Searching for a Bargain: Power of Strategic Commitment*

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Abstract

It is common to think of reputation as assets –things of value that require costly investments to build, that can deteriorate if not maintained attentively. This paper shows that under reputational concerns, the equilibrium outcome of a highly competitive market may not be Walrasian. In particular, the incentive of committing to a specific share, the opportunity of building reputation about inflexibility and the anxiety to preserve their reputation can provide significant market power to the players that are in the long side of the market, even when frictions are negligible.

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1. Introduction

Negotiators often use various bargaining tactics, manipulate the adversaries' belief and build false reputation to improve their bargaining positions and shares (Schelling 1960 and Arrow et al. 1995). A growing literature on bargaining and reputation focuses particularly on a specific tactic –standing firm and not backing down from the initial offer– and analyze its impacts on bilateral negotiations (Myerson 1991; Abreu and Gul 2000; Kambe 1999; Compte and Jehiel, 2002; Atakan and Ekmekci, 2010).[1] This paper studies an interesting and unanswered question with the help of a simple game theoretical model; does reputational concerns have any significant impact on markets where price search and negotiation are the main ingredients and frictions are negligibly small? The analyses provide an affirmative answer.

I construct a simple market setup where the long side –the sellers– has *virtually* no market power. There are three defining features of the model. First, a single buyer negotiates with two sellers over the sale of one item. Second, the sellers make initial posted-price offers in the Bertrand fashion. The buyer can accept one of these costlessly, or else try to bargain for a lower price.[2] Third, each of three players suspects that the opponents might have some kind of irrational commitment forcing them to insist on a specific allocation.[3] That is, the players can be obstinate with small probabilities which affects their negotiating tactics and provides incentives to build reputation on their resoluteness.[4] For analytical clarity, I construct the model with negligibly small frictions

[1]Atakan and Ekmekci (2010) is the most related work to this paper as they study a market environment with multiple players. However, their main focus is substantially different. They show –in a market with large numbers of buyers and sellers– that existence of commitment types and endogenous outside options provide enough incentive for the rational players to create false reputation on obstinacy. On the other hand, in this paper, I aim to answer how reputational concerns affect the market participants' pricing and search decisions. For further discussion on this related literature, please see Section 7.

[2]Bertrand paradigm has been extensively used to study competitive markets. Bertrand (1883) assumes that each seller can supply the entire market as the sellers have constant average costs, and that buyers can freely accept one price that the sellers post simultaneously. As a result of these specifications, the presence of two price-setting firms suffices to yield the perfectly competitive outcome. Because of this result, many accepted models in the information economics and the industrial organization literatures have employed the Bertrand approach to reproduce competitive markets. See, for example, Spence (1973), Rothschild and Stiglitz (1976) and Stiglitz and Weiss (1981).

[3]Shelling (1960) points out the potential benefits of commitment in strategic and dynamic environments and asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. Crawford (1982), Muthoo (1996) and Ellingson and Miettinen (2008) follow this approach and show that commitment can be rationalized in equilibrium if (revoking) it is costly. However, I adopt the approach following Kreps and Wilson (1982) and Milgrom and Roberts (1982) where commitments are modeled as behavioral types that exist in the society so that the rational players can mimic if they like to do so. Abreu and Sethi (2003) supports the existence of commitment types from evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such behavioral types is not compatible with evolutionary stability in bargaining environments.

[4]Obstinate (or commitment) types take an extremely simple form. Parallel to Myerson (1991) and Abreu and Gul (2000) a commitment player always demands a particular share and accepts an offer

and then take their limit to zero (vanishing frictions). Therefore, I assume that the initial priors of each player being obstinate is small but positive and the search cost that the rational buyer incurs at each time he switches his bargaining partner is very small but positive.

The analysis of the model shows that even in the limit where the frictions vanish, a range of prices that includes the monopoly price and zero are compatible in equilibrium.[5] Therefore, reputational concerns may give the sellers significant market power in a highly competitive market environment.

The formalization I propose in this article has three major benefits. First, the model facilitates the investigation of the roles of strategic commitment and reputation that are elements missing in existing formal models of search and multilateral bargaining. For example, the important finding of bargaining models in search markets is that an outside option plays a limited or no role when the continuation of negotiation is at least as valuable as that of the outside option. The current model, however, makes this prediction invalid by showing that the availability of an endogenous outside option substantially affects the outcome in the bargaining between a buyer and a pair of sellers, if reputational concerns are present.

In the model, the rational buyer can costlessly learn and accept the sellers' posted prices. Therefore, price search is indeed costless. However, for analytical convenience, searching for a bargain price is assumed to be costly as the buyer suffers very small but positive switching cost each time he changes his bargaining partner. Regardless of his initial reputation, the rational buyer believes that he can achieve a lower price by haggling with the sellers, and low cost for searching a deal makes haggling more attractive than accepting a seller's posted price. Indeed, the rational buyer strictly prefers to visit sellers if his initial reputation is high, i.e. the buyer is *strong*, and is indifferent between visiting stores and immediate acceptance of the lowest price if the rational buyer is *weak*, that is his initial reputation is low enough.

Equilibrium analysis shows that sellers have no bargaining power when they fail to coordinate on their initial offers or when the buyer's initial reputation is sufficiently high, i.e. the buyer is strong. The reason behind this finding is simple. First, in equilibrium, the buyer's outside option of leaving a seller is high means that he prefers to walk away from

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if and only if it weakly exceeds that share. An obstinate seller, for example, always offers his original posted price, and never accepts an offer below that price. Similarly, an obstinate buyer always offers a particular amount, and will never agree to pay more. Thus, a rational player must choose either to mimic an inflexible type, or reveal his rationality and continue negotiation with no uncertainty regarding his actual type. Therefore, reputation of a player is the posterior probability (attached to this player) of being the obstinate type.

[5] This is true regardless of the players' time preferences. By vanishing frictions I mean that initial priors and the buyer's search cost converge to zero.

this seller's store rather than to accept the seller's price. Clearly, this is the case when the buyer's reputation is sufficiently higher than the sellers' reputation or the other seller posts a lower price.[6] Second, in standard models where obstinate types are nonexistent, a seller can always offer the buyer his continuation value and prevent the buyer leaving him empty-handed. However, when commitment types are present, offering something different than his posted price reveals a seller's type (flexibility), which yields surplus smaller than what he can achieve by accepting the buyer's offer (see Myerson 1991; Compte and Jehiel 2002). As a result, if the buyer's outside option is sufficiently high, then the buyer's bargaining power becomes substantially strengthened so that the sellers accept any positive share the buyer is about to offer.

However, when the buyer is weak, then the rational buyer's desire or hope to make a better deal turns into a trap. This trap drags the rational buyer into a situation where he may get much less than what he would achieve if he would have committed himself to accept the lowest posted price. The problem is that the rational buyer cannot commit himself to immediate acceptance because searching for a bargain is equally attractive to him. For this reason, the rational sellers do not have to compete with each other over their posted prices when the buyer is weak, yielding non-Walrasian outcomes consistent with equilibrium.[7] High search cost clearly makes this trap go away as the rational buyer knows that high cost decreases the attractiveness of searching for a deal.

Arguably, this trap –caused by the buyer's reputational weakness and low search cost– probably is the reason for significant markups in some markets, e.g. oriental bazaars, where there are many stores next to one another, selling (almost) identical products.

The second significant benefit of the formalization is that given the sellers' initial offers, the equilibrium strategies in the multilateral bargaining game is essentially unique.[8]

[6] If the buyer's reputation is sufficiently high, then he can sustain a long delay to convince a seller about his resoluteness. In this case, the rational buyer expects to receive some surplus from the seller, that is closer to his own terms.

[7] If the buyer is weak, then reputation has a *lock-in* effect (see Klemperer, 1987) which provides leverage to the sellers. On the one hand, for the rational buyer, conceding to the first seller is at least as good as visiting the second seller when the buyer is weak and the sellers post the same price. The rational buyer can credibly threaten the first seller to terminate the negotiation only if he maintains enough reputation to make his obstinacy credible against the second seller. But, this is possible if the rational buyer is playing a strategy in which he accepts the seller's price with a positive probability. Therefore, the rational buyer cannot abandon a seller unless he guarantees a positive expected surplus to that seller. On the other hand, price undercutting is not optimal for the sellers. We reach this conclusion in two steps. First, if a seller price undercuts, then he would be perceived as obstinate. Second, as I argued previously, posting different prices will improve the buyer's bargaining power remarkably. As a result, in a competitive environment, being perceived as an obstinate seller reduces the chance that his offer is accepted because the rational buyer prefers to visit the seller, who is very likely to be flexible, first and this restrains a rational seller from underbidding his competitor. This contrasts with the prediction in the two-person bargaining model of Abreu and Gul (2000). In their model, being perceived as an obstinate type causes the concession by the rational opponent.

[8] This finding differs from the standard conclusion in non-cooperative bargaining games that informational asymmetries give rise multiplicities. See, for example, Osborne and Rubinstein (1990)

This makes the model a fruitful ground to answer further questions regarding the impacts of reputation on market outcomes and structures. One immediate extension I examine in the paper investigates the effects of reputation in "large markets". The current model presumes that the buyer's moves throughout the haggling process are observable by the sellers. Therefore, the buyer can use his reputation that is built in one store against the other seller. This might be a strong assumption for large markets where the buyers are usually anonymous. For this reason, in Section 4, I relax this condition and suppose that the buyer's arrival time to stores, initial offers and the time he spends in each store are not publicly observable. The simple model in this section shows that anonymity increases the sellers' market power even further.

The third advantage of the formalization is that its predictions are robust in many aspects. For instance, in Section 3 (Proposition 3.4), I check if the impacts of reputation decrease in "larger" markets where the number of sellers is greater than two, and show that a range of prices including the monopoly price and zero are still consistent with equilibrium. In addition, Section 5 shows that the premises on the obstinate buyer's store selection has no significant effect. That is, even if the obstinate buyer is committed to immediately leave a seller's store once his offer is not accepted, then the lock-in effect of the reputation will still be in play and lead to non-Walrasian equilibrium prices. Finally, in Section 6, I show that reputational concerns of the players overwhelm their behaviors so that equilibrium has a war of attrition structure –each player is indifferent between accepting his opponents' initial demand and waiting for acceptance. As a result, given the sellers' posted prices, the equilibrium of the haggling process is unique and robust in the sense that it is "independent" of the exogenously assumed bargaining protocols (unlike more familiar but relatively less sophisticated models).[9]

[9]Likewise, Chatterjee and Samuelson (1987), Samuelson (1992), Caruana, Eirav and Quint (2007) and Caruana and Einav (2008) show that credible commitment to certain promises, threats or actions would wash out technical specifications of the bargaining procedures.

2. The Competitive-Bargaining Game in Continuous-Time

Here I define the *competitive-bargaining game* in continuous-time. Section 5 elaborates the assumptions of the model in more detail and presents some robustness results.

The Players: There are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit.[10] The valuation of the good is one for the buyer and zero for the sellers. Both the buyer and the sellers have some small, positive probability of being a "commitment" type. An obstinate (or commitment) type of player $n \in \{1, 2, b\}$, where b represents the buyer, 1 and 2 represents the sellers, is identified by a number $\alpha_n \in [0, 1]$. A type α_i of seller $i \in \{1, 2\}$ always demands α_i, accepts any price offer greater or equal to α_i and rejects all smaller offers. On the other hand, a type α_b of the buyer always demands α_b, accepts any price offer smaller or equal to α_b and rejects all greater offers. I use the terms rational (flexible) or obstinate (inflexible) with the identity of a player (buyer or seller) whenever I want to differentiate the types of the player. Not mentioning these terms with the identity of a player should be understood that I mean both rational and obstinate types of that player.

I denote by $C \subset [0, 1)$ with $0 \in C$ the **finite** set of obstinate types for all three players and by $\pi(\alpha_n)$ the conditional probability that player n is obstinate of type α_n given that he is obstinate.[11] Thus, π is a probability distribution on C satisfying $\pi(\alpha) > 0$ for all $\alpha \in C$. For simplicity, I assume that π is a uniform distribution, and so common for all three players. In case I need to emphasize different obstinate types of player n, I use α_n, α'_n and so on. The initial probability that n is obstinate (i.e. player n's initial reputation) is denoted by z_n. I restrict my attention to the case where the sellers' initial reputations are the same (that is $z_i = z_s$ for $i = 1, 2$) and that z_b and z_s take sufficiently small values. Finally, I denote by r_b and r_s the rate of time preferences of the rational buyer and the sellers, respectively.

The Timing of the Game: The competitive-bargaining game between the sellers and the buyer is a two-stage, infinite horizon, continuous-time game. The sellers make initial posted-price offers; the buyer can accept one of these costlessly (over the phone, say), or else visit one of the stores and try to bargain for a lower price. The buyer can negotiate only with the seller whom he is currently visiting. The buyer is free to walk out of one store and try the other, but at a cost (delay) of switching which is assumed

[10] At the end of Section 3, I consider the case where the number of sellers is some $N > 2$. In Section 3 I show that non-Walrasian prices can be supported in equilibrium even though the buyer has monopsony power. In this respect, having more than one buyer can only strengthen the main findings of the paper.

