Financial Distress and Endogenous Uncertainty

preliminary and incomplete

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Abstract

What is the macroeconomic effect of having a substantial number of firms close to default? This paper studies financial distress costs in a model where customers, suppliers and workers suffer losses if their employer goes bankrupt. I show that this mechanism generates amplification of fundamental shocks through procyclical TFP and countercyclical labor wedge. Because the strength of this amplification depends on the share of firms that are in financial distress, it operates mostly in recessions, when equity values are low. This leads macroeconomic volatility to be endogenously countercyclical. The cross-sectional distribution of firms' equity values affects directly aggregate macroeconomic volatility. Empirical evidence consistent with the model is provided.

JEL: E32, E44, G12.

Keywords: time-varying uncertainty, uncertainty shocks, distance-to-default, leverage.

1 Introduction

Economic recessions and financial crises sometimes lead a large number of firms to become close to default. This arises either because of contraction in credit supply, or because equity values fall and become more volatile, reducing the cushion protecting solvency.[1] Intuitively, the fact that a nontrivial share of the nonfinancial corporate sector is close to default would seem to have significant negative macroeconomic effects. This paper is concerned with the modeling and measuring the costs of such financial distress.

In particular, the paper focuses on a specific channel: when a firm becomes close to default, it becomes harder to find and retain employees, suppliers and customers, as they worry that they would

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[1] See Atkeson, Eisfeldt and Weil (2013) for a recent empirical analysis of the business cycle variation in the distance to insolvency.

suffer losses if the firm goes under. These losses include direct missed payments,[2] but also capture more broadly the costs of the loss of a working relationship - employees, customers and suppliers must look for another match, and they may lose some relationship-specific capital. The anticipation of these losses make it more costly for firms to operate, and hence leads directly to lower production and employment. A "default wedge" whereby the marginal revenue exceeds the marginal cost.

This financial distress mechanism stands in contrast to alternative mechanisms which work chiefly through investment, such as limited pledgeability -a smaller distance to default makes it more costly to raise external finance- or debt-overhang -a smaller distance to default reduces incentives for investment on the part of equityholders as debtholders stand to reap the gains of investment. A key challenge in macroeconomic models is to generate large variation in output and employment, making the default wedge mechanism appealing. In particular, I show that it can generate an apparent "labor wedge", so that an econometrician that looks at the data through the lens of the neoclassical model would find an excessive employment contraction, making it look as if labor income taxes were countercyclical. As argued by Mulligan (1997), Hall (1997), and Chari, Kehoe and McGrattan (2005), this is a desirable feature of the data. Furthermore, the mechanism also reduces aggregate total factor productivity by allocating labor across firms in part based on their default likelihood rather than their productivity. As a result of these two wedges, financial distress amplifies macroeconomic fluctuations.

Moreover, this amplification effect operates only under certain circumstances, leading to nonlinearities. In goods times, when productivity or equity values are high, a small aggregate shock does not change substantially the likelihood of default of most firms, which remains very low. But in bad times, the same-size shock could be sufficient to increase significantly the likelihood of default of many firms, with larger aggregate effects. The time-varying elasticity implies time-varying macroeconomic volatility even if the underlying, fundamental shock is homoskedastic. As a result, financial distress can generate endogenously time-varying second moments. This is of interest in light of the large recent literature on uncertainty shocks, which for the most part takes the variation of uncertainty as exogenous.[3]

There is a lot of anecdotal evidence that customer/supplier and employee relationships are deteriorated when firms become close to default. For instance, in November 2008, Circuit City, the second largest electronics retailer in the U.S., was forced to file for bankruptcy when news of its deteriorating financial position led its suppliers (such as Samsung, Sony, and other big electronic manufacturers) to refuse extending credit for deliveries.[4] These phenomena are likely frequent and important, because trade credit is large: in Compustat, the median account receivables/assets is around 0.15, versus 0.21 for debt/assets. The impact of bankruptcy risk on customers was an important element for Chrysler and

[2] For instance, in the United States, employee wages have a priority in bankruptcy up to 10,000$, if they were earned in the past six months. The excess over 10,000$ is treated as a regular unsecured claim, payable in proportion to the assets recovered, after administrative expenses are paid, and possibly with a substantial delay. Furthermore, bonuses, sick and vacation days are lost if they were earned over 180 days ago, and may be hard to recover otherwise. Finally, while company pension plans benefit from the backstop of the Pension Benefit Guarantee Corporation, this guarantee is incomplete (i.e. capped).

[3] For some models with shocks to aggregate uncertainy, see Bloom (2009), Fernandez-Villaverde et al. (2010, 2013), or Gourio (2012). Studies that attempt to endogenize volatility include, among others, Bachmann and Moscarini, Berger and Vavra, Brunnermeier and Sannikov, Bianchi and Mendoza.

[4] See for instance http://online.wsj.com/news/articles/SB122632305224313513

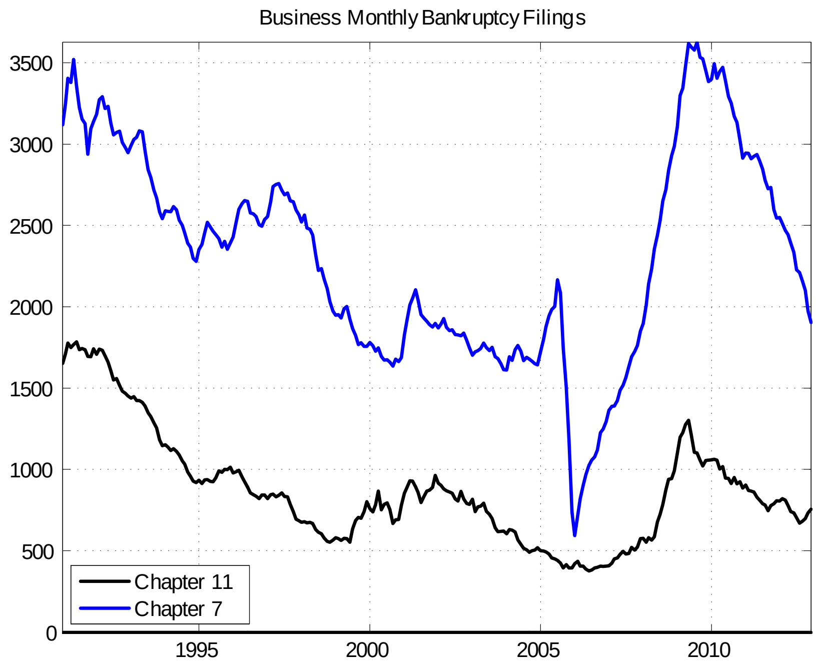

Figure 1: US business bankruptcy filings. Note: series are smoothed using a 6-month moving average. The 2005 spike due to the bankruptcy reform is removed.

GM - there was widespread concern that consumers would stop buying these manufacturers' cars once they realized the companies were likely to file for bankruptcy, since product warranties and auto parts might not be available later. This motivated the U.S. Federal government to introduce the warranty guarantee program (see Hortacsu et al. (2012)). More systematic evidence has been provided by the corporate finance literature. For instance, Brown and Matsa (2012) find that (financial) firms in financial distress during the recent recession received fewer applications to job openings than did financially healthy firms. Moreover, the quality of the applicants was worse.

A key challenge for the mechanism is to generate significant macroeconomic impact in spite of the fairly low observed default rates. Two observations are important here. First, this mechanism applies not only to firms that are *actually* in default or restructuring their debt, but rather to all firms that have some significant likelihood of default. Second, for some purposes, the relevant default rate is the exit rate. For instance, an employee who is let go faces a loss from the firm discontinuing its operations, even if the firm exits without financial default. For instance, even if his wages and benefits are paid, the worker will spend time searching for a new job, which might not be as good, and there is some lost firm-specific human capital.

To illustrate the cyclicality of default and bankruptcies, figure 1 presents the time series of business chapter 7 and chapter 11 filings in the United States.[5] Figure 2 presents the S&P corporate bond default rate.

[5] Unfortunately, these are the raw numbers of filings, i.e. they are not weighted by the firm size or the debt in bankruptcy. These data do not appear to exist.

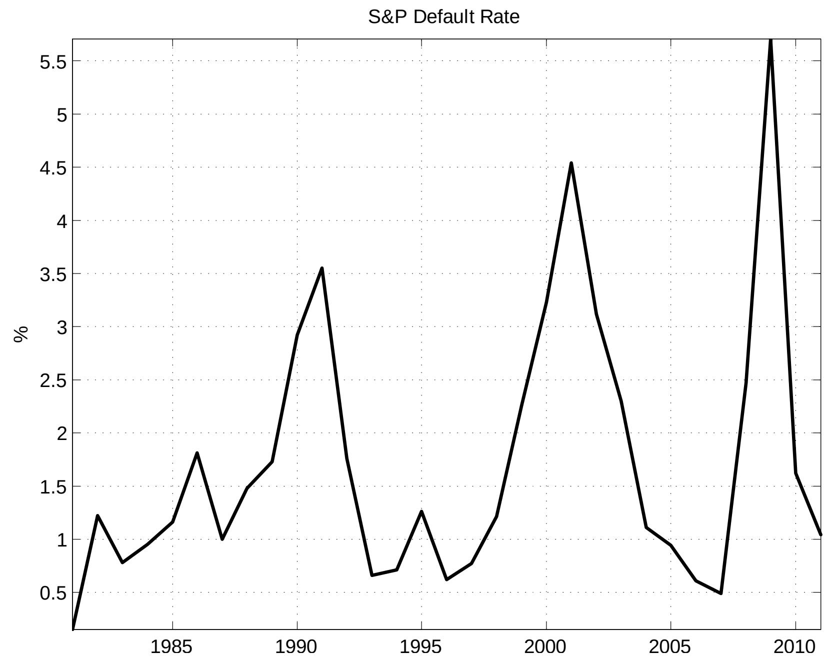

Figure 2: S&P default rate. Note: this is the percentage of S&P rated issuers that defaulted in a given year.

1.1 Related Literature (incomplete)

- A large literature in corporate finance studies the costs of financial distress (e.g. Opler and Titman). A motivation for this literature is that it is difficult to understand the low observed leverage of most nonfinancial firms in light the large tax savings that firms could generate, and the relatively small bankruptcy costs. Or to put it a different way, most models require a loss upon default of nearly 50% to be roughly consistent with observed leverage choices and default probabilities. This deadweight loss seems too large in light of estimates of actual bankruptcy costs of 10% or less. This suggests there must be some other costs to being in financial distress. A natural interpretation is that some costs arise prior to default, in the form of higher costs of financing, inefficiencies driven by debt overhang, or the "default wedge" studied in this paper (and which is not original to this paper). Despite the large empirical work in corporate finance, there is less work in the macroeconomic literature that studies in general equilibrium the effects of these frictions. Moreover, the social and private costs of these frictions could be quite different. One limitation of some studies (including, to some extent, the Brown and Masa study discussed above) is that it is difficult to disentangle the effects of economic from financial distress.
- Literature on financial accelerator with labor (Chugh, Petrovsky-Nadeau)
- Literature on domino effects and bankruptcy waves.
- Literature on working capital

2 Model

This section presents a simple model without capital, that demonstrates and evaluates quantitatively the key mechanism. A simpler static model is relegated to the appendix.

