding rate). It would be interesting to see how robust my findings are in a setting with endogenous separations. Extending my model further by introducing precautionary savings, skill heterogeneity and skill loss during unemployment would allow us to analyze in more detail the welfare implications of the Hartz IV reform. Finally, it seems that in an environment with multi-worker firms, the way in which hours per worker are chosen affects the quantitative implications of the model. In particular, the level of employment is much more sensitive to changes in unemployment benefits when firms choose hours per worker than when firms bargain over both hourly wages and hours with each individual worker. It is worth exploring this further to identify how hours per worker are actually determined, based on what we observe in the data.

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Appendix

Appendix A. Bargaining

The first-order conditions of the bargaining game are

$$\omega J_{St} = (1-\omega)(V_t^{WS} - V_t^{US}) \tag{9}$$

$$\omega J_{Lt} = (1-\omega)(V_t^{WL} - V_t^{UL}) \tag{10}$$

From (1) and (2), we get

$$V_t^{WS} - V_t^{WL} = w_t^S h_t - w_t^L h_t \tag{11}$$

From (7) and (8) we get

$$J_{St} - J_{Lt} = -w_t^S h_t + w_t^L h_t \tag{12}$$

Subtract (10) from (9) to get

$$\omega(J_{St} - J_{Lt}) = (1-\omega)(V_t^{WS} - V_t^{US} - V_t^{WL} + V_t^{UL}) \tag{13}$$

To derive the equilibrium wage for long-term unemployed, subtract (4) from (2) and rearrange

$$V_t^{WL} - V_t^{UL} = w_t^L h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta \mathbb{E}_t[(1-\rho^x)V_{t+1}^{WS} + \rho^x V_{t+1}^{US} - \theta_t q(\theta_t) V_{t+1}^{WL} - (1-\theta_t q(\theta_t))V_{t+1}^{UL}] \Rightarrow$$

$$\begin{aligned} V_t^{WL} - V_t^{UL} = {} & w_t^L h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta \mathbb{E}_t[V_{t+1}^{WS} - \rho^x (V_{t+1}^{WS} - V_{t+1}^{US}) \\ & -V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{WL} - (1-\theta_t q(\theta_t))V_{t+1}^{UL}] \Rightarrow \end{aligned}$$

$$\begin{aligned} V_t^{WL} - V_t^{UL} = {} & w_t^L h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta \mathbb{E}_t[V_{t+1}^{WS} - V_{t+1}^{WL} - \rho^x (V_{t+1}^{WS} - V_{t+1}^{US}) \\ & +(1-\theta_t q(\theta_t))(V_{t+1}^{WL} - V_{t+1}^{UL})] \end{aligned} \tag{14}$$

Using (9)-(11), rewrite (14) as

$$\begin{aligned} V_t^{WL} - V_t^{UL} = {} & w_t^L h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta \mathbb{E}_t \Bigg[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x \frac{\omega}{1-\omega} J_{St+1} \\ & +(1-\theta_t q(\theta_t)) \frac{\omega}{1-\omega} J_{Lt+1} \Bigg] \end{aligned} \tag{15}$$

Plug (15) and (8) into (10) and solve for w_t^L

$$(1-\omega)\left(w_t^L h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^L + \beta\mathbb{E}_t[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x\frac{\omega}{1-\omega}J_{St+1}\right.$$

$$\left.+(1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}]\right) = \omega\left(\alpha z_t h_t^\alpha[n_t^S + n_t^L]^{\alpha-1} - w_t^L h_t - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}]\right) \Rightarrow$$

$$w_t^L h_t = \omega(\alpha z_t h_t^\alpha[n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}])$$

$$+(1-\omega)\left(\zeta\frac{h_t^{\mu+1}}{\mu+1} + b^L - \Gamma_t - \beta\mathbb{E}_t[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x\frac{\omega}{1-\omega}J_{St+1} + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}]\right)$$

To derive the equilibrium wage for current workers or short-term unemployed, subtract (3) from (1) and rearrange