[11] Having $1 \notin C$ does not affect the analyses and the results of the paper but eliminates additional cases that produce nothing new.

to be very small. The reader may wish to picture this market as an environment where the sellers' stores are located at opposite ends of a town, and so changing the bargaining partner is costly for the buyer because it takes time to move from one store to the other and the buyer discounts time.

More formally, stage 1 starts and ends at time zero and the timing within the first stage is as follows. Initially, each seller simultaneously announces (posts) a demand (price) from the finite set C and it is observable by the buyer.[12] After observing the sellers' demands, the buyer has two options. He can accept one of the posted prices and finish the game. Or, he can make a counter offer that is observable by the sellers and visit one of the sellers to start the second stage (the bargaining phase).

Note that if seller i is rational and posting the price of $\alpha_i \in C$ in stage 1, then this is his strategic choice. If he is the obstinate type, then he merely declares the demand corresponding to his type. Given the description of the obstinate players, if the buyer accepts α_i and finishes the game at time zero, then he is either rational and finishing the game strategically or obstinate of type α_b such that $\alpha_b \geq \alpha_i$. Likewise, if the buyer makes a counter offer $\alpha_b \in C$ which is incompatible with the sellers' demands, i.e., $\alpha_b < \min\{\alpha_1, \alpha_2\}$, then this may be because the buyer is rational and strategically demanding this price or because the buyer is the obstinate type α_b.[13]

Upon the beginning of the second stage (at time zero) the buyer and seller i, who is visited by the buyer first, immediately begin to play the following *concession game*: At any given time, a player either accepts his opponent's initial demand or waits for a concession. At the same time, the buyer decides whether to stay or leave store i. If the buyer leaves store i and goes to store $j \in \{1, 2\}$ with $j \neq i$, the buyer and seller j start playing the concession game upon the buyer's arrival at that store.[14] Assuming that the sellers are spatially separated, let δ denote the discount factor for the buyer that occurs due to the time, $\Delta > 0$, required to travel from one store to the other. That is, $\delta = e^{-r_b \Delta}$. Note that $1 - \delta$ (the search friction) is the cost that the buyer incurs at each time he switches his bargaining partner.[15] I assume that the search friction is very small, i.e., $1 - \delta$ is very close to zero, and thus the finite set C is coarse relative to the search friction.[16] More specifically, I assume that for all $\alpha, \alpha' \in C$ with $\alpha > \alpha'$ we have

[12] For analytical simplicity, I assume that the set of offers is common for all the players and is equal to the set of obstinate types C. This restriction is dispensable and can be removed with no impact on equilibrium outcomes.

[13] Therefore, if the buyer makes a counter offer and demands α_b that is greater than or equal to the minimum of the posted prices, then the buyer is rational and strategically demanding this price.

[14] After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again. However, the buyer will never behave that way in equilibrium.

[15] One may assume a switching cost for the buyer that is independent of the "travel time" Δ, but this change would not affect our results. However, incorporating the search friction in this manner simplifies the notation substantially.

[16] In some markets, search friction may shape the market participants' behavior significantly. However,

$(1-\alpha) < \delta(1-\alpha')$. The idea behind this assumption is very simple; the friction should not prevent the rational buyer to walk away from a store if he knows that the other seller has posted a lower price.[17] Concession of the buyer or seller i, while the buyer is in store i, marks the completion of the game; if the agreement $\alpha \in \{\alpha_b, \alpha_i\}$ is reached at time t, then the payoffs to seller i, the buyer and seller j are $\alpha e^{-r_s t}$, $(1-\alpha)e^{-r_b t}$ and 0, respectively. In case of simultaneous concession, surplus is split equally.[18]

I denote the two stage competitive-bargaining game in continuous-time by G. The competitive-bargaining game is modeled as a modified war of attrition game. This model is justified in Section 6. There, I show that under some restrictions, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers converge to zero, and this limit is equivalent to the unique outcome of the second stage of the game G.

The Information Structure: There is no informational asymmetry regarding the players' valuations and time preferences. Moreover, all three players' initial offers, the buyer's timing and store selection are observable by the public.[19] However, players have private information about their resoluteness. That is, each player knows its own type but does not know the opponents' true types.

More Details on Obstinate Types: The obstinate types are defined by the strategies they pursue, and so they are *strategy types*. Details of their strategies are important in determining equilibrium behavior of the rational players. The critical assumption for our results is that an obstinate player never backs down from his initial offer during the

there are many examples where search cost is negligible. For example, a growing number of global e-commerce platforms, such as alibaba.com, importers.com, makes it easier for millions of buyers and suppliers around the world to do trade online. If the buyer is, for example, a small sized enterprise that is planning to buy electronic chips from the producers located in Hong Kong and Kuala Lumpur, then it is most likely that the offers and concessions between the parties will be exchanged via e-mail, fax or phone.

[17]This inequality follows from the dynamics of the rational buyer's haggling activities. Suppose that the buyer is in store 1 and playing the concession game with seller 1 whose posted price is α. If the buyer concedes to seller 1, the buyer's instantaneous payoff will be $1-\alpha$. However, if the buyer (immediately) leaves 1 and goes directly to the second seller to accept his posted price α' (where $\alpha' < \alpha$), his discounted payoff will be $\delta(1-\alpha')$. Hence, the inequality $(1-\alpha) < \delta(1-\alpha')$ ensures that the rational buyer will not hesitate to walk away from a store to accept the other seller's lower price.

[18]This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

[19]I consider an extreme case where the buyer's actions (demands) are perfectly observable. Clearly, in some circumstances, e.g. in large markets where traders are rather anonymous, the sellers may not be able to attain all the information nor can the buyer convey it perfectly. For this reason, in Section 4, I consider the other extreme case where the buyer's arrival to the market and moves in negotiating with a seller is unobservable by the public. The simple model in that section shows that anonymity increases sellers' market power further.

concession games. Remaining details of the obstinate players' strategies have minor impact on the main results in Sections 3 and 4, and I prove this by analyzing some possible alternatives in Section 5.

The remaining details of the strategies of the obstinate types are as follows. The obstinate buyer of any type (or demand) $\alpha_b \in C$ understands the equilibrium and leaves his bargaining partner permanently when he is convinced that his partner will never concede. If the sellers' posted prices (α_1 and α_2) are the same, or the obstinate buyer's type (α_b) is incompatible with these prices, then the obstinate buyer visits each seller with equal probabilities. Moreover, if a seller's posted price is compatible with the obstinate buyer's type α_b, that is $\min\{\alpha_1, \alpha_2\} \leq \alpha_b$, then he immediately accepts the lowest price and finishes the game at time zero. Finally, the obstinate buyer with demand α_b never visits a seller who is known to be the commitment type with demand $\alpha > \alpha_b$.[20]

The assumption on the obstinate buyer's departure habit seems a strong one since it eliminates the possibility that the rational buyer would increase his bargaining power by committing to a particular pattern of store choice. Consider, for example, the case where the obstinate buyer is more aggressive. That is, he commits himself to immediately switch to another seller if the first seller does not concede right away. In some situations, it will increase the rational buyers payoff. However, as the results in Section 5 show, it does not alter the main message of the paper. That is, multiple, non-Walrasian prices can be supported in equilibrium.

Strategies of the Rational Players: In the first stage of the competitive-bargaining game G, a strategy for rational seller i, μ_i, is a distribution function over the set C. For any $\alpha_i \in C$, $\mu_i(\alpha_i)$ is the probability that rational seller i announces the demand α_i.

A first-stage strategy for the rational buyer consists of two parts; μ_b and σ_i. Although the strategy μ_b is a function of the sellers' announcements, α_1 and α_2, and σ_i is a function of all three players' announcements, these connections are omitted for notational simplicity. Given that each seller posts α_i, $\mu_b(\alpha_b)$ is the probability that the rational buyer announces the demand $\alpha_b \in C$ with $\alpha_b \leq \alpha$ where $\alpha = \min\{\alpha_1, \alpha_2\}$. That is, μ_b is a probability measure over $C_\alpha = \{x \in C | x \leq \alpha\}$. I require that the game G ends in stage 1 when the rational buyer announces α. That is, immediate concession of the buyer is represented by the buyer's announcement of α. Moreover, σ_i denotes the probability of the rational buyer visiting seller i first, and so $\sigma_1 + \sigma_2 = 1$.

If the competitive-bargaining game proceeds to stage 2 and the first stage strategies of the players are μ_1, μ_2, σ_1 and μ_b, then the Bayes' rule implies the followings: The

[20] This assumption is consistent with the story that the obstinate buyer can understand the equilibrium; he knows that visiting an obstinate seller with a demand higher than α_b has no point because it is impossible to reach an agreement with him.

probability of seller i being obstinate conditional on posting price α_i is

$$\frac{z_s\pi(\alpha_i)}{z_s\pi(\alpha_i)+\mu_i(\alpha_i)(1-z_s)} := \hat{z}_i(\alpha_i)$$

Furthermore, the probability that the buyer is the commitment type conditional on announcing his demand as $\alpha_b < \alpha$ and visiting seller i first is[21]

$$\frac{\frac{1}{2}z_b\pi(\alpha_b)}{\frac{1}{2}z_b\pi(\alpha_b)+(1-z_b)\sigma_i\mu_b(\alpha_b)\left[\sum_{x<\alpha}\pi(x)\right]} \tag{1}$$

Second stage strategies are relatively more complicated. A nonterminal history of length t, h_t, summarizes the initial demands chosen by the players in stage 1, the sequence of stores the buyer visits and the duration of each visit until time t (inclusive). For each $i = 1, 2$, Let $\hat{H}_t^i$ be the set of all nonterminal histories of length t such that the buyer is in store i at time t. Also, let H_t^i denote the set of all nonterminal histories of length t with which the buyer just enters store i at time t.[22] Finally, set $\hat{H}^i = \bigcup_{t\geq 0}\hat{H}_t^i$ and $H^i = \bigcup_{t\geq 0} H_t^i$.

The buyer's strategy in the second stage has three parts. The first part determines the buyer's location at any given history. For the other two parts, $\mathscr{F}_b^i$ for each i, let $\mathbb{I}$ be the set of all intervals of the form $[T,\infty]$ ($\equiv [T,\infty)\cup\{\infty\}$)for $T\in\mathbb{R}_+$, and $\mathbb{F}$ be the set of all right-continuous distribution functions defined over an interval in $\mathbb{I}$. Therefore, $\mathscr{F}_b^i : H^i \to \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F_b^{i,T} : [T,\infty] \to [0,1]$ *representing the probability of the buyer conceding to seller i by time t (inclusive).* Similarly, seller i's strategy $\mathscr{F}_i : H^i \to \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F_i^T : [T,\infty] \to [0,1]$ *representing the probability of seller i conceding to the buyer by time t (inclusive).*

Player n's reputation $\hat{z}_n$ is a function of histories and n's strategies, representing the probability that the other players attach to the event that n is obstinate. It is updated according to the Bayes' rule. At the beginning of the game we have $\hat{z}_b(\emptyset) = z_b$ and $\hat{z}_i(\emptyset) = z_s$ for each seller i, where $\emptyset$ represents the null history. Given the rational buyer's first stage strategies and a history h_0 where the buyer announces α_b and visits seller i first, the buyer's reputation at the time he enters store i, i.e. $\hat{z}_b(h_0)$, is given by Equation (1). Following the history h_0, if the buyer plays the concession game with seller i until some time $t > 0$, and the game has not ended yet (call this history h_t), then the buyer's reputation at time t is $\frac{\hat{z}_b(h_0)}{1-F_b^{i,0}(t)}$, assuming that the buyer's strategy in the concession game is $F_b^{i,0}$.

[21]Given the sellers' announcements α_1 and α_2, the obstinate buyer of type $\alpha_b \geq \alpha = \min\{\alpha_1,\alpha_2\}$ accepts the seller's price α and finalizes the game. Therefore, conditional on the buyer visiting seller i first and demanding some $\alpha_b < \alpha$, the probability that the buyer is obstinate of type α_b should be $\frac{\pi(\alpha_b)}{\sum_{x<\alpha}\pi(x)}$. Moreover, $\frac{1}{2}z_b$ is the probability that the buyer is obstinate and he visits seller i first.

[22]That is, there exits $\epsilon > 0$ such that for all $t' \in [t-\epsilon, t)$, $h_{t'} \notin \hat{H}_t^i$ but $h_t \in \hat{H}_t^i$.

Note from the last arguments that the buyer's reputation at time t reaches 1 when $F_b^{i,0}(t)$ reaches $1-\hat{z}_b(h_0)$. This is the case because $F_b^{i,0}(t)$ **is the sellers' belief about the buyer's play during the concession game with seller** i. That is, it is the strategy of the buyer from the point of view of the sellers. More generally, the upper limit of the distribution function $F_b^{i,T}$ is $1-\hat{z}_b(h_T)$ where $\hat{z}_b(h_T)$ is the buyer's reputation at time $T \geq 0$, the time that the buyer (re)visits store i. That is, $\lim_{t\to\infty} F_b^{i,T}(t) \leq 1-\hat{z}_b(h_T)$. Same arguments apply to the sellers' strategies.