2.1 Firms

There is a continuum of measure one of firms. All firms produce the same good under perfect competition in input and output markets. Each firm operates a labor-only production function:

$$y = zxn^{\alpha},$$

where n is labor, z is aggregate productivity, which follows an exogenous stochastic process, and x is idiosyncratic productivity, which follows an exogenous stationary stochastic process with invariant distribution $\mu(x)$.

The entry and exit process is kept simple to simplify aggregation. Each period, firms are faced with some costs that they must pay if they are to survive. First, they must pay b. Second, they must pay a random "liquidity" (or endowment) shock η, with mean 0 and cdf $H(.)$. These costs can be interpreted as debt repayment, or as fixed costs.[6] The randomness may arise because of a pure profit windfall, that could be either positive or negative (e.g., a lawsuit that must be settled, or a one-time capital gain or profit from special circumstances). Each period, some firms will decide to default rather than continue operating and pay these costs. These exiters are replaced with new firms, which we assume have exactly the same idiosyncratic productivity x as the old firms. (One possible interpretation is that the new firms are the same as the old firms because they are the same firms, after restructuring.) This assumption keeps the analysis very simple since the cross-sectional distribution of firms according to x is constant as a result, equal to $\mu(x)$. Firms will turn out to be heterogeneous only in this dimension.

The timing of the model is as follows. At the beginning of each period, the productivity shocks z and x are realized. Firm x then decides to hire $n(x)$ workers. The "liquidity" shock η is then realized. Two possibilities arise: either the firm decides to continue operating, and pay $\eta + b$; in this case, the residual is paid as dividend to the equityholders. Note that this dividend may be negative: there is no financial friction preventing recapitalization. Or the firm defaults, in which case equityholders lose everything. We assume that the firm keeps producing, but the workers face a loss $\theta_w w(x)$ where $w(x)$ is their wage, and θ_w is a parameter. This parameter is the novelty of the paper. A literal interpretation of the parameter is missed wages (and time/legal costs of recovering wages and benefits). A broader interpretation would include all costs of default that are borne by workers, such as time spent unemployed, or long-term wage losses because of displacement. Obviously, the broader interpretation is outside the model - θ_w is in this case a reduced form parameter.

Note that the wage $w(x)$ depends on the firm's characteristics, i.e. here its productivity: since the labor market is competitive and workers care about their employer's likelihood of distress, firms will have to offer different wages in equilibrium to compensate for their default risk. To obtain labor demand,

[6] It would be possible to endogenize the choice of b for instance by appealing to a tax advantage of debt.

consider the problem of the equityholder operating the firm,

$$V(x,\omega)=\max_{n\geq 0}E_{\eta}\left(\max\left(0,\pi-b-\eta+E_{\omega',x'}M(\omega,\omega')V(x',\omega')\right)\right). \tag{1}$$

Here ω is the aggregate state of this economy, to be discussed below, $M(\omega,\omega')$ is the stochastic discount factor, which in equilibrium equals the marginal rate of substitution of the household, and π is the operating profit:

$$\pi(n,x,\omega)=zxn^{\alpha}-w(x,\omega)n.$$

The equityholder problem has a default option that allows him to let the firm fall apart after observing the realization of η. Dividends $\pi-b-\eta$ are allowed to be negative, so that equityholders will inject funds in the firm if it is profitable. Hence, we do not model "liquidity driven defaults" i.e. situations where firms end up in default for lack of cash, despite positive net present value.[7]

The wages differ by firm $w(x,\omega)$, and will be determined in equilibrium. From equation (1), we obtain the both optimal labor choice $n^*(x,\omega)$ and default threshold $\eta^*(x,\omega)$. First, the firm defaults if $\eta\geq\eta^*(x,\omega)$, where

$$\eta^*(x,\omega)=zxn(x,\omega)^{\alpha}-w(x,\omega)n(x,\omega)-b+E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right),$$

and the probability of default is hence

$$PD(x,\omega)=1-H\left(\eta^*(x,\omega)\right).$$

To find labor demand, rewrite the problem as:

$$V(x,\omega)=\max_{n\geq 0}\int_{-\infty}^{\eta^*(x,\omega)}zxn^{\alpha}-w(x,\omega)n-b-\eta+E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')dH(\eta)\right)$$

or

$$V(x,\omega)=\max_{n\geq 0}H\left(\eta^*(x,\omega)\right)\left(zxn^{\alpha}-w(x,\omega)n-b+E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right)\right)-\int_{-\infty}^{\eta^*(x,\omega)}\eta dH(\eta).$$

This implies that the standard first-order condition holds:

$$\alpha zxn\left(x,\omega\right)^{\alpha-1}=w(x,\omega).$$

To find the equilibrium wages, we turn to the household problem.

2.2 Households

There is a continuum of measure 1 of workers, all of whom have utility $U(c)-\gamma n$. Following Hansen (1985) and Rogerson (1988), I assume indivisible labor: $n\in\{0,1\}$. In the interest of simplicity, I assume that there is perfect risk-sharing, so the household decisions can be thought of as arising from a big family perspective: the household head decides to allocate family members to work or leisure, and to work at different firms x with wages $w(x,\omega)$.

[7] Liquidity driven defaults can lead to an alternative interesting mechanism, with "negative spirals": as the firm becomes closer to default, its profitability deteriorates because of the default wedge, leading to lower cash flows and further increases in the likelihood of default.

The household problem is to send $n^s(x, w)$ people to work at firms with productivity x, taking into account the wages offered by these firms and that there will be a loss in the event that the firm goes under. Summarizing,

$$\begin{aligned} &\max_{\{n^s(x)\}} U(C(\omega)) - \gamma \int_0^\infty n^s(x, w) d\mu(x), \\ s.t. \quad : \quad & C(\omega) = \Pi(\omega) + \int_0^\infty n^s(x, w) w(x, w)(1 - \theta_w PD(x, \omega)) d\mu(x). \end{aligned}$$

Note that consumption is equalized across household members given the separability of preferences. This problem leads directly to the first-order conditions:

$$\forall x \geq 0, \frac{\gamma}{U'(C(\omega))} = w(x, \omega)(1 - \theta_w PD(x, \omega))). \tag{2}$$

This equations reflects a compensating differential: workers require higher wages to work in more risky firms. In any competitive equilibrium, they must be indifferent between working at two firms with different productivities x and x' :

$$w(x, \omega)(1 - \theta_w PD(x, \omega))) = w(x', \omega)(1 - \theta_w PD(x', \omega))). \tag{3}$$

The other first-order condition with respect to asset holdings shows that the household prices all assets

in this economy, leading to the usual expression for the stochastic discount factor,

$$M(\omega, \omega') = \frac{\beta U'(C(\omega'))}{U'(C(\omega))}.$$

2.3 Resource constraint

There is no capital in this economy, hence the aggregate resource constraint reads

$$C(\omega) = Y(\omega) - \zeta DC(\omega),$$

where $DC(\omega)$ are default costs, and $\zeta \in [0, 1]$ is a parameter that measures the fraction of default losses that are real resource costs (rather than transfers). Default costs include both the (standard) losses born by bondholders, and the (novel to this paper) losses born by workers. Mathematically,

$$\begin{aligned} DC(\omega) \quad = \quad & \theta_w \int_0^\infty w(x, \omega) n(x, \omega)(1 - H(\eta^*(x, \omega))) d\mu(x) \\ & + \theta_b \int_0^\infty A(x, \omega)(1 - H(\eta^*(x, \omega))) d\mu(x), \end{aligned}$$

and $A(x, \omega)$ is the enterprise value of the firm. Since bonds are not priced ex-ante in this economy, the only effect of bondholder losses ($\theta_b < 1$) is to generate a wealth effect. In the interest of clarity and simplicity, I will focus on the case where $\zeta = 0$. This implies that there is no wealth effect of default losses, and

$$C(\omega) = Y(\omega).$$

2.4 Aggregation and equilibrium

We can find an expression for aggregate labor given the cross-sectional distribution of probability of defaults:

$$\begin{aligned}
N(\omega) &= \int_0^\infty n(x,\omega)d\mu(x), \\
&= \int_0^\infty \left(\frac{\alpha z x}{w(x,\omega)}\right)^{\frac{1}{1-\alpha}} d\mu(x), \\
&= \int_0^\infty \left(\frac{\alpha z x}{\overline{w}(\omega)}(1-\theta_w PD(x,\omega))\right)^{\frac{1}{1-\alpha}} d\mu(x), \\
&= \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x),
\end{aligned}$$

where $\overline{w}(\omega) = \gamma/U'(C(\omega))$ is the wage of an hypothetical risk-free firm.

Similarly, we can obtain aggregate output as

$$\begin{aligned}
Y(\omega) &= \int_0^\infty zxn(x,\omega)^\alpha d\mu(x), \\
&= \left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x).
\end{aligned}$$

Note that these two expressions allows deriving a formula for average labor productivity, which will prove useful later:

$$\begin{aligned}
\frac{Y(\omega)}{N(\omega)} &= \frac{\left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x)}{\left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x)}, \\
&= \frac{\overline{w}(\omega)}{\alpha} \frac{\int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x)}{\int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x)}, \\
&= \frac{\gamma}{\alpha} \frac{1}{U'(C(\omega))} \frac{\int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x)}{\int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x)}.
\end{aligned}$$

With log utility, $U(C) = \log(C)$, and given that $C(\omega) = Y(\omega)$, we can further simplify to:

$$N(\omega) = \frac{\alpha}{\gamma} \frac{\int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x)}{\int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x)}. \quad (4)$$

2.5 Computation

First, note that the only aggregate state is $z : \omega = z$. This results from (i) net worth is not a state variable for firm since there is no constraint on equity issuance; (ii) the distribution of x is constant (rather than truncated).

Second, the set of equations characterizing the equilibrium is

$$N(\omega) = \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x),$$

$$Y(\omega) = \left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}}(1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x),$$

$$\overline{w}(\omega) = \frac{\gamma}{U'(C(\omega))},$$

$$C(\omega) = Y(\omega),$$
$$M(\omega, \omega') = \frac{\beta U'(C(\omega'))}{U'(C(\omega))},$$

and for all $x \geq 0$:

$$w(x, \omega)(1 - \theta_w PD(x, \omega))) = \overline{w}(\omega),$$
$$\alpha z x n\left(x, \omega\right)^{\alpha - 1} = w(x, \omega),$$
$$\eta^*(x, \omega) = z x n(x, \omega)^\alpha - w(x, \omega) n(x, \omega) - b + E_{\omega', x'}\left(M(\omega, \omega') V(x', \omega')\right),$$
$$PD(x, \omega) = 1 - H\left(\eta^*(x, \omega)\right),$$

$$\begin{aligned} V(x, \omega) &= (1 - PD(x, \omega))\left\{z x n(x, \omega)^\alpha - w(x, \omega) n(x, \omega) - b + E_{\omega', x'}\left(M(\omega, \omega') V(x', \omega')\right)\right\} \\ &- \int_{-\infty}^{\eta^*(x, \omega)} \eta dH(\eta). \end{aligned}$$

It is important for our examination of nonlinear dynamics to solve precisely the equilibrium of this economy. Hence, we solve for the equilibrium functions that characterize the equilibrium, $C(\omega)$, $N(\omega)$, $Y(\omega)$, $M(\omega, \omega')$, $\overline{w}(\omega)$ and $n(x, \omega)$, $V(x, \omega)$, $PD(x, \omega)$, $\eta^*(x, \omega)$, $w(x, \omega)$. Specifically, both x and z are discretized, and we iterate on the system of equations until a fixed point is reached.