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t[(1-\rho^x)V_{t+1}^{WS} + \rho^x V_{t+1}^{US}$$

$$-\lambda[\theta_t q(\theta_t)V_{t+1}^{WS} + (1-\theta_t q(\theta_t))V_{t+1}^{US}] - (1-\lambda)[\theta_t q(\theta_t)V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{UL}]] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t[(1-\rho^x)V_{t+1}^{WS} + \rho^x V_{t+1}^{US}$$

$$-[\theta_t q(\theta_t)V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{UL}] - \lambda[\theta_t q(\theta_t)(V_{t+1}^{WS} - V_{t+1}^{WL}) + (1-\theta_t q(\theta_t))(V_{t+1}^{US} - V_{t+1}^{UL})]] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t\Bigg[V_{t+1}^{WS} - \rho^x(V_{t+1}^{WS} - V_{t+1}^{US})$$

$$-[\theta_t q(\theta_t)V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{UL}] - \lambda\left(\theta_t q(\theta_t)(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) + (1-\theta_t q(\theta_t))\frac{w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}}{1-\omega}\right)$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t\Bigg[V_{t+1}^{WS} - \rho^x(V_{t+1}^{WS} - V_{t+1}^{US})$$

$$-V_{t+1}^{WL} - [-(1-\theta_t q(\theta_t))V_{t+1}^{WL} + (1-\theta_t q(\theta_t))V_{t+1}^{UL}]$$

$$-\lambda\left(\theta_t q(\theta_t)(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) + (1-\theta_t q(\theta_t))\frac{w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}}{1-\omega}\right)\Bigg] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t\Bigg[V_{t+1}^{WS} - \rho^x(V^{WS}(S_{t+1}) - V_{t+1}^{US}) - V_{t+1}^{WL}$$

$$+(1-\theta_t q(\theta_t))(V_{t+1}^{WL} - V_{t+1}^{UL}) - \lambda\left(\theta_t q(\theta_t)(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) + (1-\theta_t q(\theta_t))\frac{w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}}{1-\omega}\right)\Bigg] \Rightarrow$$

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta\frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta\mathbb{E}_t\Bigg[V_{t+1}^{WS} - V_{t+1}^{WL} - \rho^x(V_{t+1}^{WS} - V_{t+1}^{US}) +$$

$$(1-\theta_t q(\theta_t))(V_{t+1}^{WL} - V_{t+1}^{UL}) - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\Bigg] \tag{16}$$

Using (9)-(11), rewrite (16) as

$$V_t^{WS} - V_t^{US} = w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t \Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} \tag{17}$$

$$-\rho^x \frac{\omega}{1-\omega} J_{St+1} + (1-\theta_t q(\theta_t)) \frac{\omega}{1-\omega} J_{Lt+1} - \lambda (w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) \frac{1-\omega\theta_t q(\theta_t)}{1-\omega} \Big] \tag{18}$$

Plug (17) and (7) into (9) and solve for w_t^S

$$(1-\omega)\bigg(w_t^S h_t + \Gamma_t - \zeta \frac{h_t^{\mu+1}}{\mu+1} - b^S + \beta \mathbb{E}_t \Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}$$

$$-\rho^x \frac{\omega}{1-\omega} J_{St+1} + (1-\theta_t q(\theta_t)) \frac{\omega}{1-\omega} J_{Lt+1} - \lambda (w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) \frac{1-\omega\theta_t q(\theta_t)}{1-\omega} \Big] \bigg) =$$

$$= \omega(\alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - w_t^S h_t - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}]) \Rightarrow$$

$$w_t^S h_t = \omega \bigg(\alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1}] \bigg) +$$

$$(1-\omega)\bigg(\zeta \frac{h_t^{\mu+1}}{\mu+1} + b^S - \Gamma_t - \beta \mathbb{E}_t \Big[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}$$

$$-\rho^x \frac{\omega}{1-\omega} J_{St+1} + (1-\theta_t q(\theta_t)) \frac{\omega}{1-\omega} J_{Lt+1} - \lambda (w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}) \frac{1-\omega\theta_t q(\theta_t)}{1-\omega} \Big] \bigg)$$

Using expressions for w_t^S and w_t^L, the first order condition for hours per worker can be rewritten as

$$z_t \alpha h_t^{\alpha-1} [n_t^S + n_t^L]^{\alpha-1} - \frac{\partial g(h_t)}{\partial h_t} = \omega \alpha^2 z_t h_t^{\alpha-1} [n_t^S + n_t^L]^{\alpha-1} - \omega \frac{\partial g(h_t)}{\partial h_t} + (1-\omega)\zeta h_t^\mu$$