Since I will use z_b, z_s and $\hat{z}_b^i$ extensively in the paper, it is crucial to emphasize what they refer to. I will denote the buyer's and the sellers' initial reputations by z_b and z_s, respectively. The term $\hat{z}_b^i$ represents the buyer's reputation at the beginning of the second stage conditional on him visiting store i first. Clearly, $\hat{z}_b^i$ is a function of the rational buyer's strategy and the realized history of the first stage, however I omit this connection only for notational simplicity.

Given $F_b^{i,T}$, rational seller i's expected payoff of conceding to the buyer at time t (conditional on not reaching a deal before time t where $T \leq t$,) is

$$\begin{aligned} U_i(t, F_b^{i,T}) &:= \alpha_i \int_0^{t-T} e^{-r_s y} dF_b^{i,T}(y) \\ &\quad + \frac{1}{2}(\alpha_i+\alpha_b)[F_b^{i,T}(t) - F_b^{i,T}(t^-)]e^{-r_s(t-T)} + \alpha_b[1-F_b^{i,T}(t)]e^{-r_s(t-T)} \end{aligned} \tag{2}$$

with $F_b^{i,T}(t^-) = \lim_{y\uparrow t} F_b^{i,T}(y)$.

In a similar manner, given F_i^T, the expected payoff of the rational buyer who concedes to seller i at time t is

$$\begin{aligned} U_b^i(t, F_i^T) &:= (1-\alpha_b) \int_0^{t-T} e^{-r_b y} dF_i^T(y) \\ &\quad + \frac{1}{2}(2-\alpha_i-\alpha_b)[F_i^T(t) - F_i^T(t^-)]e^{-r_b(t-T)} + (1-\alpha_i)[1-F_i^T(t)]e^{-r_b(t-T)} \end{aligned} \tag{3}$$

where $F_i^T(t^-) = \lim_{y\uparrow t} F_i^T(y)$.[23]

[23] Expected payoffs are evaluated at time T, and they are conditional on the event that the buyer visits seller i at time $T \geq 0$.

3. Main Results

In this section, I present two main results of the paper. For this purpose, I first fix the values of δ, r_b, r_s and the set of obstinate types C, and show by Theorem 1 that all demands in the set C can be supported in equilibrium for some small z_b and z_s. Then, by Theorem 2, I prove that even in the limit where the frictions vanish, that is z_b and z_s converge to zero, a range of prices that includes the monopoly price and zero are compatible in equilibrium. For any $z_b, z_s \in (0,1)$, let $\mathrm{G}(z_b, z_s)$ denote the competitive-bargaining game G where the initial reputations of the sellers and the buyer are z_b and z_s, respectively.

Theorem 1. *For all $\alpha_s \in C$, there exists some small $z_b, z_s \in (0,1)$ such that α_s is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$.*

A sketch of the proof. A series of statements that I will prove in this section suffice to prove Theorem 1. In this short sketch I will provide the main idea behind these statements. First, note that a seller has no incentive to increase his price if his opponent announces his demand as 0 because the buyer will certainly accept the price of 0 and finalize the game in stage 1. Hence, for any values of the primitives (e.g. z_b and z_s), $0 \in C$ is an equilibrium price of the competitive-bargaining game $\mathrm{G}(z_b, z_s)$.

Now, consider a demand $\alpha_s \in C \setminus \{0\}$, and pick z_s and z_b small enough so that the following condition holds: For all $\alpha_b \in C$ with $\alpha_b < \alpha_s$ we have $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ where $\hat{z}_s = \frac{z_s \pi(\alpha_s)}{z_s \pi(\alpha_s)+1-z_s}$, $A = 1 - \frac{1-\delta}{\delta}\frac{1-\alpha_s}{\alpha_s-\alpha_b}$, $\lambda_b = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha_b}$ and $\lambda_s = \frac{\alpha_b r_s}{\alpha_s-\alpha_b}$. Remark that the parameters A, λ_b and λ_s depend on the sellers' and the buyer's announced demands α_s and α_b, although these notations omit this connection for simplicity.

The following strategy profile σ^G constitute a sequential equilibrium as I will prove it through Propositions 3.1 to 3.3:

1. In stage 1 both rational sellers post the same demand α_s. That is, $\mu_i^*(\alpha_s) = 1$ and $\mu_i^*(\alpha_s') = 0$ for all $\alpha_s' \in C \setminus \{\alpha_s\}$.
2. In stage 1 the rational buyer visits each seller with equal probabilities, i.e. $\sigma_1^* = 1/2$, and declares a demand $\alpha_b < \alpha_s$ according to $\mu_b^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$.
3. (As characterized in Proposition 3.1 and Lemma 3.1) Following a history where the buyer visits seller 1 first, the buyer leaves store 1 at time $T_1^d = -\log(\hat{z}_s)/\lambda_s$ for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller 2 may continue until the time $T_2^e = -\log(\hat{z}_s/A)/\lambda_s$. The players' concession game strategies are $F_b^1(t) = 1 - z_b(A/\hat{z}_s^2)^{\lambda_b/\lambda_s}e^{-\lambda_b t}$ and $F_1(t) = 1 - e^{-\lambda_s t}$

in store 1, and $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda_s t}$ in store 2. Symmetric strategies would work following a history where the buyer visits seller 2 first.[24]

4. (As characterized in Proposition 3.2) In case, one of the sellers, say, seller 2 undercuts his opponent and posts a price $\alpha_2 \in C$ such that $\alpha_2 < \alpha_s$, then there are two possible scenarios:

4.1. If $\alpha_2 > 0$, then the rational buyer announces his demand as 0 and visits seller 1 first (with probability one) to make the take it or leave it offer; he leaves store 1 upon his arrival at that store. Conditional on not reaching a deal, the rational buyer goes directly to seller 2 and accepts α_2. On the other hand, rational seller 1 immediately accepts the buyer's demand. Therefore, in case the game does not end in store 1, the buyer infers that 1 is the obstinate type with demand α_1.

4.2. If $\alpha_2 = 0$, then the buyer immediately accepts the second seller's posted demand and finishes the game in the first stage.

5. (As characterized in Proposition 3.2) If seller 2 deviates and posts a price $\alpha_2 > \alpha_s$, then the buyer visits seller 1 first and never goes to the second store, and the concession game with seller 1 may continue until the time $T_1^e = \frac{-\log \hat{z}_s}{\lambda_s}$ with the following strategies: $F_1(t) = 1 - e^{-\lambda_s t}$ and $F_b^1 = 1 - z_b(1/\hat{z}_s)^{\lambda_b/\lambda_s} e^{-\lambda_b t}$.

As a result of condition (*1*), the Bayes' rule implies that the posterior probability that seller i is obstinate is $\hat{z}_s = \frac{z_s \pi(\alpha_s)}{z_s \pi(\alpha_s) + 1 - z_s}$ if he posts α_s, and is 1 otherwise. Likewise, condition (*2*) implies that the posterior probability that the buyer is obstinate is z_b if he posts some α_b in the support of μ_b^*, and is 1 otherwise.

As a result of condition (*3*), the rational buyer's expected payoff in the game is simply $1 - \alpha_s$ if he announces a demand α_b in the support of μ_b^* (a detailed explanation of this statement is provided after Lemma 3.1). Thus, the rational buyer has no incentive to deviate from his strategy μ_b^*. Moreover, if a rational seller i plays according to his prescribed strategies, his expected payoff in the game is greater than $\frac{u}{2}\left[1 - z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b)\right]$ where $u = \sum_{\alpha_b < \alpha_s} \alpha_b \mu(\alpha_b)$. On the other hand, as Lemma 3.2 and condition (*4*) show, a rational seller's expected payoff is much less than $z_b + z_s$ if he deviates from μ_i^*. Hence, for sufficiently small values of z_b and z_s, posting non zero prices is an optimal strategy for the sellers since the rational sellers' expected payoff under σ^G is strictly greater than what they can achieve by price undercutting.

[24] In what follows I will skip the superscript T in players' strategies and denote them by F_1, F_2, F_b^1 and F_b^2 for notational simplicity.

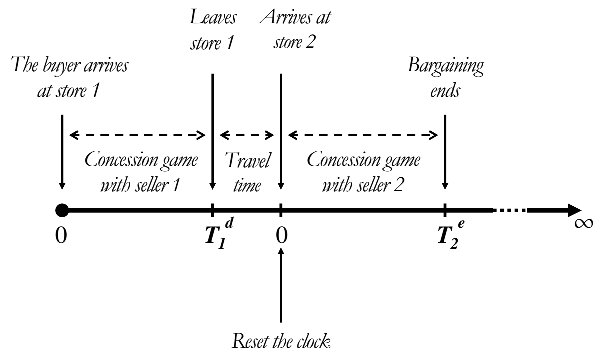

Figure 1: The time-line of the buyer's equilibrium strategy

Q.E.D.

To prove the statements that are necessary for the proof of Theorem 1, I will first start with the equilibrium characterization of stage 2 given the players' strategies of the first stage, i.e. μ_1^*, μ_2^*, μ_b^* and σ_1^*. In what follows, unless otherwise stated, I will consider continuation strategies following a history where the buyer visits seller 1 first and the demand announcements in stage 1 are $\alpha_s \in C \setminus \{0\}$ for the sellers and $\alpha_b \in C$ for the buyer where $\alpha_b < \alpha_s$.

A short descriptive summary of the equilibrium strategies in stage 2 is as follows (see *Figure 1*). The buyer visits each store at most once. When the buyer enters store 1 at time zero, his reputation at this time is $\hat{z}_b^1 = z_b$. Since the buyer's reputation, z_b is low enough (that is $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$), then the rational buyer plays the concession game with seller 1 until time $T_1^d > 0$. At time T_1^d, the buyer leaves store 1 for sure, if the game has not yet ended, and goes directly to store 2. The value of this deterministic departure time from store 1 depends on the primitives.

Note that, building reputation on inflexibility by negotiating with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. In equilibrium, the rational buyer leaves the first store when his discounted expected payoff in the second store is at least as high as his continuation payoff in the first store. Since z_b is low relative to $\hat{z}_s$, in equilibrium, the rational buyer needs to build up his reputation before leaving the first store.

During the concession game, the rational buyer and seller 1 concede by choosing the timing of acceptance randomly with constant hazard rates λ_b and λ_s respectively.

Conditional on the game lasting until time T_1^d, seller 1's reputation reaches one, and the buyer's reputation reaches $\frac{z_b}{1-F_b^1(T_1^d)}$. The last term is less than one because it provides enough incentive to the buyer to walk away from the first store and to search a deal in the second.

Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until T_2^e, the time that both players' reputations simultaneously reach 1. For notational simplicity, I manipulate the subsequent notation and reset the clock once the buyer arrives in store 2. Thus, I define each player's distribution function as if the concession game in each store starts at time zero. In the second store, the rational buyer and seller 2 concede with constant hazard rates λ_b and λ_s respectively. The next two results formally characterize the equilibrium strategies in the second stage.

Proposition 3.1. *In any (sequential) equilibrium of the competitive-bargaining game G, the rational buyer visits each store at most once. Moreover, the rational buyer leaves the first store at some finite time for sure, given that the game does not end before, and directly goes to the other store if and only if the first seller is obstinate. Finally, in an equilibrium where the rational buyer visits seller 1 first with probability* $1/2$*, leaves store 1 at time* T_1^d *and finalizes the game in store 2 at time* T_2^e *if the game has not yet ended before, the players' concession game strategies must be*

$$\begin{aligned} F_b^1(t) &= 1 - c_b^1 e^{-\lambda_b t} \quad & F_1(t) &= 1 - \hat{z}_s e^{\lambda_s (T_1^d - t)} \\ F_b^2(t) &= 1 - e^{-\lambda_b t} \quad & F_2(t) &= 1 - \hat{z}_s e^{\lambda_s (T_2^e - t)} \end{aligned}$$

satisfying

$$F_b^1(0)F_1(0) = 0 \qquad \textit{and} \qquad F_b^2(T_2^e) = 1 - \frac{z_b}{1 - F_b^1(T_1^d)}$$

where $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s - \alpha_b}$ *and* $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$.

I defer the proofs of all the results in this section to Appendix. In equilibrium, the rational buyer's continuation payoff is no more than $1 - \alpha_s$ if he reveals his rationality.[25] Since the obstinate buyer leaves a seller when he is convinced that his bargaining partner is also inflexible, leaving the first seller "earlier" (or "later") than this time would reveal the buyer's rationality. Moreover, since the cost of switching stores is positive, the rational buyer never leaves a seller if there is a positive probability that the seller is flexible, and he immediately leaves otherwise. Clearly the buyer does not revisit a seller once he knows that this seller is obstinate.