2.6 Labor and TFP Wedges

Equation (2) shows clearly also that a labor wedge will arise in this economy, and will depend on the likelihood of default. To make this explicit, I follow Shimer (2010) and define the labor wedge as

$$1 - \tau_t = \frac{U_2(C_t, N_t)/U_1(C_t, N_t)}{\alpha Y_t / N_t},$$

or

$$\tau_t = 1 - \frac{\widehat{\gamma}}{\alpha} \frac{C_t}{Y_t} N_t,$$

where $\frac{\widehat{\gamma}}{\alpha}$ is picked so that on average $\tau = .4$ (a realistic level of average labor taxes). This corresponds to what an outside econometrician would calculate, assuming he picked the correct preferences, but without knowing the leisure preference or labor share. Using first that in equilibrium $C = Y$, and second equation (4), the labor wedge implied by the model satisfies:

$$\begin{aligned} \tau(\omega) &= 1 - \frac{\widehat{\gamma}}{\alpha} N(\omega), \\ &= 1 - \frac{\widehat{\gamma}}{\alpha} \frac{\alpha}{\gamma} \frac{\int_0^\infty x^{\frac{1}{1-\alpha}} \left(1 - \theta_w PD(x, \omega)\right)^{\frac{1}{1-\alpha}} d\mu(x)}{\int_0^\infty x^{\frac{1}{1-\alpha}} \left(1 - \theta_w PD(x, \omega)\right)^{\frac{\alpha}{1-\alpha}} d\mu(x)}. \end{aligned}$$

While this expression appears complicated, some simple cases are useful to examine. First, suppose that $PD(x, \omega) = 0$ for all firms (e.g. in the case where $b = 0$ and the standard deviation of η goes to zero), then the labor wedge equals

$$\tau(\omega) = 1 - 1 - \frac{\widehat{\gamma}}{\alpha} \frac{\alpha}{\gamma}.$$

Hence, the labor wedge is constant, contrary to the data. Second, suppose that $PD(x, \omega) = PD(\omega)$ was the same for all firms (as is the case, for instance, if the unconditional standard deviation of x was small relative to the standard deviation of η). In this case, we have

$$\tau(\omega) = 1 - \widehat{\theta} \frac{\alpha}{\gamma} (1 - \theta_w PD(\omega)).$$

The labor wedge would be higher in times when the default likelihood is higher, i.e. in recessions. The magnitude of the labor wedge further depends directly on the employee losses θ_w as well as the likelihood of default. Given the sharp movements in the probability of default for some firms, and the significant losses to workers of losing their job, this variation could be important.

More generally, when the default likelihood depends both on x and ω, there will be an average wedge will be different for different firms. This in turn implies some "misallocation" of the workforce: there will be "too many" people working in high productivity firms, which have low default risk, and "too few" in low productivity firms, with high default risk. This in turn reduces aggregate total factor productivity, because decreasing returns make it inefficient to have too many workers in high x firms.

Specifically, one can define aggregate TFP implied by the model as

$$TFP = \frac{Y}{N^\alpha} = z\frac{\int_0^\infty x^{\frac{1}{1-\alpha}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{\alpha}{1-\alpha}} d\mu(x)}{\left(\int_0^\infty x^{\frac{1}{1-\alpha}}\left(1-\theta_w PD(x,\omega)\right)^{\frac{1}{1-\alpha}} d\mu(x)\right)^\alpha}.$$

This is the standard Solow residual measure. It is easy to see that if $PD(x,\omega) = 0$, then $TFP = zE(x^{\frac{1}{1-\alpha}})^{1-\alpha}$, the usual total factor productivity adjusted for allocation. Furthermore, if $PD(x,\omega) = PD(\omega)$ is the same for all x, then $TFP = zE(x^{\frac{1}{1-\alpha}})^{1-\alpha}$ still. However, in general, from the formula above, $TFP < zE(x^{\frac{1}{1-\alpha}})^{1-\alpha}$ if $PD(x,\omega)$ depends on x. The logic here is similar to Restuccia and Rogerson (2008), and comes from the fact that $Cov\left(x, PD(x,\omega)\right) < 0$. {expand this}

2.7 Multiple equilibria

{to be added}

If $\theta_w = 1$, there are multiple equilibria: $PD(x,\omega) = 1$ leads to $n(x,\omega) = 0$...

If $\theta_w < 1$, there is a range of values of x such that there are multiple equilibria...

3 Model Results

This section first discusses the model parameters, then illustrates the model implications by presenting quantitative simulations.

3.1 Parametrization

{to be added} {the current results are not final, and should be understood more as a numerical example than a full calibration at this stage}

3.2 Response to a productivity disturbance

Figure 3 presents the cross-sectional distribution of equity values implied by the model together with the wedge implied in the first-order condition. The wedge is higher for low productivity x firms, which have a correspondingly higher probability of default. The average distortion in the economy depends on the product of these two curves - how many firms are affected by the distortion, times the magnitude of the distortion. A macroeconomic shock leads to higher distortions by shifting the cross-sectional

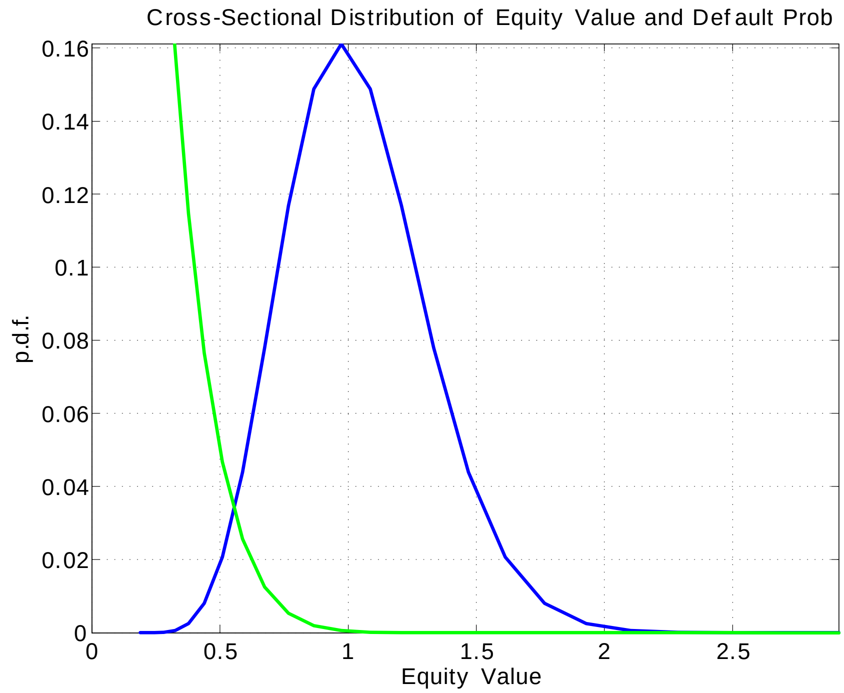

Figure 3: Cross-sectional distribution of equity values, and distortion in the labor first-order condition.

distribution of equity values to the left, and also potentially by shifting up the distortion as a function of productivity.

Figure 4 calculates the response of the economy as aggregate total factor productivity z varies. The red line depicts the response of the model without worker losses (i.e. $\theta_w = 0$); in this case, the model collapses to the standard RBC model (without capital). Given log utility, employment is independent of z. Output responds hence one-for-one, aggregate TFP is measured without error, and there is no variation in the labor wedge. The blue line shows the case where $\theta_w > 0$: in this case, as z falls, the number of firms close to default becomes larger, which leads to lower employment and output than would otherwise be, as shown in the top two graphs. The bottom graphs establish that aggregate TFP falls further than z itself implies, i.e. there is a TFP wedge, and that there is a significant countercyclical labor wedge.

Figure 5 shows the effect of varying the parameter b on the macroeconomic equilibrium. For convenience, the x-axis is aggregate leverage, $\frac{b}{\int_0^\infty V(x,\omega)d\mu(x)}$. A higher b and the ensuing higher leverage leads to a larger share of firms being close to default. This reduces employment and output, again by creating a TFP wedge and a labor wedge. This figure shows that any shock that affects equity values in this model will lead to an economic contraction through this same mechanism.

Figure 6 illustrates the time-varying elasticity to fundamental shocks that the model captures. For various values of the parameter b, I calculate the percentage change in output, employment, TFP and the labor wedge if a one percent change in z were to hit the economy. The elasticity is constant in the case of no worker losses (red line, $\theta_w = 0$): the level of leverage is immaterial for the sensitivity of the economy to shocks. But in the case with worker losses, a higher leverage renders the economy more subject to the amplification mechanism depicted above, and hence the elasticity rises, in some cases

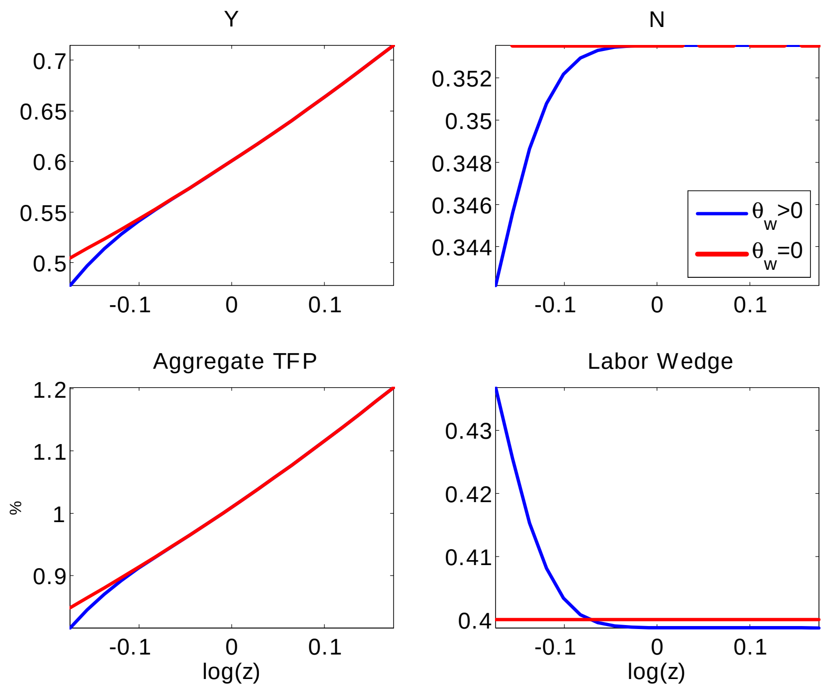

Figure 4: Effect of aggregate TFP z on output Y, employment N, measured TFP Y/N^{α}, and the measured labor wedge.