Appendix B. Equilibrium System of Equations

Equilibrium of the model is characterized by the following system of equations:

$$w_t^S h_t = \omega(\alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)])+ \tag{19}$$

$$(1-\omega)\left(\zeta\frac{h_t^{\mu+1}}{\mu+1} + b^S - \Gamma_t - \beta\mathbb{E}_t\Bigg[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1}\right.$$

$$\left.-\rho^x\frac{\omega}{1-\omega}J_S(X_{t+1}) + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_L(X_{t+1}) - \lambda(w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1})\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\Bigg]\right)$$

$$w_t^L h_t = \omega(\alpha z_t h_t^\alpha [n_t^S + n_t^L]^{\alpha-1} - g(h_t) + \beta(1-\rho^x)\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)]) \tag{20}$$

$$+(1-\omega)\Bigg(\zeta\frac{h_t^{\mu+1}}{\mu+1} + b^L - \Gamma_t - \beta\mathbb{E}_t\Bigg[w_{t+1}^S h_{t+1} - w_{t+1}^L h_{t+1} - \rho^x\frac{\omega}{1-\omega}J_S(X_{t+1}) + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_L(X_{t+1})$$

$$z_t\alpha h_t^{\alpha-1}[n_t^S + n_t^L]^\alpha - \frac{\partial g(h_t)}{\partial h_t}[n_t^S + n_t^L] = w_t^S n_t^S + w_t^L n_t^L + \frac{\partial w_t^S}{\partial h_t}n_t^S h_t + \frac{\partial w_t^L}{\partial h_t}n_t^L h_t \tag{21}$$

$$\kappa = q(\theta_t)\beta\mathbb{E}_t[p_t^S(w_{t+1}^L h_{t+1} - w_{t+1}^S h_{t+1}) + J_L(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \tag{22}$$

$$\text{where } J_S(X_t, n_t^S, n_t^L) - J_L(X_t, n_t^S, n_t^L) = -w_t^S h_t + w_t^L h_t$$

$$J_L(X_t, n_t^S, n_t^L) = \alpha z_t h_t^\alpha [n_t^L + n_t^L]^{\alpha-1} - w_t^L h_t - g(h_t) + (1-\rho^x)\beta\mathbb{E}_t[J_S(X_{t+1}, n_{t+1}^S, n_{t+1}^L)] \tag{23}$$

$$U_{t+1}^S = \rho^x N_t + \lambda(1 - q(\theta_t)\theta_t)p_t^S U_t \tag{24}$$

$$p_t^S = U_t^S / U_t \tag{25}$$

$$\theta_t = V_t / U_t \tag{26}$$

$$U_t = \bar{N} - N_t \tag{27}$$

$$N_{t+1} = (1-\rho^x)N_t + q(\theta_t)V_t \tag{28}$$

$$n_{t+1}^L = (1 - p_t^S)q(\theta_t)v_t \tag{29}$$

$$n_{t+1}^S = N_{t+1} - n_{t+1}^L \tag{30}$$

$$T_t = b_t^S p_t^S U_t + b_t^L(1 - p_t^S)U_t \tag{31}$$

$$\Gamma_t = g(h_t) \tag{32}$$

$$\Pi_t = z_t([n_t^S + n_t^L]h_t)^\alpha - [w_t^S n_t^S + w_t^L n_t^L]h_t - g(h_t)[n_t^S + n_t^L] - \kappa v_t \tag{33}$$

$|h_t - h|$ is approximated using a smooth function $f(h_t - h)$ given by

$$f(h_t - h) = \frac{2}{k}\log(1 + \exp^{k(h_t-h)}) - (h_t - h) - \frac{2}{k}\log(2),$$

where k is a parameter that controls the smoothness and is set to 1000.

Appendix C. Elasticities

Assume the economy is at the steady state and, for simplicity, that $b^S = b^L = b$. Let $n = n^S + n^L$. From the first order condition for hours per worker we obtain the equilibrium value of hours per worker

$$h = \left(\frac{\alpha z(1-\alpha\omega)n^{\alpha-1}}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}} \tag{34}$$

Plugging it into job creation condition yields

$$\kappa = q(\theta)\beta\mathbb{E}_t[J_L(X_{t+1}, n^S_{t+1}, n^L_{t+1})]$$
$$J_L(S_t, n^S_t, n^L_t) = \alpha z_t h^\alpha_t n^{\alpha-1}_t - w^L_t h_t + (1-\rho^x)\frac{\kappa}{q(\theta_t)}$$