Next, I will characterize the buyer's departure time from the first store, T_1^d, the time that the competitive-bargaining game ends in store 2, T_2^e and the rational buyer's *initial*

[25] Arguments similar to the proof of Lemma 2 in the Online Appendix and the one-sided uncertainty result of Myerson (1991, Theorem 8.4) imply this result.

probabilistic concession in the first store, i.e. $F_b^1(0)$. The rational players' equilibrium payoffs in the concession games are calculated by the equations (2) and (3). That is, for each seller i

$$\begin{aligned} v_b^i &= F_i(0)(1-\alpha_b) + [1-F_i(0)](1-\alpha_s), \text{and} \\ v_i &= F_b^i(0)\alpha_s + [1-F_b^i(0)]\alpha_b \end{aligned} \tag{4}$$

However, the rational players' equilibrium payoffs in the game G is different as they should take into account the buyer's outside option and store selection in stage 1. I will provide the rational buyer's payoffs because they are important for the subsequent analyzes.[26]

In equilibrium where the buyer first visits seller 1, the rational buyer leaves the first seller when he is convinced that this seller is obstinate. At this moment, walking out of store 1 is optimal for the rational buyer if his discounted continuation payoff in the second store, δv_b^2, is no less than $1-\alpha_s$, payoff to the rational buyer if he concedes to the obstinate seller 1. Let z_b^* denote the level of reputation required to provide the rational buyer enough incentive to leave the first store. Assuming that $z_b < z_b^*$ (i.e., the rational buyer needs to build up his reputation before walking out of store 1), the game ends in store 2 at time $T_2^e = -\log(z_b^*)/\lambda_b$.[27] Thus, given the value of $F_2(0)$ and the rational buyer's discounted continuation payoff in store 2, z_b^* must solve

$$1-\alpha_s = \delta[1-\alpha_b - \hat{z}_s(\alpha_s-\alpha_b)(z_b^*)^{-\lambda_s/\lambda_b}]$$

implying that $z_b^* = \left(\frac{\hat{z}_s}{A}\right)^{\frac{\lambda_b}{\lambda_s}}$ and $A = 1 - \frac{1-\delta}{\delta}\frac{1-\alpha_s}{\alpha_s-\alpha_b}$. Note that z_b^* is well-defined, i.e. $z_b^* \in (0,1)$, as A is positive. In fact, A is very close to 1 since the cost of traveling is assumed to be very small.

Lemma 3.1. *In equilibrium where the rational buyer visits seller 1 first with probability 1/2 and $z_b \le z_b^*(\hat{z}_s/A)^{\lambda_b/\lambda_s} = (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ holds, the buyer leaves store 1 at time $T_1^d = -\log(\hat{z}_s)/\lambda_s$ for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller 2 may continue until the time $T_2^e = -\log(\hat{z}_s/A)/\lambda_s$. The players' concession game strategies are $F_b^1(t) = 1-z_b(A/\hat{z}_s^2)^{\lambda_b/\lambda_s}e^{-\lambda_b t}$ and $F_1(t) = 1-e^{-\lambda_s t}$ in store 1, and $F_b^2(t) = 1-e^{-\lambda_b t}$ and $F_2(t) = 1-Ae^{-\lambda_s t}$ in store 2.*

I call the buyer ***strong*** if the first seller he visits makes an initial probabilistic concession and ***weak*** otherwise.[28] Similarly, seller i is called strong if the rational buyer

[26] The sellers' expected payoff calculations are more involved, and hence presented in the appendix.

[27] According to Proposition 3.1, $F_b^2(T_2^e) = 1-z_b^*$, which implies the value of T_2^e.

[28] Note that, the second seller (the one who is visited after the first seller) always makes an initial probabilistic concession in equilibrium.

concedes to him with a positive probability at the time he visits store i first at time zero, and weak otherwise.

In equilibrium, $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ implies that the rational buyer's initial reputation is very low and thus he needs to spend significant amount of time to build up his reputation before leaving the first seller. In this case, $F_1(0) = 0$, i.e. the buyer does not receive an initial probabilistic gift from seller 1, implying that the rational buyer is weak and so his expected payoff during the concession game with seller 1, v_b^1, is $1 - \alpha_s$. Therefore, the rational buyer's expected payoff of visiting seller 1 first, V_b^1, is also $1 - \alpha_s$.

Given the second stage equilibrium strategies as characterized by Proposition 3.1 and Lemma 3.1, I now turn my attention to first stage strategies. The next result provides an equilibrium strategy following a history where a seller undercuts his opponent. Lemma 3.2 calculates the deviating seller's payoff.

Proposition 3.2. *Consider a history at which sellers post the prices α_1 and α_2 with $\alpha_1 \neq \alpha_2$, seller 2 is known to be obstinate whereas the true types of seller 1 and the buyer are unknown. Then following continuation strategies form a sequential equilibrium of the continuation game followed by this history:*

(i) *If $\alpha_1 > \alpha_2 > 0$, then the rational buyer announces his demand as 0 and visits seller 1 first (with probability one) to make the take it or leave it offer; he leaves store 1 upon his arrival at that store. Conditional on not reaching a deal, the rational buyer goes directly to seller 2 and accepts α_2. On the other hand, rational seller 1 immediately accepts the buyer's demand.*[29]

(ii) *If $\alpha_1 > \alpha_2 = 0$, then the buyer immediately accepts the second seller's posted demand and finishes the game in the first stage.*

(iii) *If $\alpha_2 > \alpha_1$, then the buyer never visits store 2 and plays the concession game with seller 1 until time $\frac{-\log \hat{z}_s}{\lambda_s}$ with the following strategies: $F_1(t) = 1 - e^{-\lambda_s t}$ and $F_b^1 = 1 - z_b(1/\hat{z}_s)^{\lambda_b/\lambda_s} e^{-\lambda_b t}$.*

Therefore, if seller 2 deviates from his strategy μ_2^* and price undercuts his opponent, then the buyer infers that seller 2 is obstinate with certainty (as sellers are playing pure strategies in the first stage). Being perceived as an obstinate seller reduces the chance that his offer is accepted by the buyer. This is true because the rational buyer prefers to use the obstinate seller's low price as an "outside option" to increase his bargaining power against seller 1 whom he can negotiate and possibly get a much better deal. As a result of this, deviating from an equilibrium price leads to a very low expected payoff for a rational seller as the following result indicates.

[29]Therefore, in case the game does not end in store 1, the buyer infers that 1 is the obstinate type with demand α_1.

Lemma 3.2. *Consider the strategy profile σ^G described above where both sellers post price $\alpha_s > 0$. Suppose that rational seller 2 deviates and posts α_2 in stage 1. Then, his continuation payoff in the game will be zero if $\alpha_2 > \alpha_s$ and $\alpha_2 \left[z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b) + \hat{z}_s(1 - z_b)\right]$, which is strictly less than $(z_b + z_s)\alpha_2$, otherwise.*

Finally, the next result characterizes the set of equilibrium prices of the game $\mathrm{G}(z_b, z_s)$. The main message of the result is simple. A demand $\alpha_s \in C \setminus \{0\}$ is an equilibrium selection of the rational sellers whenever the buyer is weak for all demands $\alpha_b \in C$ with $\alpha_b < \alpha_s$. Hence, in an equilibrium where the sellers post the price of α_s, the rational buyer's expected payoff in the game is $1 - \alpha_s$.

Proposition 3.3. *Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ can be supported as an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ whenever we have $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

The Limiting Case of Complete Rationality

This section shows that a large set of equilibrium prices would be supported in equilibrium even when the frictions vanish. For this purpose, first fix the parameters C, π, r_b, r_s and the friction δ. I say the competitive-bargaining game $\mathrm{G}(z_b^m, z_s^m)$ converges to $\mathrm{G}(K)$ when the sequences $\{z_s^m\}$ and $\{z_b^m\}$ of initial priors satisfy

$$\lim z_s^m = 0, \lim z_b^m = 0 \text{ as } m \to \infty \text{ and } \log z_s^m / \log z_b^m = K \text{ for all } m \geq 0 \tag{5}$$

Theorem 2. *If the game $G(z_b^m, z_s^m)$ converges to $G(K)$, α_s^m is the equilibrium posted price of the rational sellers in the game $G(z_b^m, z_s^m)$ and $\alpha_s \in C$ is a limit point of α_s^m, then we have $2K\alpha_b r_s \leq (1 - \alpha_s) r_b$ holds for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

Proof. Recall that Proposition 3.3 implies that for any given z_b^m and z_s^m small enough the demand $\alpha_s^m \in C$ can be supported as an equilibrium posted price of the sellers in the game $\mathrm{G}(z_b^m, z_s^m)$ whenever $z_b^m \leq [(\hat{z}_s^m)^2/A]^{\frac{\alpha_b r_s}{(1-\alpha_s^m) r_b}}$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s^m$ where $\hat{z}_s^m = \frac{z_s^m \pi(\alpha_s^m)}{z_s^m \pi(\alpha_s^m) + 1 - z_s^m}$. Taking the log of both sides we have

$$\log z_b^m \leq \frac{\alpha_b r_s}{(1 - \alpha_s^m) r_b} \left(2 \log \hat{z}_s^m - \log A\right)$$

dividing both sides by $\log z_b^m$ and taking the limit as $m \to \infty$ we get $2K\alpha_b r_s \leq (1 - \alpha_s) r_b$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$, yielding the desired inequality. □

Theorem 2, together with Proposition 3.3, indicates that there are many prices that are consistent with equilibrium even for vanishing uncertainties. Given the value of $0 < K$, the set of equilibrium prices for the sellers converge to a subset of C containing all $\alpha_s \in C$ that satisfy $\alpha_s \leq \frac{r_b}{r_b+2Kr_s}$. Thus, all prices in C can be supported in equilibrium with carefully selected and vanishing initial priors. The monopoly price of 1, for example, can be arbitrarily approached if the priors z_b^m and z_s^m are selected so that K is sufficiently close to zero.

The final result of this section examines a straightforward extension of the model to the case with $N > 2$ identical sellers. Namely, let $\mathrm{G}^N(z_b^m, z_s^m)$ denote the competitive-bargaining game where the number of sellers is N; it is identical to $\mathrm{G}(z_b^m, z_s^m)$ except the number of players. Let the convergence of $\mathrm{G}^N(z_b^m, z_s^m)$ to the game $\mathrm{G}^N(K)$ be identical to the convergence of its 2-seller counterpart. Therefore,

Proposition 3.4. *If the game $G^N(z_b^m, z_s^m)$ converges to $G^N(K)$, α_s^m is the equilibrium posted price of the rational sellers in the game $G^N(z_b^m, z_s^m)$ and $\alpha_s \in C$ is a limit point of α_s^m, then we have $NK\alpha_b r_s \leq (1-\alpha_s)r_b$ holds for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

Therefore, for any large but finite number of sellers N, we can find small enough z_b^m relative to z_s^m so that $K < 1/N$, and thus prices arbitrarily close to 1 can be supported in equilibrium with vanishing frictions.

4. The Buyer's Moves are Unobservable by the Public

Next, I investigate the case where the buyer's moves and demand announcements are not public. I will show that the sellers' market power will increase further in this case. For this reason, I make three modifications on the competitive bargaining game G. First, the rational buyer announces his demand at the sellers' stores and he can offer different demands in each store.[30] Second, the buyer's moves including his arrival to the market are unknown by the public. That is, sellers can observe the buyer only when he visits their stores. Third, related to the previous one, the buyer arrives at the market according to a Poisson arrival process. Given that the rational buyer plays a strategy in which he visits both sellers with positive probabilities upon his arrival at the market, the last assumption ensures that sellers cannot learn the buyer's actual type and if they are the first or the second store visited by the buyer.[31]

[30] Parallel to the assumptions made in Section 2, the obstinate buyer also announces his demand at the sellers' store if his demand is less than the posted prices. Otherwise, he immediately accepts the lowest posted price and finalize the game in stage 1.

[31] In the modified game, the rational players' strategies, that may depend on time T indicating the

The next result shows that if z_b is sufficiently small, then the following strategies support any $\alpha_s \in C \setminus \{0\}$ as an equilibrium demand selection of the sellers. Strategies are as follows: In stage 1, both sellers post α_s. In stage 2, upon his arrival at time $T \geq 0$, the rational buyer (immediately) visits the sellers with equal probabilities. Upon the buyer's entry to store i (at time T), the rational buyer immediately declares his demand $\alpha_b < \alpha_s$ according to $\mu^T_{\alpha_i}(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$ and starts concession game with seller i. The players' strategies in the concession games are $F^T_b(t) = 1 - \frac{\hat{z}^{T,i}_b}{\hat{z}_s^{\lambda_b/\lambda_s}} e^{-\lambda_b t}$ and $F^T_i(t) = 1 - e^{-\lambda_s t}$ where $\hat{z}^{T,i}_b$ is the probability that the buyer is the commitment type α_b conditional on him visiting seller i at time T and demanding $\alpha_b < \alpha_i$. The rational players' hazard rates λ_b, λ_s are as characterized in Section 3. The concession game with a seller may last until time $T - \log(\hat{z}_s)/\lambda_s$ at which point both the buyer's and the seller's reputations simultaneously reach one.

According to these strategies, the rational buyer will visit only one seller. Moreover, due to the Poisson arrival process and Bayes' rule, the sellers will believe very highly that the buyer is rational conditional on his arrival at their stores. In particular, $\hat{z}^{T,i}_b$ is independent of i and it equals to either z_b or a number very close to z_b. In other words, sellers will learn nothing about the buyer's actual type upon his arrival at their stores because the sellers' prior belief will stay (almost) the same for the entire arrival process.[32] Given that the buyer arrives at the market at time T, the concession game with the seller does not end by the time $-\log(\hat{z}_s)/\lambda_s + T$ if both the buyer and the seller are commitment types. The obstinate buyer with demand α_b leaves the first seller at this time (if the game has not yet ended) and directly goes to the second seller. However, the rational second seller will play the concession game with the (obstinate) buyer believing that his opponent is the obstinate type with probability $\hat{z}_b^{-\log(\hat{z}_s)/\lambda_s+T,i}$ which is very close to z_b.