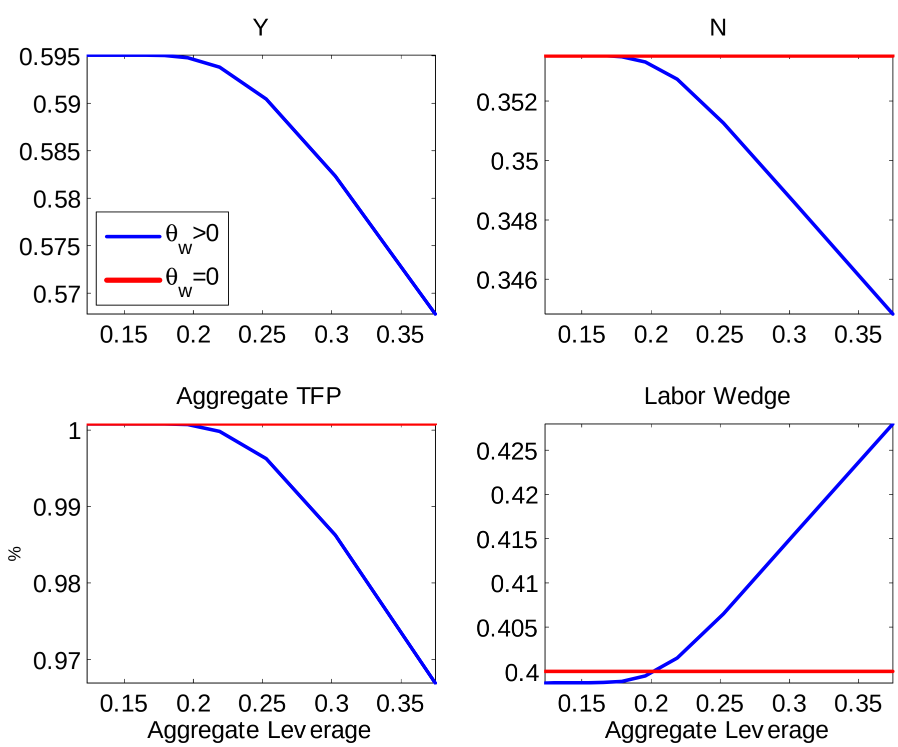

Figure 5: Effect of debt repayment b on macroeconomic equilibrium.

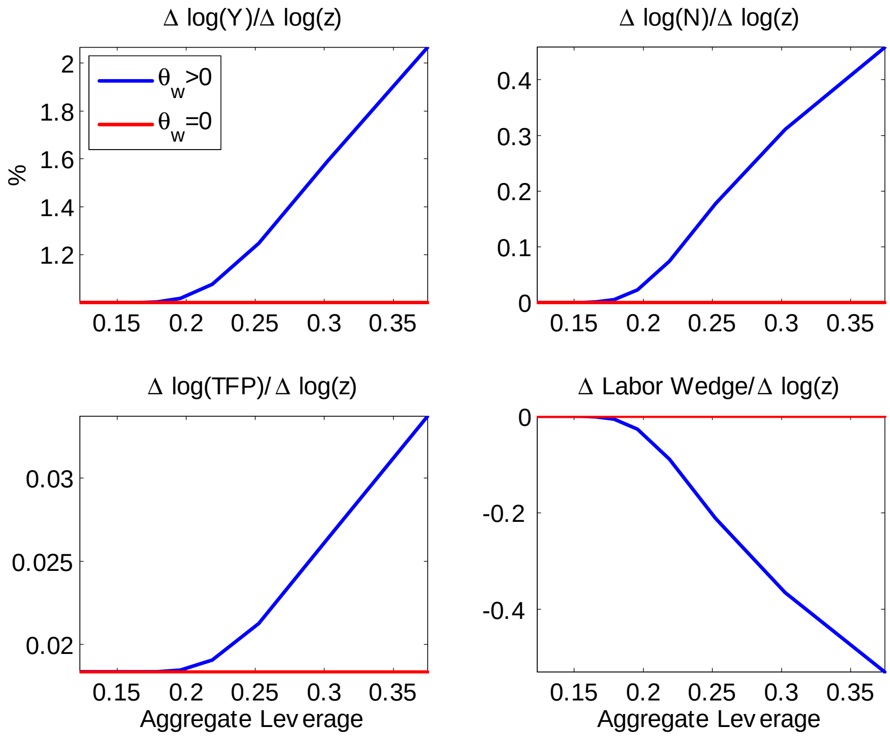

Figure 6: Elasticity of output, employment, TFP and the labor wedge to a z shock, as a function of the leverage.

substantially.

To illustrate this in a more empirically oriented way, figures 7 and 8 present the result from a long model simulation. In these figures, each dot represents a time period, with the associated leverage, and the associated macro volatility, proxied as the standard deviation of the growth rate of employment (figure 7) or output (figure 8) over the next 20 quarters. In this simulation, driven by shock to z only, leverage rises if z falls significantly. It follows that the economy is more sensitive to further shocks. The top panel is the economy with $\theta_w > 0$, while the bottom panel has $\theta_w = 0$. The regression lines illustrate that higher leverage leads to higher volatility, only if $\theta_w > 0$.

Finally, figure 9 illustrates again in a model simulation the relation between leverage and the labor wedge. This relation is flat when leverage is low and the default mechanism does not operate, then becomes positive.

3.3 Time-varying risk aversion

As a simple model extension, I introduce time-varying risk aversion in the model. To do so in a clean way, I use Epstein-Zin preferences:

$$V_t = \left((1-\beta) u(C_t, N_t)^{1-\sigma} + \beta E_t \left(V_{t+1}^{1-\gamma_t} \right)^{\frac{1-\sigma}{1-\gamma_t}} \right)^{\frac{1}{1-\sigma}},$$

and assume that γ_t follows a Markov chain that approximates an AR(1) process. A shock to risk aversion is a shortcut to proxy either a "panic" whereby agents suddenly attempt to reduce their risk

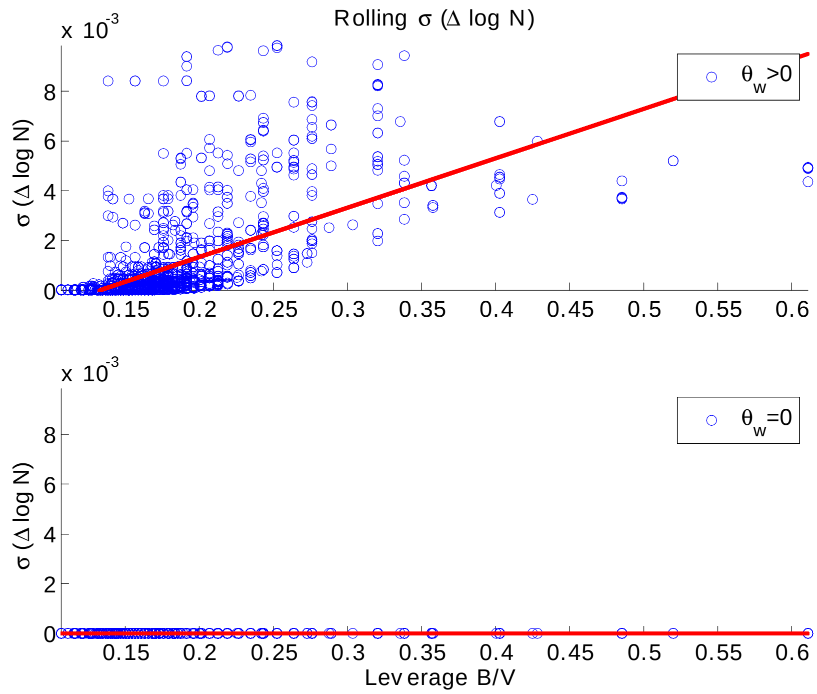

Figure 7: Leverage and employment volatility in the model. Scatter plot of leverage at time t and realized volatility between $t+1$ and $t+20$ for employment growth. Top panel: model with $\theta_w > 0$; bottom panel: $\theta_w = 0$. Regression lines in red.

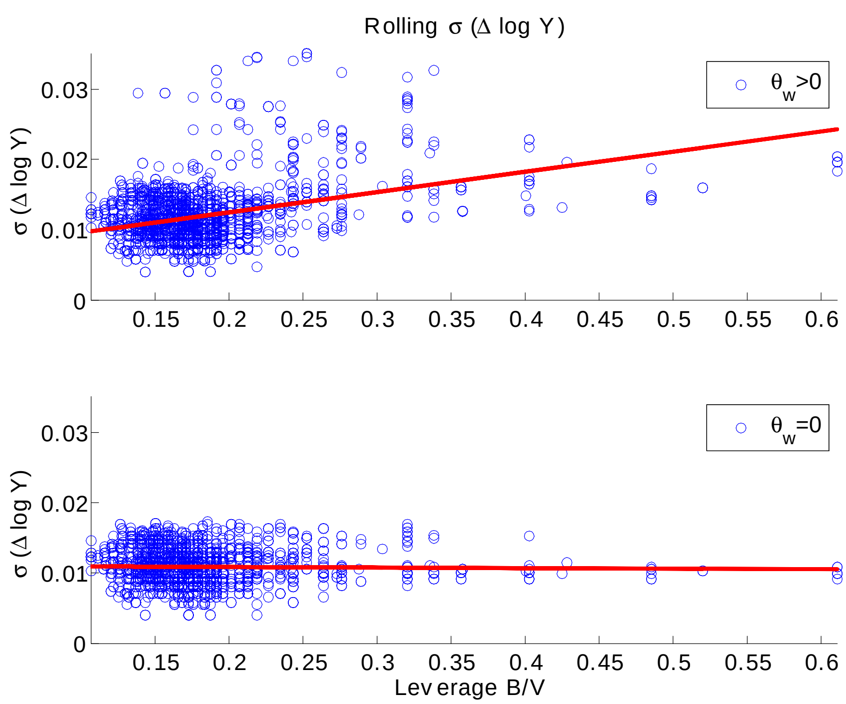

Figure 8: Leverage and output volatility in the model. Scatter plot of leverage at time t and realized volatility between $t+1$ and $t+20$ for output growth. Top panel: model with $\theta_w > 0$; bottom panel: $\theta_w = 0$. Regression lines in red.