Expression for w^L_t becomes

$$w^L h = \omega(\alpha z h^\alpha n^{\alpha-1} + \beta(1-\rho^x)\mathbb{E}_t[J_L]) + (1-\omega)\left(\zeta\frac{h^{\mu+1}}{\mu+1} + b - \beta\mathbb{E}_t\left[(1-\theta_t q(\theta_t) - \rho^x)\frac{\omega}{1-\omega}J_L\right]\right)$$

Job creation condition becomes

$$\kappa\left[\frac{1}{\beta q(\theta)} - (1-\rho^x)\frac{1}{q(\theta)} + \omega\theta\right] = (1-\omega)\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)$$

where

$$K = \alpha^{\frac{1+\mu}{\mu+1-\alpha}}\zeta^{\frac{-\alpha}{\mu+1-\alpha}}\left(\frac{1-\alpha\omega}{1-\omega}\right)^{\frac{\alpha}{\mu+1-\alpha}}\left[1 - \frac{1-\alpha\omega}{(\mu+1)(1-\omega)}\right]$$

Using implicit differentiation, we obtain

$$\frac{\partial\theta}{\partial z} = \frac{(1-\omega)\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\frac{\frac{\partial q(\theta)}{\partial\theta}}{q(\theta)^2} - (1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}} =$$

$$= \frac{\kappa\left[\frac{1}{\beta q(\theta)} - (1-\rho^x)\frac{1}{q(\theta)} + \omega\theta\right]\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)\left[\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\frac{\frac{\partial q(\theta)}{\partial\theta}}{q(\theta)^2} - (1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

$$= \frac{\kappa\left[\frac{1}{\beta q(\theta)} - (1-\rho^x)\frac{1}{q(\theta)} + \omega\theta\right]\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)\left[\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\frac{m_0\xi\theta^{-\xi-1}}{q(\theta)m_0\theta^{-\xi}} - (1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

$$= \frac{\kappa\left[\frac{1}{\beta q(\theta)} - (1-\rho^x)\frac{1}{q(\theta)} + \omega\theta\right]\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)\left[\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\frac{\xi}{q(\theta)\theta} - (1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

$$= \frac{q(\theta)\theta\kappa\left[\frac{1}{\beta q(\theta)} - (1-\rho^x)\frac{1}{q(\theta)} + \omega\theta\right]\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)\left[q(\theta)\theta\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\xi - q(\theta)\theta(1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

$$= \frac{\theta\kappa\left[\frac{1-\beta(1-\rho^x)}{\beta} + \omega\theta q(\theta)\right]\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)\left[q(\theta)\theta\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\xi - q(\theta)\theta(1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

$$= \frac{\theta M}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)},$$

where

$$M = \frac{\kappa\left[\frac{1-\beta(1-\rho^x)}{\beta} + \omega\theta q(\theta)\right]\left[K\frac{1+\mu}{\mu+1-\alpha}z^{\frac{1+\mu}{\mu+1-\alpha}-1}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}}\right]}{\left[q(\theta)\theta\omega\kappa - \kappa\left(\frac{1-\beta(1-\rho^x)}{\beta}\right)\xi - q(\theta)\theta(1-\omega)Kz^{\frac{1+\mu}{\mu+1-\alpha}}\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}-1}\frac{\partial n}{\partial\theta}\right]}$$

Thus, productivity elasticity of labor market tightness is given by

$$\varepsilon_{\theta,z} = \frac{zM}{\left(Kz^{\frac{1+\mu}{\mu+1-\alpha}}n^{\frac{(\alpha-1)(1+\mu)}{\mu+1-\alpha}} - b\right)} \tag{35}$$

Use the law of motion of the aggregate employment to calculate the productivity elasticity of employment

$$\varepsilon_{n,z} = \frac{\partial n}{\partial z}\frac{z}{n} = \frac{\rho m_0(1-\xi)\theta^{-\xi}\frac{\partial\theta}{\partial z}}{(\rho^x + \theta q(\theta))^2}\frac{z}{n} = \frac{\rho^x m_0(1-\xi)\theta^{-\xi}\frac{\partial\theta}{\partial z}}{(\rho^x + \theta q(\theta))}\frac{z}{\theta q(\theta)} \Rightarrow$$

$$\varepsilon_{n,z} = \varepsilon_{\theta,z}\frac{\rho^x(1-\xi)}{(\rho^x + \theta q(\theta))} \tag{36}$$