Proposition 4.1. *Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ can be supported as an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ whenever we have $z_b \leq \frac{\hat{z}_s^{\lambda_b/\lambda_s}}{1+\hat{z}_s(1-\hat{z}_s^{\lambda_b/\lambda_s})}$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

I defer the proof to Appendix. Similar to the analyses in Section 3, the following result shows that a large set of prices can be supported in equilibrium under vanishing frictions.

buyer's arrival time, are equivalent to the strategies defined in Section 2 with one exception. Now, $\mu^T_{\alpha_1}, \mu^T_{\alpha_2}$ are parts of the buyer's second stage strategies and functions of the sellers' posted prices and the arrival time $T \geq 0$. Note that, the first stage is time 0 where the sellers announce their demands and the buyer observes these prices. The second stage starts at the time that the buyer arrives at the market.

[32] I calculate $\hat{z}^{T,i}_b$ formally in the proof of Proposition 4.1

Proposition 4.2. *If the game $G(z_b^m, z_s^m)$ converges to $G(K)$, α_s^m is the equilibrium posted price of the rational sellers in the game $G(z_b^m, z_s^m)$ and $\alpha_s \in C$ is a limit point of α_s^m, then we have $K\alpha_b r_s \leq (1-\alpha_s) r_b$ for all $\alpha_b \in C_{\alpha_s}$.*

Proof. Recall that Proposition 4.1 implies that for any given z_b^m and z_s^m small enough the demand α_s^m is the equilibrium posted price of the sellers in the game $G(z_b^m, z_s^m)$ whenever $z_b^m \leq \frac{(\hat{z}_s^m)^{\lambda_b/\lambda_s}}{1+(\hat{z}_s^m)[1-(\hat{z}_s^m)^{\lambda_b/\lambda_s}]}$ for all $\alpha_b \in C_{\alpha_s^m}$, where $\hat{z}_s^m = \frac{z_s^m \pi(\alpha_s^m)}{z_s^m \pi(\alpha_s^m)+1-z_s^m}$. Taking the log of both sides we have

$$\log z_b^m \leq \frac{\alpha_b r_s}{(1-\alpha_s^m) r_b} \left(\log \hat{z}_s^m - \log\left[1 + \hat{z}_s^m[1-(\hat{z}_s^m)^{\lambda_b/\lambda_s}]\right]\right)$$

dividing both sides by $\log z_b^m$ and taking the limit as $m \to \infty$ we get $K\alpha_b r_s \leq (1-\alpha_s) r_b$ for all $\alpha_b \in C_{\alpha_s}$. $\square$

Finally, since the buyer cannot carry his improved reputation when he leaves a seller, the buyer is weak, regardless of the number of sellers in the market, if $z_b \leq \frac{\hat{z}_s^{\lambda_b/\lambda_s}}{1+\hat{z}_s(1-\hat{z}_s^{\lambda_b/\lambda_s})}$. Therefore, the immediate counterpart of Proposition 3.4 will be as follows.

Corollary 4.1. *If the game $G^N(z_b^m, z_s^m, C^m, \delta^m)$ converges to $G^N(K, [0,1])$, α_s^m is the equilibrium posted price of the rational sellers in the game $G^N(z_b^m, z_s^m, C^m, \delta^m)$ and $\alpha_s \in C$ is a limit point of α_s^m, then we have $K\alpha_b r_s \leq (1-\alpha_s) r_b$ for all $\alpha_b \in C_{\alpha_s}$.*

Note that a demand $\alpha_s \in C$ satisfying the inequality provided in Theorem 2 (or Proposition 3.4) also satisfies the inequality provided in Proposition 4.2 (or Corollary 4.1), but the converse is not true. Thus, if the buyer's moves are unobservable by the public, then the sellers' market powers may increase as higher prices can be supported in equilibrium.

5. Some Extensions

An obstinate player is a man of unyielding perseverance. Sellers may manifest such a steadfast attitude because they might be confined to do so. A company may be inflexible in a wage negotiation due to some regulations within the company. For example, a car dealer, a sales clerk or a realtor may be restricted by the owner regarding how flexible he can be in his demands while negotiating with a buyer. Steady persistence in adhering to a course of action as assumed for an obstinate (type) buyer would be reasonable when, for example, the "buyer" is looking to advance his position. A worker (negotiating with more than one firm) may accept the new job offer if it provides a significant jump in his salary or title relative to the position he is already holding. On the other hand, an entrepreneur who is running a successful small business may commit to his initial demands while negotiating with investors to sell his business or a franchise because of his overly optimistic expectations about the future of his business.

To justify the current assumptions on the obstinate buyer, one may suppose that the obstinate buyer is a player that is the least aggressive or naive in terms of store choice and timing of departure, or a man who "plays it cool." Alternatively, one may assume that the obstinate buyer is a rational player (utility type) who (1) is forced to commit to a specific demand, (2) does not discount time and (3) incurs a positive (but very small) switching cost ($\epsilon_b > 0$) every time he switches his bargaining partner, and thus his aforementioned strategy endogenously occurs in equilibrium.[33]

The assumption that the obstinate buyer visits each seller at time zero with equal probabilities is a simplification assumption. It can be generalized with no impact on the main messages of our results. For example, one may assume that there are multiple types for the obstinate buyer (regarding the initial store selection) such that some always choose a fix seller and some visit the sellers according to their announcements while the rest are possibly a combination of these two.

The assumption on the obstinate buyer's departure habit seems a strong one since it eliminates the possibility that the rational buyer would increase his bargaining power by committing to a particular pattern of store choice. In the next two subsections, I

[33]Therefore, according to (1), the time of an agreement is not a concern for the obstinate buyer, and thus he does not feel the need to distinguish himself from the rational buyer who wishes to reach an agreement as quickly as possible. Since the obstinate buyer does not discount time, ϵ_b is the only search friction that the obstinate buyer is subject to and it would have no impact on our analysis –the switching cost ϵ_b would work as a tie-breaking device. Moreover, the assumption "the obstinate buyer understands the equilibrium and leaves his bargaining partner when he is convinced that his partner is also obstinate" can be interpreted as an implication rather than an assumption. Since the obstinate buyer does not value time, he should be indifferent between staying with his current partner or visiting the other seller at any time (ignoring the switching cost). However, if he leaves his current partner before being convinced that he is obstinate, he will revisit this seller later if he exhausts all his hope to reach an agreement with the other seller. Therefore, since the switching cost ϵ_b is positive, the obstinate buyer will switch his partner just once and thus leaves a store when he is convinced that his opponent is also obstinate.

show that the main message of the paper will not change if the obstinate buyer is "more strategic" in the sense that he commits to immediately switch or leave his bargaining partner in case his demand is not accepted.

The Case With More Aggressive Obstinate Buyer

I suppose now that the obstinate buyer (of any demand) leaves the first store he visits at time $T = 0$. The next result shows that any $\alpha_s \in C$ with $0 < \alpha_s$ is an equilibrium price for the sellers if the buyer is weak in equilibrium. The equilibrium strategies are as follows. In stage 1, rational sellers post the same demand α_s, the rational buyer visits each seller with equal probabilities and declares his demand as $\alpha_b < \alpha_s$ according to $\mu_b^*(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$. At the beginning of stage 2, assuming that the buyer visits seller 1 first, the rational buyer immediately accepts seller 1's demand at time zero with probability $P_b = \frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s} - z_b}{(1-z_b)(\hat{z}_s/A)^{\lambda_b/\lambda_s}}$ and immediately leaves store 1 with probability $1 - P_b$. Rational seller 1 never concedes to the buyer. The buyer and seller 2 play the concession game in the second store until time $T_2^e = -\frac{log(\hat{z}_s/A)}{\lambda_s}$ with the following strategies $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda_s t}$ where the terms λ_b, λ_s and A are as characterized in Section 3. Finally, in case one of the sellers deviate in stage 1, then the strategies of the continuation game are given in Proposition 3.2.

Proposition 5.1. *Suppose that the obstinate buyer leaves the first store he visits immediately following his arrival. Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ whenever $z_b \leq \frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s}(\alpha_s - \alpha_b)}{\alpha_s + \alpha_b}$ holds for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

I defer all the proofs in this section to Appendix. Parallel to our results in Section 3, the last statement shows that if z_b and z_s are selected carefully, then all prices in the set C can be supported in equilibrium.

The Case With the Most Aggressive Obstinate Buyer

Now suppose that the obstinate buyer (of any demand) leaves all stores immediately following his arrival. The following strategies ensure that all demands in the set C can be supported in equilibrium for small values of z_b and z_s. Rational sellers post the price of $0 < \alpha_s \in C$ and the rational buyer visits each seller with equal probabilities and declares his demand as $\alpha_b < \alpha_s$ according to μ_b^* that is given above. At the beginning of stage 2, assuming that the buyer visits seller 1 first, the rational buyer immediately accepts seller 1's demand at time zero with probability $P_b = \frac{\alpha_s(1-z_b)-\alpha_b}{(1-z_b)(\alpha_s-\alpha_b)}$ and immediately leaves store 1 with probability $1 - P_b$. Rational seller 1 never concedes to the buyer. In store 2, rational seller 2 accepts the buyer's demand upon his arrival with probability $P_s = \frac{(1-\alpha_s)(1-\delta)}{\delta(1-\hat{z}_s)(\alpha_s-\alpha_b)}$ and never concedes to the buyer with probability $1 - P_s$.[34] The rational buyer does not leave store 2 immediately. Instead he waits for the seller's concession. However, if the game does not end at time zero by seller 2's concession, the rational buyer concedes to the buyer immediately. Finally, in case one of the sellers deviate in stage 1, then the strategies of the continuation game are given in Proposition 3.2.

Proposition 5.2. *Suppose that the obstinate buyer leaves both stores immediately following his arrival. Take any z_b and z_s small enough. Then, $\alpha_s \in C \setminus \{0\}$ is an equilibrium demand selection of the rational sellers in the first stage of the competitive-bargaining game $G(z_b, z_s)$ whenever $z_b \leq \frac{(\alpha_s-\alpha_b)^2}{\alpha_s(\alpha_s+\alpha_b)}$ holds for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$.*

Different Initial Reputations for the Sellers

Suppose for now that the probability distribution π_i over C is different for each seller i and the sellers' initial reputations are not equal, i.e. $z_1 \neq z_2$. These assumptions would not change the essence of our results as long as z_1 and z_2 are small enough. Similar to Proposition 3.3, in equilibrium, rational sellers post the same price α_s whenever the buyer is weak, which would mean $z_b \leq \left(\frac{\hat{z}_1\hat{z}_2}{A}\right)^{\lambda_b/\lambda_s}$ for all $\alpha_b \in C$ with $\alpha_b < \alpha_s$ and $\hat{z}_i = \frac{z_i\pi_i(\alpha_s)}{z_i\pi_i(\alpha_s)+1-z_i}$. As the rational buyer is weak, his expected payoff is independent of the seller's initial reputations, and so these particular sources of heterogeneity do not change the fundamentals of the competition between the sellers.

Sequential Price Quoting

Suppose now that the price announcement in the game G is sequential. Seller 1 announces his demand first. Then, the second seller posts his price after observing the first seller's announcement. Finally, the buyer declares his demand after observing the sellers' prices and the rest of the game follows as it was before. Note that, this change in the first stage does not alter the equilibrium strategies of the players in the concession game (the second stage), and so they are the same as those provided in Section 3.

Similar to the previous arguments, if the buyer is weak, that is $z_b \leq (\hat{z}_s/A)^{\lambda_b/\lambda_s}$, then the rational sellers' expected payoff in the game increases with the price they post if z_b and z_s are sufficiently small.[35] Hence, in equilibrium, both sellers will post the same price which will be the highest price available in the set C. As a result, when all the frictions vanish, the unique equilibrium price will converge to $\frac{r_b}{r_b+NKr_s}$ (the upper bound we found

[34] Note that P_s is in $(0, 1)$ as $\hat{z}_s < \frac{(1-\alpha_s)(1-\delta)}{\delta(\alpha_s-\alpha_b)} < 1$.

[35] See the rational sellers' expected payoff, for example, in the proof of Proposition 3.3.

in Proposition 3.4) if the buyer is weak and 0 otherwise.

6. The Discrete-Time Model and Convergence

In this section, I consider the competitive-bargaining game in discrete time and investigate the structure of its equilibria as players can make their offers increasingly frequent. I show that given the symmetric obstinate types, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers approach to zero, and this limit is equivalent to the unique outcome of the continuous-time game partially investigated in Section 3. I characterize the second stage equilibrium strategies of the game G (given that both sellers post the same demand $0 < \alpha_s \in C$) in Online Appendix.