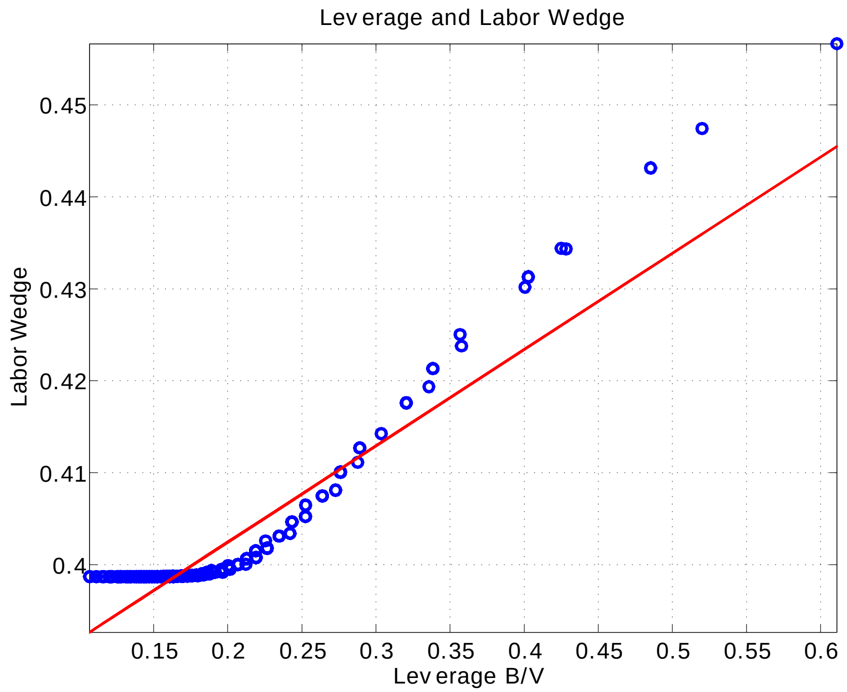

Figure 9: Relation between leverage and the labor wedge in the model. Simulated path and regression line.

exposure, or perhaps a shock to the ability of financial intermediaries to carry risk. As illustrated in figure 10, higher risk aversion pushes the cross-sectional distribution of equity values to the left, and hence amplifies the distortions. Consequently, output and employment are reduced, as shown in figure 11.

3.4 Summary: model implications

To summarize, this section has demonstrated three model implications: (1) the effect of a fundamental shock (such as a z shock) is larger if there are financial distress costs; (2) more generally, any shock that affects equity value or likelihood of default will generate a contraction in economic activity; (3) the amplification effect of financial distress costs depends on the economy's leverage, or more generally the share of firms with high leverage. Overall, we expect the labor wedge, economic activity, the share of firms with high leverage, and macroeconomic volatility to be correlated.

4 Empirical Evidence

This section is a first attempt at assessing the mechanism discussed in the previous section. I proceed in three steps. First, panel data regressions provide some support for the basic idea that firms with higher leverage have sales and employment more sensitive to aggregate fluctuations. Second, I construct the cross-sectional distribution of (market) leverage and find that it varies substantially over time. In particular, the number of firms "close to default" (e.g. with a leverage above a threshold value) is strongly procyclical. This result holds for various definitions of leverage and various thresholds. In

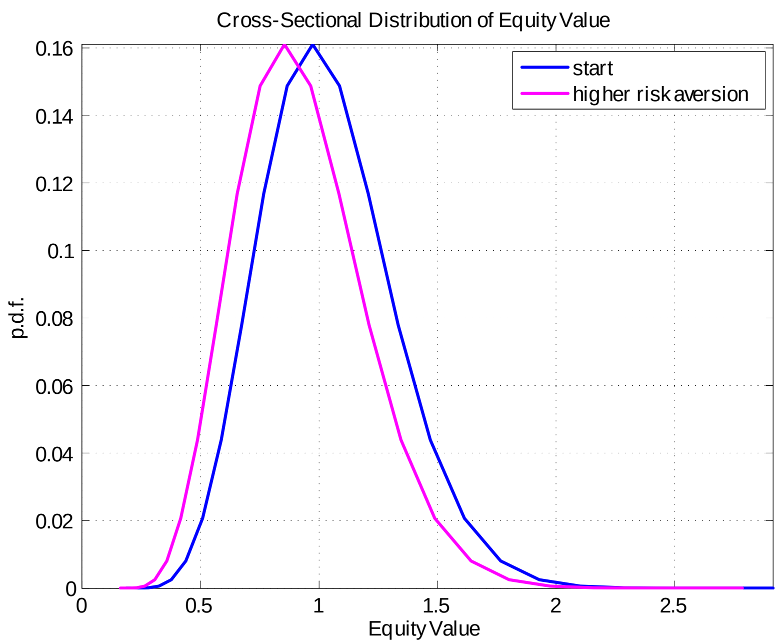

Figure 10: Cross-sectional distribution of equity values in the model, for low and high risk aversion.

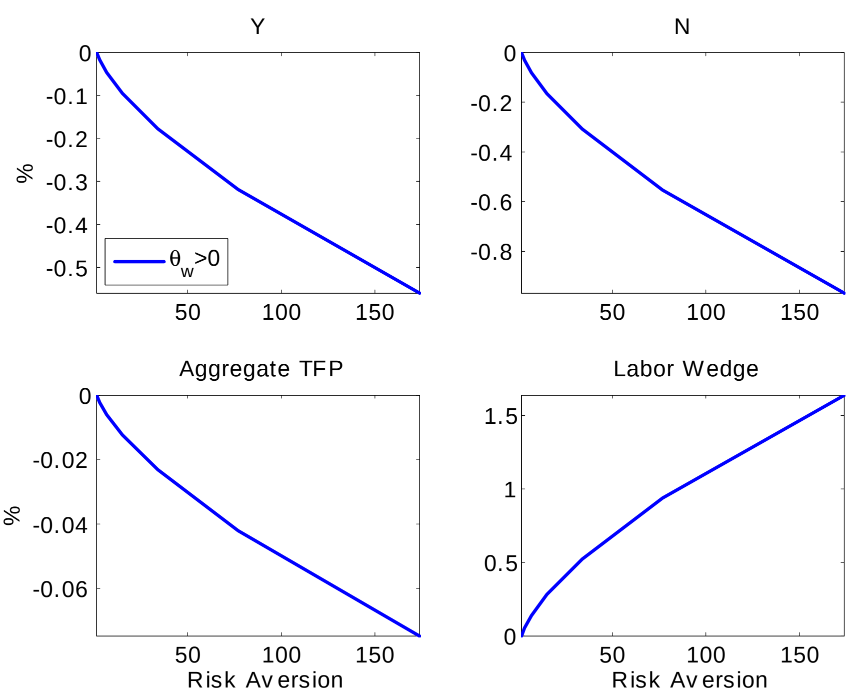

Figure 11: Effect of risk aversion on output, employment, TFP and the Labor Wedge.

contrast, the median leverage is not clearly cyclical. Third, I investigate the time series relations between (i) aggregate uncertainty, (ii) the labor wedge, (iii) measures of economic activity, and (iv) the share of firms that are close to default. While some of the correlations are consistent with the model, others are not. However, an interesting finding is that the share of firms that are close to default appears to be a useful statistical predictor of economic activity - more so than the average or median leverage that is typically used in empirical studies such as Kollman and Zeugner (2013).

4.1 Cross-sectional evidence

A key empirical implication of the theory is that firms with high leverage or high default risk are more sensitive to aggregate fluctuations. This section provides some simple reduced-form evidence consistent with this implication (see also Sharpe (1994) for related evidence). I discuss below some potential endogeneity concerns.

Using firm-level annual panel data from Compustat, I estimate the regression,

$$\Delta \log S_{it} = \delta_t + \gamma \text{HighLev}_{i,t-1} + \beta \text{HighLev}_{i,t-1} \Delta \log GDP_t + \varepsilon_{it},$$

where S_{it} is real sales of firm i in year t, $\text{HighLev}_{i,t-1}$ is a dummy variable equal to one if a firm has "high leverage", $\Delta \log GDP_t$ is the growth rate of real GDP, and δ_t include a full set of time effects.[8] The coefficient of interests are γ, which measures by how much "high leverage" reduces sales growth, and especially β, which measures the extra sensitivity to GDP growth of high leverage firms.

I also estimate a similar regression using employment growth as the dependent variable,

$$\Delta \log N_{it} = \delta_t + \gamma \text{HighLev}_{i,t-1} + \beta \text{HighLev}_{i,t-1} \Delta \log GDP_t + \varepsilon_{it}.$$

The Compustat sample used is fairly standard; it includes all non-financial, non-utilities, domestic firms, with a December fiscal-year, from 1970 to 2011. Leverage is defined as the ratio of total debt (long-term debt plus short-term) over equity market value (i.e. (dltt+dlc)/(csho*prcc_f)), but we also consider alternative definitions including net-of-cash leverage (substracting cash and short-term investments from debt on the denominator ((dltt+dlc-che)) and net-of-cash and trade credit leverage (substracting receivables and adding receivables, (dltt+dlc+ap–rectp-che)). A high leverage firm is defined as a firm with leverage above 0.45. The results continue to hold, however, if one uses other thresholds.

Table 1 reports the result. First, estimates of γ are consistently negative and highly significant. Being high leverage is associated with lower sales growth of 5-8% going forward. Obviously, this coefficient does not measure the "effect" of high leverage, since leverage is endogenous: negative shocks to expected sales drive equity value lower and leverage higher, so the causation runs "both ways", and we expect a negative coefficient even in the absence of any financial distress cost. On the other hand, one might have expected some mean-reversion of sales for firms that had negative shocks the previous year.

Second, the coefficient β is positive and significant. The typical firm in Compustat has a sensitivity of sales growth to GDP around 2; a coefficient β of 0.5 reflects that high leverage firms have a sensitivity

[8] As a result, it is not necessary to include $\Delta \log GDP_t$ on the right-hand side.

Dependent variable: $\Delta \log S_{it}$	(1)	(2)	(3)	(4)	(5)	(6)
Leverage definition	(debt)		(debt-cash)		(debt-cash-trade)	
γ	-0.075	-0.068	-0.063	-0.051	-0.057	-0.055
t-stat	14.3	10.5	12.4	8.1	10.8	8.2
β	0.52	0.50	0.49	0.49	0.38	0.37
t-stat	3.6	3.4	3.5	3.4	2.6	2.5
Firm fixed-effects	n	y	n	y	n	y
Observations	69377					

Table 1: Sales growth sensitivity as a function of leverage. Robust standard errors.

Dependent variable: $\Delta \log N_{it}$	(1)	(2)	(3)	(4)	(5)	(6)
	(debt)		(debt-cash)		(debt-cash-trade)	
γ	-0.062	-0.060	-0.055	-0.051	-0.049	-0.051
t-stat	15.1	11.3	13.2	9.4	11.4	8.8
β	0.44	0.51	0.44	0.51	0.38	0.45
t-stat	3.7	4.2	3.7	4.2	3.1	3.6
Firm fixed-effects	n	y	n	y	n	y
Observations	69377					

Table 2: Employment growth sensitivity as a function of leverage.

around 2 instead. This is economically important.[9] Finally, note that both coefficients β and γ are fairly stable across different definitions of leverage.

Table 2 turns to the employment results, which are very similar overall. In Compustat, the typical firm has a sensitivity of employment to GDP around 1.5. Hence the estimated β is, if anything, even more economically important for employment than for sales.