Finally, the productivity elasticity of hours per worker is given by

$$h = \left(\frac{\alpha z(1-\alpha\omega)n^{\alpha-1}}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}}$$

$$\varepsilon_{h,z} = \left(\frac{\alpha(1-\alpha\omega)}{\zeta(1-\omega)}\right)^{\frac{1}{\mu+1-\alpha}}\frac{z^{\frac{\alpha-\mu}{\mu+1-\alpha}}n^{\frac{2(\alpha-1)-\mu}{\mu+1-\alpha}}}{\mu+1-\alpha}\left[n + (\alpha-1)z\frac{\partial n}{\partial\theta}\frac{\partial\theta}{\partial z}\right]\frac{z}{h}$$

$$= \left(\frac{\alpha(1-\alpha\omega)}{\zeta(1-\omega)} \right)^{\frac{1}{\mu+1-\alpha}} \frac{z^{\frac{1}{\mu+1-\alpha}} n^{\frac{2(\alpha-1)-\mu}{\mu+1-\alpha}}}{\mu+1-\alpha} \left[n + (\alpha-1) z \frac{\partial n}{\partial \theta} \frac{\partial \theta}{\partial z} \right] \left(\frac{\alpha z(1-\alpha\omega) n^{\alpha-1}}{\zeta(1-\omega)} \right)^{\frac{-1}{\mu+1-\alpha}} \Rightarrow$$

$$\varepsilon_{h,z} = \frac{1}{\mu+1-\alpha} \left[1 - (1-\alpha) \varepsilon_{n,\theta} \varepsilon_{\theta,z} \right] \tag{37}$$

The productivity elasticity of output is given by

$$\varepsilon_{y,z} = \frac{\partial [z(hn)^\alpha]}{\partial z} \frac{z}{y} = \left[(hn)^\alpha + z\alpha h^\alpha n^{\alpha-1} \frac{\partial n}{\partial z} + z\alpha h^{\alpha-1} n^\alpha \frac{\partial h}{\partial z} \right] \frac{z}{y} =$$

$$= 1 + \alpha \frac{\partial n}{\partial z} \frac{z}{n} + \alpha \frac{\partial h}{\partial z} \frac{z}{h} = 1 + \alpha (\varepsilon_{n,z} + \varepsilon_{h,z}) \tag{38}$$

Appendix D. Model with One-Worker Firms

The economy consists of a large number of infinitely lived firms. A firm-worker pair produces $y_t = z_t h_t^\alpha$ of a single homogeneous good. The values of being employed and unemployed are the same as in the benchmark model.

The value of a short-time unemployed worker is given by

$$J_S(X_t) = \max_{h_t^S} z_t(h_t^S)^\alpha - w_t^S h_t^S - g(h_t^S) + \beta\mathbb{E}_t[(1-\rho^x)J_S(X_{t+1}) + \rho^x J_V(X_{t+1})],$$

where h_t^S denotes hours worked and $J_V(\cdot)$ is the value of unfilled vacancy. The first order condition for h_t^S is

$$z_t\alpha(h_t^S)^{\alpha-1} - \frac{\partial g(h_t^S)}{\partial h_t^S} = w_t^S + \frac{\partial w_t^S}{\partial h_t^S} h_t^S$$

The value of a long-time unemployed worker is

$$J_L(X_t) = \max_{h_t^L} z_t(h_t^L)^\alpha - w_t^L h_t^L - g(h_t^L) + \beta\mathbb{E}_t[(1-\rho^x)J_S(X_{t+1}) + \rho^x J_V(X_{t+1})].$$

The first order condition for h_t^L is

$$z_t\alpha(h_t^L)^{\alpha-1} - \frac{\partial g(h_t^L)}{\partial h_t^L} = w_t^L + \frac{\partial w_t^L}{\partial h_t^L} h_t^L$$

The value of unfilled vacancy is given by

$$J_V(X_t) = -\kappa + \beta\mathbb{E}_t[q(\theta_t)(p_t^S J_S(X_{t+1}) + (1-p_t^S)J_L(X_{t+1})) + (1-q(\theta_t))J_V(X_{t+1})]$$