To be more specific, I suppose that each player has a single commitment type; some $\alpha_s \in C$ for the sellers and $\alpha_b \in C$ for the buyer where $0 < \alpha_b < \alpha_s$. In stage 1, first the sellers and then the buyer announces their types. Then the buyer chooses a store to visit first. Upon the buyer's arrival at store i, beginning of stage 2, the buyer and seller i bargain in discrete time according to some protocol g^i that generalizes Rubinstein's alternating offers protocol. A bargaining protocol g^i between the buyer and seller i is defined as $g^i : [0, \infty) \to \{0, 1, 2, 3\}$ such that for any time $t \geq 0$, an offer is made by the buyer if $g^i(t) = 1$ and by seller i if $g^i(t) = 2$.[36] Moreover, $g^i(t) = 3$ implies a simultaneous offer whereas $g^i(t) = 0$ means no offer is made at time t. An infinite horizon bargaining protocol is denoted by $g = (g^1, g^2)$. The bargaining protocol g is discrete. That is, for any seller i and for all $\bar{t} \geq 0$, the set $I^i := \{0 \leq t < \bar{t} | g^i(t) \in \{1, 2, 3\}\}$ is countable. Notice that this definition for a bargaining protocol is very general and accommodates non-stationary, non-alternating protocols.

In stage 2, the rational players are free to choose any offer from the set $[0, 1]$. An offer $x \in [0, 1]$ denotes the share the seller is to receive. If the proposer's opponent accepts his offer, the game ends with agreement x where xe^{-tr_s} denotes the payoff to seller i, 0 is the payoff to seller j and finally $(1 - x)e^{-tr_b}$ is the payoff to the buyer. If the proposer's opponent rejects his offer, the game continues. Prior to the next offer, the rational buyer decides whether to stay or leave the store. If the rational buyer decides to stay, the next offer is made at time $t' := \min\{\hat{t} > t | \hat{t} \in I^i\}$, for example, by the buyer if $g^i(t') = 1$. The two-stage competitive-bargaining game in discrete-time is denoted by $\mathrm{G}\langle g, (z_n, r_n)_{n \in \{b,s\}} \rangle$ (or $\mathrm{G}(g)$ in short). The competitive-bargaining game $\mathrm{G}(g)$ ends if

[36] Time 0 denotes the beginning of the bargaining phase.

the offers are compatible. In the event of strict compatibility the surplus is split equally. Throughout the game, both sellers can perfectly observe the buyer's moves. Thus, the players' actual types remain to be the only source of uncertainty.

I am particularly interested in equilibrium outcome(s) of the competitive-bargaining game $\mathrm{G}(g)$ in the limit where the players can make sufficiently frequent offers. Therefore, for $\epsilon > 0$ small enough, let $\mathrm{G}(g_\epsilon)$ denote discrete-time competitive-bargaining game where the buyer and the sellers bargain, in stage two, according to the protocol $g_\epsilon = (g^1_\epsilon, g^2_\epsilon)$ such that for all $t \geq 0$ and i, both seller i and the buyer have the chance to make an offer, at least once, within the interval $[t, t+\epsilon]$ in the bargaining protocol g^i_ϵ.[37] In this sense, the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ converges to continuous time as $\epsilon \to 0$.[38]

Now, let σ_ϵ denote a sequential equilibrium of the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ and σ_i be the rational buyer's equilibrium strategy for store selection at time zero. Given σ_i, the random outcome corresponding to σ_ϵ is a random object $\theta_\epsilon(\sigma_i)$ which denotes any realization of an agreed division as well as a time and store at which agreement is reached.

The next result shows that in the limit as ϵ converges to zero $\theta_\epsilon(\sigma_i) \to \theta(\sigma_i)$ in distribution, where $\theta(\sigma_i)$ is the unique equilibrium distribution of the continuous-time game G, that is fully characterized in the online appendix for $\sigma_1 = 1/2$. Therefore, the outcome of the discrete-time competitive-bargaining game, independent of the bargaining protocol g_ϵ, converge in distribution to the unique (given the buyers initial choice of store) equilibrium outcome of the competitive-bargaining game analyzed in Section 3.

Proposition 6.1. *As ϵ converges to 0, $\theta_\epsilon(\sigma_i)$ converges in distribution to $\theta(\sigma_i)$.*

I defer the proof to the online appendix.

7. Related Literature and Closing Remarks

This paper investigates the impacts of reputation (in contact with inflexibility) on competitive search markets where the sellers announce their initial demands prior to the buyer's visit and the buyer directs his search for a better deal. The buyer facing multiple sellers can negotiate with only one at a time and can switch his bargaining partner with some delay. A modified war of attrition structure is derived in the equilibrium (Section

[37] More formally, either $g^i(\hat{t}) = 3$ for some $\hat{t} \in [t, t+\epsilon]$, or $g^i(t') = 1$ and $g^i(t'') = 2$ for some $t', t'' \in [t, t+\epsilon]$.

[38] One may assume that the travel time is discrete and consistent with the timing of the bargaining protocols so the buyer never arrives a store at some non-integer time.

6). In equilibrium, if the sellers' posted prices are the same, then the buyer will never visit one seller more than once. Unlike standard conclusions regarding multilateral bargaining games, the equilibrium of the second stage of the game G is unique. This makes the model a fruitful setup for further investigations. In Sections 3 and 4, I show that the range of prices including the monopoly price and zero are compatible in equilibrium even when frictions vanish. Further investigations of the model in Sections 4 and 5 show that the main message of the paper is robust in many aspects.

This paper is directly related to the reputation and bargaining literature initiated by Myerson (1991). Myerson investigates the impacts of one sided reputation building on bilateral negotiations. Abreu and Gul (2000), Kambe (1999) and Compte and Jehiel (2002) consider two sided versions of it. Compte and Jehiel (2002) consider a discrete-time bilateral bargaining problem in Abreu-Gul setting and explore the role of (exogenous) outside options. They show that if both agents; outside options dominate yielding to the commitment type, then there is no point in building a reputation for inflexibility and the unique equilibrium is again the Rubinstein outcome. Atakan and Ekmekci (2010) consider a two-sided search market with a large number of buyers and sellers who wait to be matched (randomly) to an opponent to bargain over the unit surplus, so the bargaining parties' outside options are endogenous. Atakan and Ekmekci (2010) analyze the steady state of this market, and in agreement with my results, they show that the endogenous outside options of the rational agents are never large enough to deter the effect of commitment types. However, their main focus is substantially different and so, they do not answer which commitment type (or price) rational players mimic (pick) in equilibrium?

This paper also adds to the literature initiated by Rubinstein and Wolinsky (1985). They consider a market, in steady state, where at each period, finite but large number of buyers and sellers are matched with an exogenous matching mechanism to negotiate over the price, and new players enter as some leave the market after agreement. Their main result suggests that the unique outcome is not Walrasian even when search and bargaining frictions vanish. Gale (1986a/b) objects to this result by arguing that supply and demand in such market setups should be treated in terms of "flows" (not "stocks") of agents into the market at any period, and then shows that the bargaining approach indeed supports the Walrasian equilibrium. Binmore and Herrero (1989) support this point and show that frictionless markets will clear period by period. That is, the short side of the market will appropriate the whole surplus if and only if entry into the market is negligible relative to exit from it. Satterthwaite and Shneyerov (2007) reinforce this finding by achieving an analogous result when there is incomplete information (regarding the players' valuations) on both sides.

Rubinstein and Wolinsky (1990) show that the controversial result in their earlier paper does not occur if there is no new entry into the market. In this case, players' fear that they may not find a bargaining partner tomorrow if they reject their current offer today forces the long side of the market to compete fiercely, thus yielding a Walrasian outcome as frictions vanish. Bester (1988) employs a model similar to Rubinstein and Wolinsky (1985) with an infinite number of buyers and sellers and shows that if there is uncertainty regarding the sellers' product quality, then relative speed of convergence for bargaining friction and search friction determines whether the limit approaches Walrasian outcome. However, if the quality uncertainty is not in play along with the other frictions, then the market outcome is clearly Walrasian, as argued in Bester (1989).

In contrast, Shaked and Sutton (1984) examines a labor market with one firm and multiple workers (similar to the one I investigate in this paper), showing that the unique equilibrium outcome is non-Walrasian. This conclusion is correct under the assumption that the firm cannot switch its bargaining partner unless some time ($T > 1$ periods), which is exogenously set, passes. However, it is hard to motivate whether a firm would commit itself to such haggling protocols in a competitive environment. In this paper, however, I show that the buyer's reputation concern may lock him in with a seller. In equilibrium, when the buyer has a low initial reputation, he cannot leave his bargaining partner before his reputation reaches a certain point (optimal departure time).

Appendix

Proof of Proposition 3.1. First, I will study the properties of equilibrium strategies (distribution functions) in concession games. For this purpose, take any $i \in \{1, 2\}$ and history $h_{T_i} \in H^i$, and consider a pair of equilibrium distribution functions $(F_b^{i,T_i}, F_i^{T_i})$ defined over the domain $[T_i, T_i']$ where $T_i' \leq \infty$ depends on the buyers' equilibrium strategy. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and Wilson (1988) and are analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

Lemma A.1. *If a player's strategy is constant on some interval $[t_1, t_2] \subseteq [T_i, T_i')$, then his opponent's strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. F_b^{i,T_i} *and* $F_i^{T_i}$ *do not have a mass point over* $(T_i, T_i']$.

Lemma A.3. $F_i^{T_i}(T_i) F_b^{i,T_i}(T_i) = 0$

Therefore, according to Lemma A.1 and A.2, both $F_i^{T_i}$ and F_b^{i,T_i} are strictly increasing and continuous over $[T_i, T_i']$. Recall that

$$U_i(t, F_b^{i,T_i}) = \int_{T_i}^{t} \alpha_s e^{-r_s y} dF_b^{i,T_i}(y) + \alpha_b e^{-r_s t}(1 - F_b^{i,T_i}(t))$$

denote the expected payoff of rational seller i who concedes at time $t \geq T_i$ and

$$U_b(t, F_i^{T_i}) = \int_{T_i}^{t} (1-\alpha_b)e^{-r_b y} dF_i^{T_i}(y) + (1-\alpha_s)e^{-r_b t}(1 - F_i^{T_i}(t))$$

denote the expected payoff of the rational buyer who concedes to seller i at time $t \geq T_i$. Therefore, the utility functions are also continuous on $[T_i, T_i']$.

Then, it follows that $D^{i,T_i} := \{t | U_i(t, F_b^{i,T_i}) = \max_{s \in [T_i, T_i']} U_i(s, F_b^{i,T_i})\}$ is dense in $[T_i, T_i']$. Hence, $U_i(t, F_b^{i,T_i})$ is constant for all $t \in [T_i, T_i']$. Consequently, $D^{i,T_i} = [T_i, T_i']$. Therefore, $U_i(t, F_b^{i,T_i})$ is differentiable as a function of t. The same arguments also hold for $F_i^{T_i}$. The differentiability of $F_i^{T_i}$ and F_b^{i,T_i} follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i^{T_i}(t) = 1 - c_i e^{-\lambda_s t}$ and $F_b^{i,T_i}(t) = 1 - c_b^i e^{-\lambda_b t}$ where $c_i = 1 - F_i^{T_i}(T_i)$ and $c_b^i = 1 - F_b^{i,T_i}(T_i)$ such that $\lambda_b = \frac{\alpha_b r_s}{\alpha_s - \alpha_b}$ and $\lambda_s = \frac{(1-\alpha_s) r_b}{\alpha_s - \alpha_b}$.

Therefore, the rational buyer's expected payoff of playing the concession game with seller i during $[T_i, T_i']$ is $[F_i^{T_i}(T_i))(1-\alpha_b) + (1 - F_i^{T_i}(T_i))(1-\alpha_s)]$. Moreover, by Lemma A.3, we know that if the buyer is strong in a concession game with seller i (starting at time T_i), then seller i is weak. Hence, there is no sequential equilibrium of the game G such that the buyer visits a store multiple times. Suppose on the contrary that there is a strategy in which, without loss of generality, the buyer visits store 1 twice. Then, the buyer must be strong in his second visit to seller 1. Otherwise the buyer would prefer to concede to seller 2 and finish the game before making the second visit to store 1 (because $\delta < 1$). Thus, since seller 1 is weak, his expected payoff is α_b when the buyer visits his store for the second time. However, in equilibrium, this continuation payoff contradicts the optimality of seller 1's strategy because seller 1 would prefer to accept the buyer's offer (for sure) when the buyer first attempts to leave his store to eliminate a further delay.

As a result, in equilibrium, rational sellers will not allow the buyer to leave their stores. On the other hand, the rational buyer will eventually leave the first store he visits if that seller is obstinate. The reason for this is clear. Since the players' concession game strategies are increasing and continuous, the seller's reputation will eventually converge to one at some finite time. The rational buyer has no incentive to continue the concession game with an obstinate seller, and so he must either concede to the seller at that time or leave the store. However, Lemma A.2 implies that concession game strategies must be continuous in their domain, eliminating the possibility of mass acceptance at the time that the seller's reputation reaches one.