4.2 Business cycle variation in the cross-sectional distribution of leverage

A second key piece of the mechanism is that the number of firms close to default varies substantially over time. To provide some light on this topic, Figures 12 and figure 13 provide the distribution of net-of-cash leverage in 2006Q1 and 2009Q1. First, note that there is a wide distribution, with the typical firms having essentially zero net leverage. But the fanning out of the distribution during the recession is

[9] In this case too, a potential source of bias is that firms with different cyclical sensitivities in their real activities (in finance language, firms that have high asset beta) may choose a different leverage. For instance, a firm with high sensitivity may choose a lower leverage. This would tend to bias our estimated β towards zero. On the other hand, in a business cycle downturn, the market values of more cyclical firms may fall more, leading them to have higher leverage ex-post, which would bias our estimated β up. Hence, the overall effect is unclear.

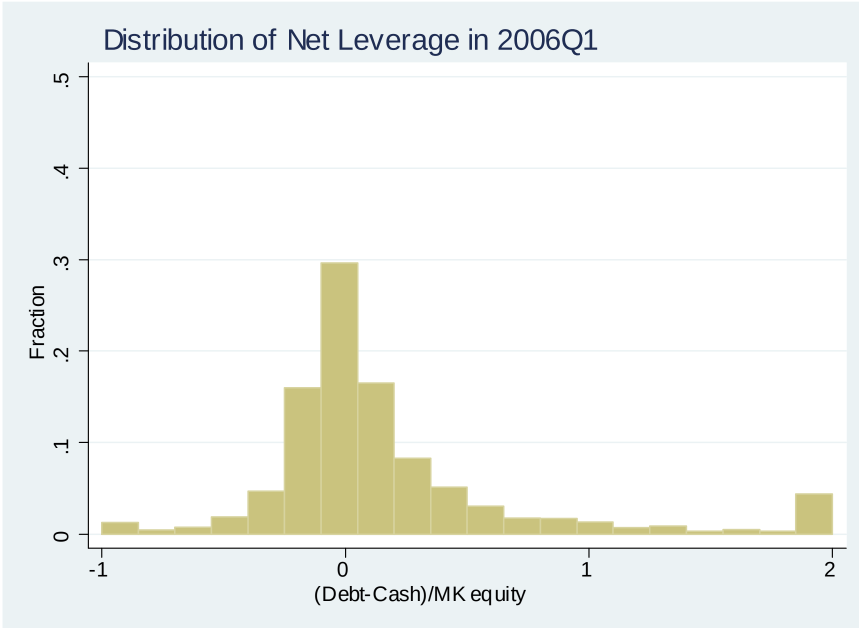

Figure 12: Distribution of net-of-cash leverage in 2006q1 in Compustat (nonfinancial firms). Net-of-cash leverage is (debt-cash)/(market value of equity).

impressive: there were many more firms with high net leverage in early 2009 than in early 2006. There were also significantly more firms with negative net leverage. This result also continues to hold for the alternative definitions of leverage.

One possible interpretation of these figures is that for the typical firm, financial distress is not a concern. But it is a concern for a small numbers of firms in "good times" like 2006, and for a significant fraction of firms in "bad times" like 2009. Hence, one really wants to track the time series of the number of firms with high leverage. Figure 14 plots the time series, which clearly exhibit a strong countercyclicality. This series remains very similar if we weight firms by sales rather than just counting firm units, as shown in figure 15; the main difference is some additional noise, as some large firms may go above or below the threshold.

An interesting fact is that this share variable behaves quite differently from the median or average leverage, depicted in figure 16. This suggests that there is some interesting information in the "tail" of the leverage distribution.

4.3 Time series evidence

The model makes strong prediction regarding the association of the following variables: (i) the share of firms close to default, (ii) the labor wedge, (iii) macroeconomic uncertainty, and (iv) economic activity. This section discusses the empirical correlation between these time series.

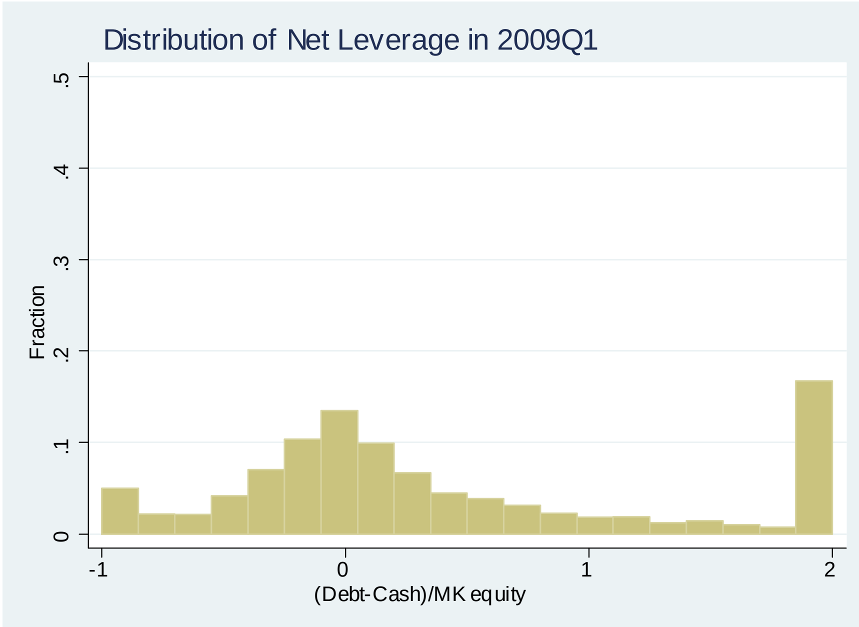

Figure 13: Distribution of net-of-cash leverage in 2009q1 in Compustat (nonfinancial firms). Net-of-cash leverage is (debt-cash)/(market value of equity).

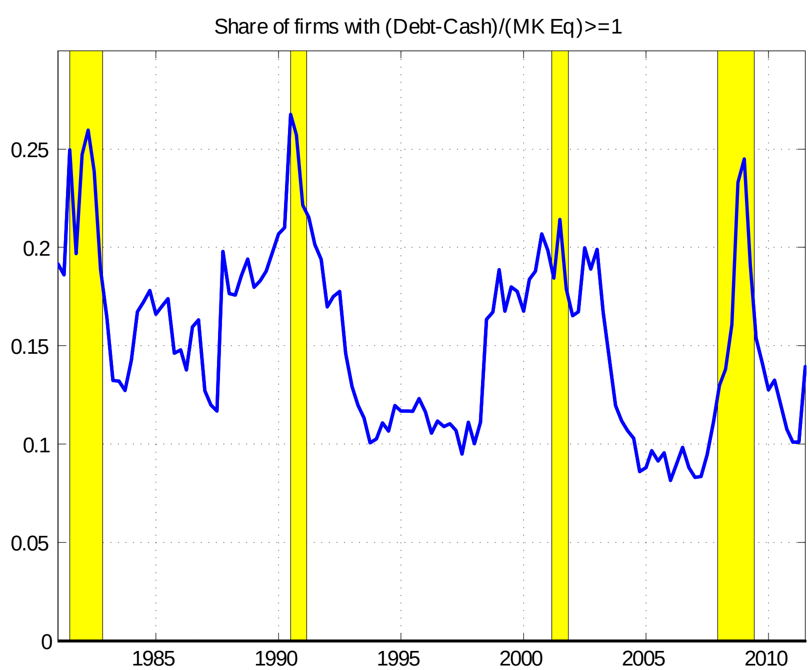

Figure 14: Share of firms with (debt-cash)/)(market value of equity) $\geq$ 1. Compustat, nonfinancial firms.

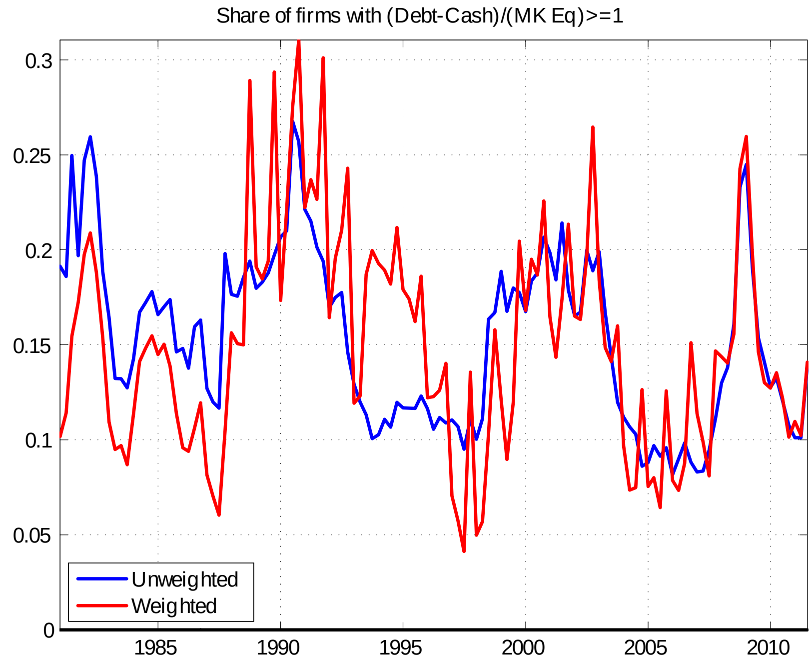

Figure 15: Share of firms with (debt-cash)/)(market value of equity) ≥ 1, both weighted by sales and unweighted. Compustat, nonfinancial firms.

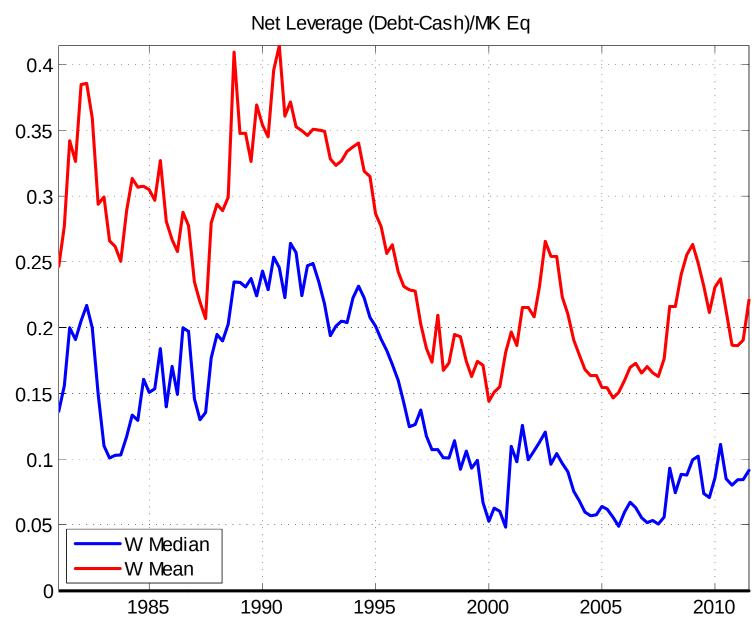

Figure 16: Weighted mean and median of net-of-cash leverage (debt-cash)/(mkt value of equity). Compustat, nonfinancial firms.