Free entry of firms implies that firms post vacancies until the value of doing so is equals to zero, $J_V(X_t) = 0$. Therefore, in equilibrium the following condition holds

$$\kappa = \beta q(\theta_t)\mathbb{E}_t[p_t^S J_S(X_{t+1}) + (1-p_t^S)J_L(X_{t+1})]$$

Each firm-worker pair bargains over hourly wages, $w_t^S(X_t, h_t^S)$ or $w_t^L(X_t, h_t^L)$, that are given by

$$\begin{gathered} w_t^S(X_t, h_t^S)h_t^S = \omega(z_t(h_t^S)^\alpha - g(h_t^S) + \beta(1-\rho^x)\mathbb{E}_t[J_{St+1})]) + \\ (1-\omega)\left(\zeta\frac{(h_t^S)^{\mu+1}}{\mu+1} + b^S - \Gamma_t^S - \beta\mathbb{E}_t\left[w_{t+1}^S h_{t+1}^S - w_{t+1}^L h_{t+1}^L\right.\right. \\ \left.\left. -\rho^x\frac{\omega}{1-\omega}J_{St+1}) + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}) - \lambda(w_{t+1}^S h_{t+1}^S - w_{t+1}^L h_{t+1}^L)\frac{1-\omega\theta_t q(\theta_t)}{1-\omega}\right]\right) \end{gathered}$$

$$w_t^L(X_t, h_t^S)h_t^L = \omega(z_t(h_t^L)^\alpha - g(h_t^L) + \beta\mathbb{E}_t[(1-\rho^x)J_{St+1})]])$$
$$+(1-\omega)\left(\zeta\frac{(h_t^L)^{\mu+1}}{\mu+1} + b^L - \Gamma_t^L - \beta\mathbb{E}_t[w_{t+1}^S h_{t+1}^S - w_{t+1}^L h_{t+1}^L - \rho^x\frac{\omega}{1-\omega}J_{St+1} + (1-\theta_t q(\theta_t))\frac{\omega}{1-\omega}J_{Lt+1}]\right)$$

Using expressions for wages, first order conditions for hours per worker can be rewritten as

$$z_t\alpha(h_t^S)^{\alpha-1} - \frac{\partial g(h_t^S)}{\partial h_t^S} = \omega\alpha z_t(h_t^S)^{\alpha-1} - \omega\frac{\partial g(h_t^S)}{\partial h_t^S} + (1-\omega)\zeta(h_t^S)^\mu$$
$$z_t\alpha(h_t^L)^{\alpha-1} - \frac{\partial g(h_t^L)}{\partial h_t^L} = \omega\alpha z_t(h_t^L)^{\alpha-1} - \omega\frac{\partial g(h_t^L)}{\partial h_t^L} + (1-\omega)\zeta(h_t^L)^\mu.$$

From the two equations above it follows that $h_t^S = h_t^L$.

Assuming that the economy is at the steady state, the equilibrium value of hours per worker is given by

$$h = \left(\frac{z\alpha}{\zeta}\right)^{\frac{1}{\mu+1-\alpha}}.$$

The productivity elasticity of hours per worker is given by

$$\varepsilon_{h,z} = \frac{1}{\mu+1-\alpha}.$$

Appendix E. Welfare

Suppose the economy is in the steady state. From the first order condition of the bargaining game, we have that

$$V^{WL} = V^{UL} + \frac{\omega}{1-\omega} J_L \tag{39}$$

From the job creation condition, we obtain the expression for J_L

$$J_L = \frac{\kappa}{q(\theta)\beta} + p^S(w^S h - w^L h) \tag{40}$$

Plug (39) and (40) into equation (4) to obtain

$$V^{UL} = b^L - T + \Pi + \beta[\theta q(\theta) V^{WL} + (1-\theta q(\theta)) V^{UL}] \Rightarrow$$

$$V^{UL} = \frac{1}{1-\beta}\left[b^L - T + \Pi + \beta\theta q(\theta)\frac{\omega}{1-\omega} J_L\right] \Rightarrow$$

$$V^{UL} = \frac{1}{1-\beta}\left[b^L - T + \Pi + \beta\theta q(\theta)\frac{\omega}{1-\omega}\left(\frac{\kappa}{q(\theta)\beta} + p^S(w^S h - w^L h)\right)\right] \tag{41}$$

Knowing V^{UL}, V^{WL} can be obtained from (39), V^{WS} from (11). To solve for V^{US}, use equations (10) and (12).