Next, for notational simplicity, I reset the clock each time the buyer arrives at a store, and denote the buyer's concession game strategy against seller i by F_b^i and i's strategy by F_i. Now, consider an equilibrium where the rational buyer visits seller 1 first with probability σ_1, leaves store 1 at time T_1^d and finalizes the game in store 2 at time T_2^e if the game has not yet ended before. Then, rational buyer visits seller 2 only if $F_2(0) > 0$ is true. Suppose $F_2(0) = 0$. Then, the rational buyer's discounted continuation payoff in store 2, $\delta[F_2(0)(1-\alpha_b)+(1-F_2(0))(1-\alpha)]$, will be $\delta(1-\alpha)$. In this case, the rational buyer prefers to concede to seller 1 instead of traveling store 2, yielding the required contradiction. By lemma A.3., as $F_2(0) > 0$, we must have $F_b^2(0) = 0$, implying that $c_b^2 = 1$. That is, $F_b^2(t) = 1 - e^{-\lambda_b t}$. Furthermore, assuming that the rational buyer leaves store 1 at time T_1^d and the concession game in store 2 ends at time T_2^e, we must have $F_1(T_1^d) = 1 - z_s$ and $F_1(T_2^e) = 1 - z_s$. Thus we have $c_1 = z_s e^{\lambda T_1^d}$ and $c_2 = z_s e^{\lambda T_2^e}$ as required.

Finally, Lemma A.3 implies that $F_b^1(0)F_1(0) = 0$. Since seller 2's reputation reaches 1 at time T_2^e, then the rational buyer will not continue the game G after this time. Thus, his reputation must also reach 1 at that time, implying that $F_b^2(T_2^e) = 1 - z_b^*$ where $z_b^* = \frac{z_b}{1-F_b^1(T_1^d)}$ is the buyer's reputation at the time he arrives at store 2 and z_b is the buyer's reputation at the time he arrives at store 1.

Proof of Lemma 3.1. Consider an equilibrium where the rational buyer visits seller 1 first with probability 1/2 and $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s} < z_b^*$. Then, the rational buyer prefers to play the concession game with seller 1 over going to store 2 at time zero. Since the buyer leaves store 1 if and only if seller 1 is obstinate, seller 1's reputation reaches one at time $T_1^d = \tau_1 = \min\{\tau_b^1, \tau_1\}$ where $\tau_b^1 = \inf\{t \geq 0 | \ F_b^1(t) = 1 - z_b\} = -\frac{\log z_b}{\lambda_b}$ and $\tau_1 = \inf\{t \geq 0 | \ F_1(t) = 1 - \hat{z}_s\} = -\frac{\log \hat{z}_s}{\lambda_s}$ denote the times that the buyer's and seller 1's reputations reach 1, respectively.

However, leaving 1 is optimal for the rational buyer if and only if the buyer's reputation at time T_1^d reaches z_b^*, implying that

$$c_b^1 e^{-\lambda_b T_1^d} = \frac{z_b}{z_b^*} \tag{6}$$

Given the value of T_1^d, solving the last equality yields the buyer's equilibrium strategy in store 1. Finally, the game ends in store 2 at time $T_2^e = \tau_b^2 = \min\{\tau_b^2, \tau_2\}$ for sure where $\tau_b^2 = -\frac{\log z_b^*}{\lambda_b}$ and $\tau_2 = -\frac{\log \hat{z}_s}{\lambda_s}$, at which points both players' reputation simultaneously reach one. Given the value of T_2^e, Proposition 3.1 implies the concession game strategies in the second store.

Proof of Proposition 3.2. First note that $1 - \alpha_1 < \delta(1 - \alpha_2)$ because the search friction is assumed to be sufficiently small. Therefore, it is optimal for the rational buyer to go to store 2 and to accept α_2 instead of accepting α_1. Moreover, regardless of the buyer's announcement α_b, postponing concession or not accepting α_b is not optimal for rational seller 1 since the buyer will never accept α_1 in equilibrium. Thus, it is a best response for rational seller 1 to accept the buyer's demand upon his arrival at store 1, and so it is a best response for the rational buyer to choose $\alpha_b = 0$.

For the last part, if $\alpha_2 > \alpha_1$, then the buyer never visits seller 2. Therefore, in any equilibrium, the continuation game is identical to the Abreu and Gul (2000) setup and the equilibrium strategies are characterized by the following three conditions: (i) $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t}$ and $F_1(t) = 1 - c_1 e^{-\lambda_s t}$ for all $t \leq T^e = \min\{\frac{-\log \hat{z}_s}{\lambda_s}, \frac{-\log z_b}{\lambda_b}\}$, (ii) $(1 - c_b^1)(1 - c_1) = 0$, and (iii) $F_b^1(T^e) = 1 - z_b$ and $F_1(T^e) = 1 - \hat{z}_s$. Note that these strategies form an equilibrium for small values of z_s, in particular for the values of z_s such that $z_s < A$. The rest of the strategies are optimal given the belief that seller 2 is known to be obstinate.

Proof of Lemma 3.2. Recall that rational sellers' price posting strategies are pure in σ^G. Therefore, if rational seller 2 deviates to α_2 at time zero, then other players will conclude that seller 2 is obstinate of type α_2. Given the assumptions on obstinate types, the rational buyer's expected payoff of posting $\alpha_2 > \alpha_s$ is zero. Proposition 3.2 gives the strategies of the continuation game following a history where seller 2 price undercuts his opponent. Deviation to $\alpha_2 = 0$ clearly implies expected payoff of 0. However, if $\alpha_2 > 0$, then the second seller's expected payoff will be $\alpha_2 \left[z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b) + \hat{z}_s(1 - z_b) \right]$ where $z_b \sum_{\alpha_b \geq \alpha_2} \pi(\alpha_b)$ is the probability that the buyer is an obstinate type with demand higher than or equal to α_2. Finally, note that $\hat{z}_s = \frac{z_s \pi(\alpha_s)}{z_s \pi(\alpha_s) + 1 - z_s} < z_s$.

Proof of Proposition 3.3. Suppose that $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$ holds for all $\alpha_b \leq \alpha_s$. Then, I want to show that there exists an equilibrium strategy where both sellers post the price of $\alpha_s > 0$. Given that both sellers choose α_s, the equilibrium strategies of the rational buyer in the first stage, σ_i^* and μ_b^* must satisfy the followings.

1. σ_i^* is the probability of visiting seller i first with $\sigma_1^* + \sigma_2^* = 1$ and μ_b^* is a probability distribution over the set $D \subset C_{\alpha_s} = \{\alpha_b \in C | \alpha_b \leq \alpha_s\}$ with $\sum_{x \in D} \mu_b^*(x) = 1$.

2. For all $i \in \{1, 2\}$ and $\alpha_b \in D$ we must have $V_b^i(\alpha_b) = V$. By Lemma 3.1 and by the assumption that $z_b \leq (\hat{z}_s^2/A)^{\lambda_b/\lambda_s}$, we have $V_b^i(\alpha_b) = 1 - \alpha_s$.

3. $V \geq 1 - \min\{C \setminus D\}$. That is, the rational buyer should have no incentive to deviate and declare some other demand α_b' which is not in the support of μ_b^*.

Therefore, in equilibrium μ_b^* and σ_i^* are solutions of $\#D+1$ (nonlinear) equations for $\#D+1$ unknowns. For small values of z_b (relative to $\hat{z}_s$), existence of these strategies is easy to show. Consider the strategy profile σ^G that is given in the main text. The strategies μ_b^* and σ_i^* satisfy the requirements 1-3. Moreover, by Lemma 3.1 and Proposition 3.2, the second stage strategies also form an equilibrium.

Lastly, we need to show that the first stage strategies μ_1^* and μ_2^* are optimal. That is, I will show that posting the demand α_s at time zero is an optimal strategy for a seller if the other seller also posts α_s. For this reason, I will first calculate each sellers expected payoff under the strategy profile σ^G that is given in the main text. Let V_i denote seller i's expected payoff in the game. Since a deviating seller's equilibrium payoff is less than $(z_b + z_s)$ (by Lemma 3.2), I will argue that price undercutting is not optimal if we choose z_b and z_s sufficiently small. Moreover, following the assumptions on obstinate types, if a seller deviates and posts a price above α_s, then his expected payoff in the game will be simply zero.

Under the strategy σ^G, we have $V_i = p\alpha_s + (\frac{1}{2} - p)(a + b)$ and we calculate it as follows:

Case 1. The buyer picks store i first and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2} z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b) := p$. Rational seller i's expected payoff in this case is α_s.

Case 2. The buyer picks store i second and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is p and rational seller i's expected payoff in this case is 0.

Case 3. The buyer picks store i first and he is either rational or obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$, $[\frac{1}{2}(1 - z_b) + z_b \frac{1}{2} - p]$, and rational seller i's expected payoff in this case is $\sum_{\alpha_b < \alpha_s} [\frac{\pi(\alpha_b)}{\sum_{x<\alpha_s} \pi(x)}][\alpha_b + F_b^i(0)(\alpha_s - \alpha_b)] := a$ where $F_b^i(0) = 1 - z_b(A/\hat{z}_s^2)^{\frac{\alpha_b r_s}{(1-\alpha_s)r_b}}$.

Case 4. The buyer picks store i second and he is either rational or obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$ and rational seller i's expected payoff in this case is $\frac{e^{-\Delta r_s} \hat{z}_s}{\sum_{x<\alpha_s} \pi(x)} \sum_{\alpha_b < \alpha_s} \hat{z}_s^{\frac{r_s(\alpha_s - \alpha_b)}{(1-\alpha_s)r_b}} \alpha_b \pi(\alpha_b) := b$. Note that the buyer will visit the second store only if the first seller is obstinate and the rational buyer announces $\alpha_b < \alpha_s$. Therefore, seller i's expected payoff in this case is discounted by the travel time $e^{-\Delta r_s}$ and $\hat{z}_s^{\frac{r_s(\alpha_s - \alpha_b)}{(1-\alpha_s)r_b}}$ - the discount due to the delay in the first store j, i.e. T_j^d.

Note that V_i is strictly greater than $(\frac{1}{2} - p)u$ where u is the convex combination of the demands in $C_{\alpha_s} \setminus \{\alpha_s\}$, i.e., $u = \sum_{\alpha_b < \alpha_s} \alpha_b \mu_b(\alpha_b)$, and it is much higher than $(z_b + z_s)$ if z_b and z_s are sufficiently small. Hence, posting α_s is optimal for each seller. This completes the proof.

Proof of Proposition 3.4. Recall that the proof of Theorem 2 relies solely on the fact that the buyer must be weak for each α_b in the support of μ_b^*. Same arguments in the proof of Proposition 3.3 shows that if there are N identical sellers and the buyer is weak in equilibrium, then we can support non-Walrasian prices in equilibrium. Next, I will show that being weak in equilibrium with N sellers means $z_b \leq \left(\hat{z}_s^N / A^{N-1}\right)^{\lambda_b/\lambda_s}$.

For the ease of exposition, I will derive this condition for the 3-sellers case, which can be extended to N-sellers case by iterating the same process. For this reason, suppose now that there are three sellers all of which choose the same demand α_s in stage 1 and the buyer declares his demand as $\alpha_b < \alpha_s$. Without loss of generality, I assume that the buyer visits seller 1 first and seller 3 last (if no agreement have been reached with the sellers 1 and 2). The following arguments are straightforward extensions of the approach that I use in the proof of Proposition 3.1. Therefore, let T_i^d denote the time that the buyer leaves seller $i \in \{1, 2\}$ and $\hat{z}_b(T_i^d)$ denote the buyer's reputation at the time he leaves store i.

The rational buyer leaves seller 2 when his discounted continuation payoff in store 3, i.e. $\delta[1 - \alpha_b - \hat{z}_s[\hat{z}_b(T_2^d)]^{-\lambda_s/\lambda_b}(\alpha_s - \alpha_b)]$, equals to $1 - \alpha_s$. This equality implies that $\hat{z}_b(T_2^d) = (\hat{z}_s/A)^{\lambda_b/\lambda_s}$. As a result, the buyer's expected payoff in store 2 at the time he enters this store is $v_b^2 = 1 - \alpha_b - \hat{z}_s \left[\frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s}}{\hat{z}_b(T_1^d)}\right]^{\lambda_s/\lambda_b} (\alpha_s - \alpha_b)$. Similarly, the buyer leaves seller 1 when his discounted continuation payoff in store 2, i.e. δv_b^2, equals to $1 - \alpha_s$. Then we have $\hat{z}_b(T_1^d) = \left(\hat{z}_s^2/A^2\right)^{\lambda_b/\lambda_s}$.

Also, note that we have $\hat{z}_b(T_1^d) = \frac{\hat{z}_b^1}{1 - F_b^1(T_1^d)}$, $F_b^1(T_1^d) = 1 - c_b^1 e^{-\lambda_b T_1^d}$ and $c_b^1 = 1$ because the buyer is weak. Thus, it must be true that $T_1^d = -\frac{\log(\hat{z}_b^1/(\hat{z}_s^2/A^2)^{\lambda_b/\lambda_s})}{\lambda_b} \geq \frac{-\log \hat{z}_s}{\lambda_s}$ again because the buyer is weak. The last inequality implies $\hat{z}_b^1 \leq \left(\hat{z}_s^3/A^2\right)^{\lambda_b/\lambda_s}$. In equilibrium, the last inequality must hold for all $\hat{z}_b^i$ with $i = 1, 2, 3$, implying that it must hold for z_b as well. The rest directly follows from the parallel arguments of the proof of Theorem 2. Iterating the above arguments suffice to prove the claim for any finite N.