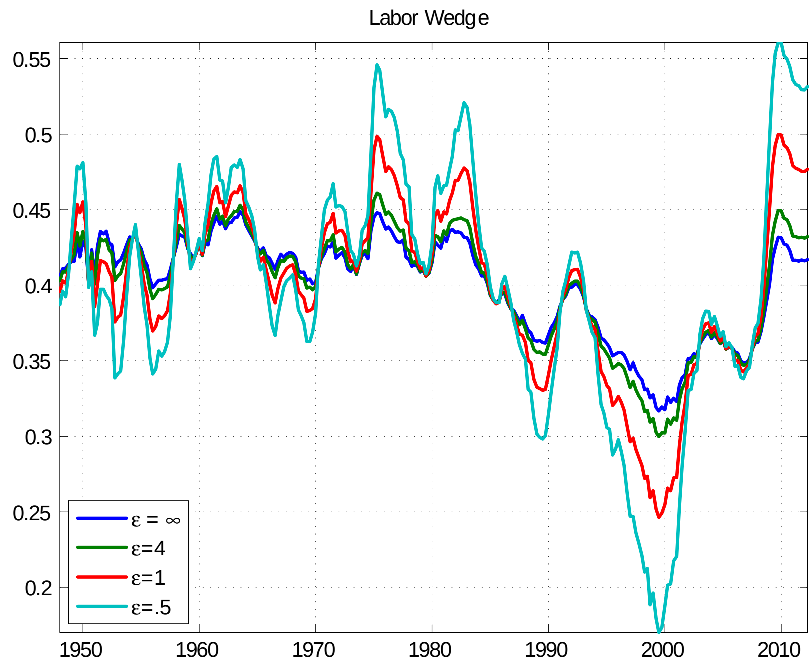

Figure 17: Labor wedge, constructed as in Shimer (2010) and as explained in the text.

4.3.1 Data construction

The share of firms close to default is constructed as in the previous section. The labor wedge is constructed as in Shimer (2010). Specifically, assuming a representative agent with utility

$$E\sum_{t=0}^{\infty}\beta^t\left(\log(c_t)-\gamma\frac{n_t^{1+\phi}}{1+\phi}\right),$$

and a production function $y_t = k_t^{\alpha}\left(z_t n_t\right)^{1-\alpha}$, the first-order condition for labor implies

$$w_t(1-\tau_t) = (1-\alpha)\frac{y_t}{n_t}(1-\tau_t) = \gamma c_t n_t^{\phi},$$

so that

$$\tau_t = 1 - \frac{\gamma}{1-\alpha}\frac{c_t}{y_t}n_t^{1+\phi}.$$

Using nondurable per-capita consumption, hours worked per-capita, and real GDP, we construct the right-hand-side for given value of ϕ. The coefficient $\frac{\gamma}{1-\alpha}$ is picked so that τ_t is on average equal to 0.4. Obviously, the elasticity of labor supply (the inverse of the parameter ϕ) matters for this construction, but the labor wedge is a puzzle regardless of the value of ϕ. Figure 17 depicts the labor wedge implied by different values of ϕ. This series is highly countercyclical with respect to employment, and quite countercyclical with respect to output. Figure 18 illustrates this by plotting together HP filtered log labor wedge and HP filtered log hours.

We consider two macroeconomic uncertainty measures. First, we use the stock market volatility, constructed as the standard deviation of realized daily returns within a quarter. Second, we use the uncertainty measure constructed by Jurado, Ludvigson and Ng (2013). They use a large dataset of macro and financial indicators and estimate the average standard deviation of the unforecastable component

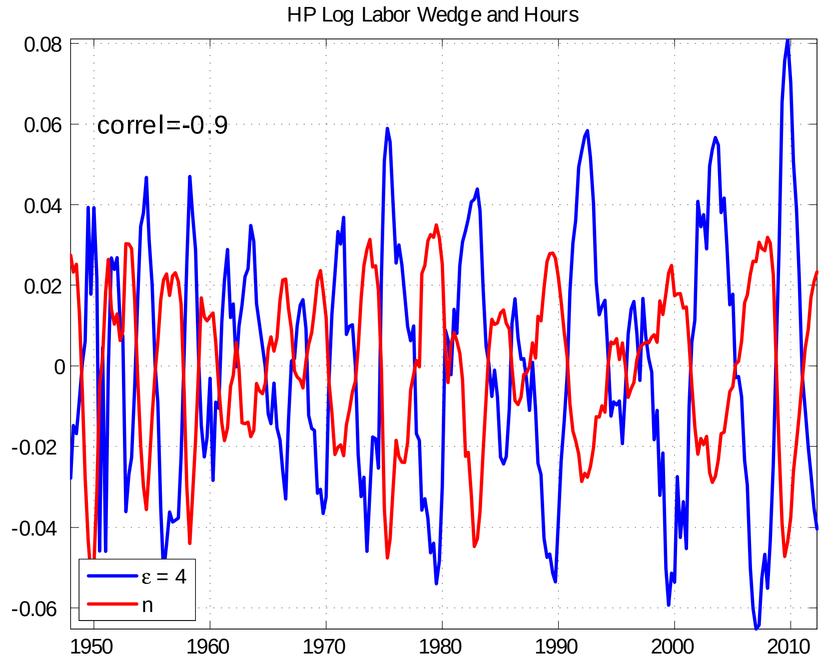

Figure 18: HP filtered log labor wedge and HP filtered log hours. US data.

of these time series. This uncertainty measure is depicted in figure 19. It has spikes in 2008, 1981, and 1975, but not much action in the 1991 or 2001 recessions.

4.3.2 Relations between time series (very preliminary and incomplete)

This section currently presents some reduced-form relations

Correlations

Table 3 presents the correlation between the key macroeconomic time series studied here. Interesting, and consistent with the model, uncertainty is strongly correlated with the share of firms with high leverage (0.48 or 0.46 depending on the measure of uncertainty). In contrast, the median or average leverage have much weaker correlation (from -0.03 to 0.33). Moreover, the labor wedge is significantly correlated with macro volatility (0.42). The correlation between leverage and the labor wedge is weaker however, and the share of firms with high leverage does not outperform here the median or average leverage.

Forecasting GDP growth

Kollman and Zeugner (2012) show that average leverage forecasts negatively GDP growth. I show that this relation is significantly stronger if one uses as measure of leverage not the average leverage, but the share of firms with high leverage. To show this, run the regression

$$\Delta \log GDP_{t+1} = a + b\Delta \log GDP_t + cZ_t + \varepsilon_{t+1},$$

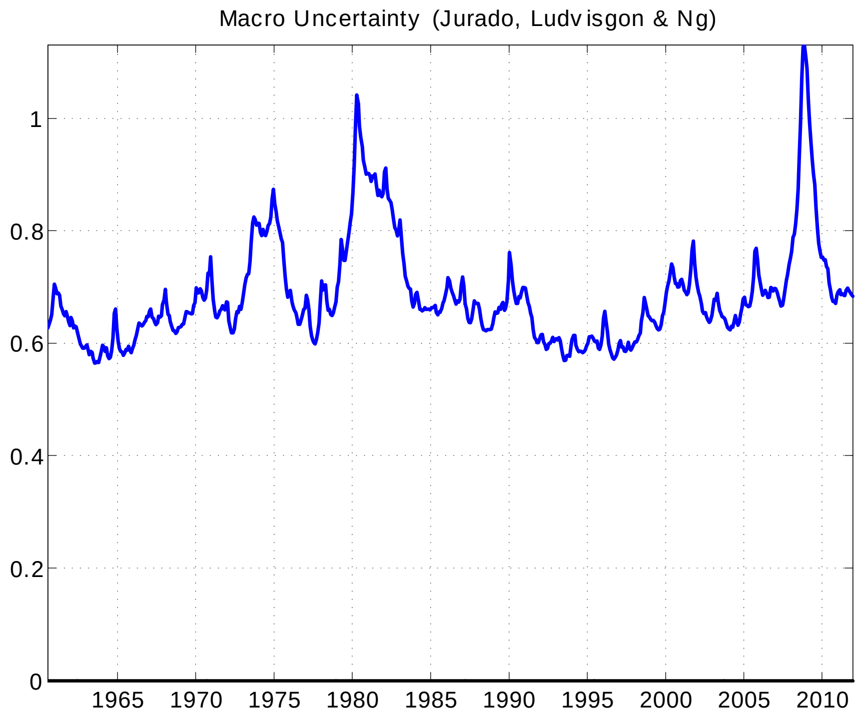

Figure 19: Macroeconomic uncertainty measure. Source: Jurido, Ludvisgon, and Ng (2013).

	StockVol	Unc	Share	Median	Average	GDP	Hours	Labor Wedge
Stock market volatility	1	0.52	0.46	-0.03	0.15	-0.16	0.02	0.00
Uncertainty JLN		1	0.48	0.24	0.33	-0.20	-0.05	0.42
Share high leverage			1	0.62	0.84	-0.23	-0.10	0.11
Median leverage				1	0.93	-0.07	0.04	0.25
Average leverage					1	-0.13	-0.01	0.17
GDP (HP)						1	0.88	-0.53
Hours (HP)							1	-0.61
Labor Wedge								1

Table 3: Correlations. 1980q1-2011q4.

	$100\times c$	t-stat	R^2
Average (unweighted)	-0.0046	-0.66	.235
Average (weighted)	-0.0024	-0.27	.233
Median (unweighted)	-0.0052	-0.92	.238
Median (weighted)	-0.0024	-0.27	.233
Share (unweighted)	-0.0330	-2.20	.262
Share (weighted)	-0.0276	-2.47	.269

Table 4: Forecasting GDP growth using its own lag and measures of leverage.

where Z_t is either average or median leverage, or the share of firms with high leverage. Table 5 summarizes the results. Clearly, the share of high-leverage firms has more explanatory power than just the average or median.