Proof of Proposition 4.1. Suppose that the Poisson arrival rate of the buyer is κ. First, if the players play the strategies described in the main text, then the Bayes' rule implies that the probability of the buyer being the commitment type α_b conditional on him visiting seller i during the period of $[T, T+dt]$ and demanding $\alpha_b < \alpha_i$ is

$$\hat{z}_b^{(T+dt),i} = \frac{\frac{1}{2}z_b\pi(\alpha_b)\kappa dt + \frac{1}{2}z_b\hat{z}_s\pi(\alpha_b)\kappa dt}{\frac{1}{2}z_b\pi(\alpha_b)\kappa dt + \frac{1}{2}z_b\hat{z}_s\pi(\alpha_b)\kappa dt + (1-z_b)\mu^T_{\alpha_i}(\alpha_b)\sigma_i\left(\sum_{x<\alpha_i}\pi(x)\right)\kappa dt}$$

The first term in the numerator corresponds to the probability that the obstinate buyer with demand α_b is visiting seller i first and arriving at the market in a short period dt. Likewise, the second term denotes the probability that the obstinate buyer visits seller i second, implying that the buyer should have arrived at the market $-\log(\hat{z}_s)/\lambda_s + \Delta$ units of time ago during the short period dt.[39]

Given the strategies of the players, if the buyer arrives at the market at the period $0+dt$, then the obstinate buyer's arrival time at the second store is $\bar{T} = -\log(\hat{z}_s)/\lambda_s + \Delta + dt$. Therefore, the second term in the numerator does not exists if $T < \bar{T}$. Moreover, the limiting case where dt approaches zero implies that $\hat{z}_b^{T,i}$ equals to z_b for all $T < \log(\hat{z}_s)/\lambda_s + \Delta$ and to $\frac{z_b(1+\hat{z}_s)}{1+z_b\hat{z}_s}$ otherwise.

Second, for any $0 < \alpha_b < \alpha_s$, we have $\hat{z}_b^{T,i} < \hat{z}_s^{\lambda_b/\lambda_s}$ because $z_b < \frac{\hat{z}_s^{\lambda_b/\lambda_s}}{1+\hat{z}_s(1-\hat{z}_s^{\lambda_b/\lambda_s})}$. Moreover, according to the strategies, the rational buyer never leaves the sellers' stores. This implies that the buyer and the seller will play the concession game according to the strategies F_b and F_i's until the time $-\frac{\log(\hat{z}_s)}{\lambda_s} = \min\{-\frac{\log\hat{z}_s}{\lambda_s}, -\frac{\log\hat{z}_b^{T,i}}{\lambda_b}\}$ (this directly follows from Abreu and Gul (2000), Proposition 1.) As a result, the buyer's expected payoff in each store is $1-\alpha_s$ because independent of the buyer's arrival time at either store, the buyer will be weak in both. Hence, visiting each seller with equal probabilities is an optimal strategy for the rational buyer. Furthermore, if the rational buyer leaves his current bargaining partner at any point of time and goes to the other seller, then his continuation payoff will be $\delta(1-\alpha_s)$. Hence, not leaving a seller's store and playing the concession game until the time $-\log(\hat{z}_s)/\lambda_s$ are also optimal strategies.

Third, independent of α_b ($\leq \alpha_s$), the rational buyer's expected payoff is $1-\alpha_s$ in each store. Thus, the mixed strategy $\mu^T_{\alpha_s}(\alpha_b) = \frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}$ is an optimal strategy for the rational buyer.

Finally, I will show that posting the demand α_s at time zero is an optimal strategy for a seller if the other seller also posts α_s. For this person, I will first calculate each seller's expected payoff under the strategies given in the main text. Let $V_i(T)$ denote seller i's expected payoff in the game (evaluated in time T) given that the buyer arrives at the market at time $T \geq 0$. Then, I calculate a deviating seller's equilibrium payoff (again evaluated in time T assuming that the buyer arrives at the market at T) and argue that it is smaller than $V_i(T)$ if we choose z_b and z_s sufficiently small. Thus, $V_i(T) = [p\alpha_s + (\frac{1}{2} - p)(a+b)]$ where

[39]Recall that $-\log(\hat{z}_s)/\lambda_s$ is the length of the concession game in the stores where $\lambda_s = \frac{(1-\alpha_s)r_b}{\alpha_s-\alpha_b}$, and Δ is the time required to travel between the stores.

Case 1. The buyer picks store i first and he is the obstinate type with demand $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2}z_b \sum_{\alpha_b \geq \alpha_s} \pi(\alpha_b) := p$ and seller i's expected payoff is α_s.

Case 2. The buyer picks the other store j first and he is the obstinate type with demand $\alpha_b \geq \alpha_s$. Probability to this event is p and i's expected payoff is 0.

Case 3. The buyer picks store i first and he is either rational or the obstinate type with demand $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$, $[\frac{1}{2}(1-z_b) + z_b\frac{1}{2} - p]$, and seller i's expected payoff is $\sum_{\alpha_b<\alpha_s}[\frac{\pi(\alpha_b)}{\sum_{x<\alpha_s}\pi(x)}][\alpha_b + F_b^T(T)(\alpha_s - \alpha_b)] := a$ where $F_b^T(T) = 1 - \hat{z}_b^{T,i}\hat{z}_s^{-\frac{\alpha_b r_s}{(1-\alpha_s)r_b}}$.

Case 4. The remaining case is that the buyer picks store j first and he is either rational or the obstinate type with demand $\alpha_b < \alpha_s$. Probability to this event is $\frac{1}{2} - p$ and i's expected payoff is $\frac{e^{-r_s\Delta}z_b\hat{z}_s}{\sum_{x<\alpha_s}\pi(x)}\sum_{\alpha_b<\alpha_s}\alpha_b\hat{z}_s^{r_s/\lambda_s}\pi(\alpha_b)\int_0^{\frac{-\log(\hat{z}_s)}{\lambda_s}} e^{-r_s t}\frac{dF_s(t)}{1-\hat{z}_s} := b$ where $F_s(t) = 1 - e^{-\lambda_s t}$.

On the other hand, if seller i price undercuts j and posts α_i such that $0 < \alpha_i < \alpha_s$, then rational seller i's expected payoff is $\left(\left[z_b \sum_{\alpha_b\geq\alpha_i}\pi(\alpha_b)\right] + \hat{z}_s\left[1 - z_b\sum_{\alpha_b\geq\alpha_i}\pi(\alpha_b)\right]\right)\alpha_i$, and it is less than $(z_b + z_s)\alpha_i$ (see Lemma 3.2). This is true because in any equilibrium following the history where seller i price undercuts j, the rational buyer visits seller j first with certainty, makes a take-it-or-leave-it offer 0, which will be accepted by the rational seller j, and immediately leaves if seller j does not accept 0. Then, the rational buyer immediately visits seller i to accept α_i. It is clear that $(z_b + z_s)\alpha_i < V_i(T)$ for sufficiently small values of z_b and z_s.

Proof of Proposition 5.1. I will show that the strategies given in the main text constitute and equilibrium. Suppose that the rational buyer announces $\alpha_b < \alpha_s$ in stage 1 and consider the second stage. First, at time zero, the rational buyer and seller 1 has two options; accept and reject. Rejection for the buyer means leaving the store. I assume that if the buyer chooses to leave but the seller accepts, then the game will end with the seller's acceptance. If the rational buyer does not leave the first store at time zero, he reveals his rationality, in which case the buyer's expected payoff will be no more than $1-\alpha_s$ (since the buyer is discounting time). Hence, in equilibrium, the rational buyer will either concede or leave the store at time zero.

Second, if the rational buyer finishes the game in store 1 with probability P_b, then the buyer's reputation conditional on him arriving store 2 after visiting 1 is $(\hat{z}_s/A)^{\lambda_b/\lambda_s}$ as calculated by $\frac{z_b}{z_b+(1-z_b)(1-P_b)}$. Therefore, the buyer and seller 2 will play the concession game until time $T_2^e = \min\{-\frac{log(\hat{z}_s/A)}{\lambda_s}, -\frac{log\hat{z}_s}{\lambda_s}\}$ which is equal to $-\frac{log(\hat{z}_s/A)}{\lambda_s}$ as $A < 1$. Thus, the equilibrium concession game strategies in store 2 must be as given in the main text. As a result, the rational buyer's expected payoff in the second store is $\frac{1-\alpha_s}{\delta}$.

Third, the rational buyer's expected payoff of accepting α_s in store 1 is

$$V_b(accept) = \hat{z}_s(1-\alpha_s) + (1-\hat{z}_s)\left[\frac{1}{2}P_s(2-\alpha_s-\alpha_b) + (1-P_s)(1-\alpha_s)\right]$$

whereas

$$V_b(reject) = \hat{z}_s\delta V + (1-\hat{z}_s)[P_s(1-\alpha_b) + (1-P_s)\delta V]$$

where $V = \frac{1-\alpha_s}{\delta}$ is the buyer's continuation payoff when he leaves the first seller at time zero. Note that if $P_s = 0$, then $V_b(accept) = V_b(reject) = 1 - \alpha_s$, implying that the buyer's strategy P_b is a best response. Moreover, since the rational buyer's expected payoff in each store and in the game, regardless of his announcement $\alpha_b < \alpha_s$, is $1-\alpha_s$, visiting each seller with probability 1/2 and announcing α_b according to μ_b^* are also best response strategies.

Similarly, rational seller i's expected payoff is

$$V_i(accept) = z_b\alpha_b + (1 - z_b)\left[\frac{1}{2}P_b(\alpha_s + \alpha_b) + (1 - P_b)\alpha_b\right]$$

whereas

$$V_i(reject) = z_b 0 + (1 - z_b)\left[P_b\alpha_s + (1 - P_b)0\right]$$

Therefore, given the value of P_b and $z_b \leq \frac{(\hat{z}_s/A)^{\lambda_b/\lambda_s}(\alpha_s-\alpha_b)}{\alpha_s+\alpha_b}$, we have $V_i(accept) < V_i(reject)$. Hence, $P_s = 0$ is a best response as well.

Finally, I will show that posting the demand α_s at time zero is an optimal strategy for a seller if the other seller also posts α_s. For this reason, I will first calculate each sellers expected payoff in the game for the second stage strategies given in the main text. Let V^i denote seller i's expected payoff in the game. Since a deviating seller's equilibrium payoff is less than $(z_b + z_s)$ (by Lemma 3.2), I will argue that price undercutting is not optimal if we choose z_b and z_s sufficiently small. We have $V^i = \alpha_s\left[p + \frac{(1-\hat{z}_s)}{2}[P_b + e^{-r_s\Delta}(1 - P_b)]\right]$ and calculate it as follows:

Case 1. The buyer picks store i first and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is $\frac{1}{2}z_b\sum_{\alpha_b\geq\alpha_s}\pi(\alpha_b) := p$. Rational seller i's expected payoff in this case is α_s.

Case 2. The buyer picks store i second and he is obstinate of type $\alpha_b \geq \alpha_s$. Probability to this event is p and rational seller i's expected payoff in this case is 0.

Case 3. The buyer is obstinate of type $\alpha_b < \alpha_s$. Probability to this event is $z_b - 2p$ and rational seller i's expected payoff in this case is 0.

Case 4. The buyer picks store i first and he is rational. Probability to this event is $(1 - \hat{z}_s)\frac{1}{2}$ and rational seller i's expected payoff in this case is $P_b\alpha_s$.

Case 5. The buyer picks store i second and he is rational. Probability to this event is $(1 - \hat{z}_s)\frac{1}{2}$ and rational seller i's expected payoff in this case is $(1 - P_b)e^{-r_s\Delta}\alpha_s$.

Note that for small values of z_b and z_s, the value of V^i is greater than $(z_b + z_s)$ which concludes the proof.

Proof of Proposition 5.2. Similar arguments in the proof of Proposition 5.1 will prove our claim. Note that given the value of P_b, as in the main text, the buyer's reputation conditional on him announcing α_b and arriving store 2 after visiting store 1 is $z_b^* = 1 - \frac{\alpha_b}{\alpha_s}$. The value of z_b^* makes rational seller 2 indifferent between immediate concession, with payoff of α_b, and rejection with payoff of $(1 - z_b^*)\alpha_s$. Since rational seller 2 is indifferent, immediate concession with probability P_s (as given in the main text) is optimal. Moreover, P_s ensures the expected payoff of $\frac{(1-\alpha_s)}{\delta}$ to the rational buyer, and it makes the buyer indifferent between conceding to seller 1 and leaving for seller 2. Finally, with the value of P_b and $z_b \leq \frac{(\alpha_s-\alpha_b)^2}{\alpha_s(\alpha_s+\alpha_b)}$, rational seller 1's expected payoff of rejecting the buyer's demand is higher than conceding to him as $V_1(accept) = z_b\alpha_b + (1-z_b)[\frac{1}{2}P_b(\alpha_s+\alpha_b) - (1-P_b\alpha_b)]$ and $V_1(reject) = (1-z_b)P_b\alpha_s$.

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