5 Conclusion

{to be added}{asymmetries}

6 References

{incomplete}

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Quadrini

7 Appendix

7.1 Sample construction in Compustat

TBA

7.2 Algebra for more general preferences

Suppose preferences are

$$E\sum_{t=0}^{\infty}\beta^t U(C_t, N_t).$$

As a result,

$$\overline{w}(\omega) = \frac{u_2(C(\omega), N(\omega))}{u_1(C(\omega), N(\omega))}.$$

The SDF is

$$M(\omega, \omega') = \beta\frac{U_1(C(\omega'), N(\omega'))}{U_1(C(\omega), N(\omega))}.$$

We still have:

$$N(\omega) = \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} (1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x)$$

$$Y(\omega) = \left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} (1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x)$$

$$C(\omega) = Y(\omega)$$

and for all $x \geq 0$:

$$w(x,\omega)(1-\theta_w PD(x,\omega))) = \overline{w}(\omega)$$

$$\alpha z x n\left(x,\omega\right)^{\alpha-1} = w(x,\omega)$$

$$\eta^*(x,\omega) = zxn(x,\omega)^\alpha - w(x,\omega)n(x,\omega) - b + E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right)$$

$$PD(x,\omega) = 1 - H\left(\eta^*(x,\omega)\right)$$

$$V(x,\omega) = \left(1 - H\left(\eta^*(x,\omega)\right)\right)\left(zxn(x,\omega)^\alpha - w(x,\omega)n(x,\omega) - b + E_{\omega',x'}M(\omega,\omega')V(x',\omega')\right) - \int_{\eta^*(x,\omega)}^\infty \eta dH(\eta)$$

Simple initial guess: no default, i.e.

$$\begin{aligned}
PD(x,\omega) &= 0,\\
w(x,\omega) &= \overline{w}(\omega),\\
n(x,\omega) &= \left(\frac{\alpha z x}{w(x,\omega)}\right)^{\frac{1}{1-\alpha}},\\
\eta^*(x,\omega) &= +\infty,\\
V(x,\omega) &= zxn(x,\omega)^{\alpha} - w(x,\omega)n(x,\omega) - b + E_{\omega',x'}M(\omega,\omega')V(x',\omega'),\\
N(\omega) &= \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^{\infty} x^{\frac{1}{1-\alpha}}\,d\mu(x),\\
Y(\omega) &= \left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^{\infty} x^{\frac{1}{1-\alpha}}\,d\mu(x).
\end{aligned}$$

To solve for $\overline{w}(\omega)$, use the condition:

$$\overline{w}(\omega) = \frac{u_2(C(\omega),N(\omega))}{u_1(C(\omega),N(\omega))},$$

which is one eqn in one unknown, for each value of ω.

In the particular case $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - B\frac{N^{1+\phi}}{1+\phi}$, we have

$$\overline{w}(\omega) = BN(\omega)^{\phi}C(\omega)^{\sigma}$$

$$\begin{aligned}
\overline{w}(\omega) &= B\left(\left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^{\infty} x^{\frac{1}{1-\alpha}}\,d\mu(x)\right)^{\phi} \left(\left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \left(\int_0^{\infty} x^{\frac{1}{1-\alpha}}\,d\mu(x)\right)\right)^{\sigma}\\
&= B\left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\phi+\alpha\sigma}{1-\alpha}} z^{\frac{\phi+\sigma}{1-\alpha}} \left(\int_0^{\infty} x^{\frac{1}{1-\alpha}}\,d\mu(x)\right)^{\phi+\sigma}
\end{aligned}$$

hence

$$\overline{w}(\omega)^{1+\frac{\phi+\alpha\sigma}{1-\alpha}} = Bz^{\frac{\phi+\sigma}{1-\alpha}}\alpha^{\frac{\phi+\alpha\sigma}{1-\alpha}} \left(\int_0^{\infty} x^{\frac{1}{1-\alpha}}\,d\mu(x)\right)^{\phi+\sigma}.$$

7.3 Algebra with Epstein-Zin preferences

Suppose preferences are now given by

$$W(\omega) = \left(u(C,N)^{1-\sigma} + \beta E\left(W(\omega')^{1-\gamma}\right)^{\frac{1-\sigma}{1-\gamma}}\right)^{1-\gamma},$$

Here N is total employment, and u is the felicity function of the big family. For instance, $u(C,N) = C^{\upsilon}(1-N)^{1-\upsilon}$.

As a result,

$$\overline{w}(\omega) = \frac{u_2(C(\omega),N(\omega))}{u_1(C(\omega),N(\omega))}.$$

For instance, if $u(C,N) = C^{\upsilon}(1-N)^{1-\upsilon}$, then

$$\frac{u_2(C,N)}{u_1(C,N)} = \frac{1-\upsilon}{\upsilon}\frac{C}{1-N}$$

The SDF is

$$M(\omega,\omega') = \beta\frac{U_1(C(\omega'),N(\omega'))}{U_1(C(\omega),N(\omega))}\frac{U(C(\omega'),N(\omega'))^{-\sigma}}{U(C(\omega),N(\omega))^{-\sigma}}\frac{W(\omega)^{\sigma-\gamma}}{E\left(W(\omega')^{1-\gamma}\right)^{\frac{\sigma-\gamma}{1-\gamma}}}.$$

We still have:

$$N(\omega) = \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} (1-\theta_w PD(x,\omega))^{\frac{1}{1-\alpha}} d\mu(x)$$

$$Y(\omega) = \left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} (1-\theta_w PD(x,\omega))^{\frac{\alpha}{1-\alpha}} d\mu(x)$$

$$C(\omega) = Y(\omega)$$

and for all $x \geq 0$:

$$w(x,\omega)(1-\theta_w PD(x,\omega))) = \overline{w}(\omega)$$

$$\alpha z x n\left(x,\omega\right)^{\alpha-1} = w(x,\omega)$$

$$\eta^*(x,\omega) = z x n(x,\omega)^\alpha - w(x,\omega)n(x,\omega) - b + E_{\omega',x'}\left(M(\omega,\omega')V(x',\omega')\right)$$

$$PD(x,\omega) = 1 - H\left(\eta^*(x,\omega)\right)$$

$$V(x,\omega) = H\left(\eta^*(x,\omega)\right)\left(z x n(x,\omega)^\alpha - w(x,\omega)n(x,\omega) - b + E_{\omega',x'} M(\omega,\omega')V(x',\omega')\right) - \int_{-\infty}^{\eta^*(x,\omega)} \eta dH(\eta)$$

Initial guess: assume no default, i.e.

$$\begin{aligned}
PD(x,\omega) &= 0, \\
w(x,\omega) &= \overline{w}(\omega), \\
n(x,\omega) &= \left(\frac{\alpha z x}{w(x,\omega)}\right)^{\frac{1}{1-\alpha}}, \\
\eta^*(x,\omega) &= +\infty, \\
V(x,\omega) &= z x n(x,\omega)^\alpha - w(x,\omega)n(x,\omega) - b + E_{\omega',x'} M(\omega,\omega')V(x',\omega'), \\
N(\omega) &= \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} d\mu(x), \\
Y(\omega) &= \left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} d\mu(x).
\end{aligned}$$

To solve for $\overline{w}(\omega)$, use the condition:

$$\overline{w}(\omega) = \frac{u_2(C(\omega), N(\omega))}{u_1(C(\omega), N(\omega))},$$

which is one eqn in one unknown, for each value of ω.

In the particular case $U(C,N) = C^\upsilon (1-N)^{1-\upsilon}$, we have

$$\overline{w}(\omega) = \frac{u_2(C,N)}{u_1(C,N)} = \frac{1-\upsilon}{\upsilon}\frac{C}{1-N}$$

$$\overline{w}(\omega) = \frac{1-\upsilon}{\upsilon} \frac{\left(\frac{\alpha}{\overline{w}(\omega)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} d\mu(x)}{1 - \left(\frac{\alpha z}{\overline{w}(\omega)}\right)^{\frac{1}{1-\alpha}} \int_0^\infty x^{\frac{1}{1-\alpha}} d\mu(x)},$$

which is one nonlinear eqn in $\overline{w}(\omega)$, for each value of ω.

Then get SDF as

$$\begin{aligned}
M(\omega,\omega') &= \beta \frac{U_1(C(\omega'),N(\omega'))}{U_1(C(\omega),N(\omega))} \frac{U(C(\omega'),N(\omega'))^{-\sigma}}{U(C(\omega),N(\omega))^{-\sigma}} \frac{W(\omega')^{\sigma-\gamma}}{E\left(W(\omega')^{1-\gamma}\right)^{\frac{\sigma-\gamma}{1-\gamma}}}. \\
&= \beta \frac{C(\omega')^{\upsilon-1}(1-N(\omega'))^{(1-\upsilon)}}{C(\omega)^{\upsilon-1}(1-N(\omega))^{(1-\upsilon)}} \left(\frac{C(\omega')^{\upsilon}(1-N(\omega'))^{(1-\upsilon)}}{C(\omega)^{\upsilon}(1-N(\omega))^{(1-\upsilon)}}\right)^{-\sigma} \frac{W(\omega')^{\sigma-\gamma}}{E\left(W(\omega')^{1-\gamma}\right)^{\frac{\sigma-\gamma}{1-\gamma}}} \\
&= \beta \frac{C(\omega')^{\upsilon(1-\sigma)-1}(1-N(\omega'))^{(1-\upsilon)(1-\sigma)}}{C(\omega)^{\upsilon(1-\sigma)-1}(1-N(\omega))^{(1-\upsilon)(1-\sigma)}} \frac{W(\omega')^{\sigma-\gamma}}{E\left(W(\omega')^{1-\gamma}\right)^{\frac{\sigma-\gamma}{1-\gamma}}}
\end{aligned}$$

$$M(\omega, \omega') =$$

fs

Note on η

$$\begin{aligned}
\int_{-\infty}^{\eta^*} \eta dH(\eta) &= E(\eta|\eta < \eta^*) \times \Pr(\eta < \eta^*) \\
&= H(\eta^*) \times E(\eta|\eta < \eta^*) \\
&= H(\eta^*) \times \left(0 - \sigma \frac{\phi\left(\frac{\eta^*}{\sigma}\right)}{\Phi\left(\frac{\eta^*}{\sigma}\right)} \right) \\
&= -\sigma\phi\left(\frac{\eta^*}{\sigma}\right)
\end{aligned}$$

7.4 An illustrative static model

This section presents a "toy model" to illustrate some key issues that arise in thinking about the costs of financial distress.

7.4.1 Model setup

Consider a one-period economy with a representative household, who supplies work and owns debt and equity claims on firms. The household finances his consumption using his labor income as well as his financial income. This representative household has standard concave preferences $U(C, N) = \log(C) - \gamma \frac{N^{1+\phi}}{1+\phi}$.

There is a continuum of mass one of firms. Each firm has the same outstanding debt with face value B that must be repaid at the end of the period. Each firm operates a standard production function, $y = zxn^{\alpha}$, where z is aggregate productivity, x is idiosyncratic productivity and is distributed according to the cumulative distribution function $H(.)$, and n is labor. The labor market is competitive.

To maximize profits, firms pick employment, given their productivity and given the current wage:

$$\pi(x, z; w) = \max_{n \geq 0} \{zxn^{\alpha} - wn\},$$

leading to labor demand

$$n(x, z; w) = \left(\frac{\alpha z x}{w}\right)^{\frac{1}{1-\alpha}},$$

and output supply:

$$y(x, z; w) = (zx)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}},$$

and profits

$$\pi(x, z; w) = (1 - \alpha)(zx)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

It will be useful for later to note that we can also write the FOC as

$$\alpha \frac{y(x, z; w)}{n(x, z; w)} = w.$$

If a firm x is unable to pay back its debt B, it files for bankruptcy. The absolute priority rule applies: equityholders are "whiped out" and get nothing, while creditors seize the firm. This process is costly however, so that a share $1-\theta$ of firm value is lot in the process. Mathematically, the market value of equity V and debt D are:

$$V(x,z;w)=\max\left\{\pi(x,z;w)-B,0\right\},$$

and

$$\begin{aligned}D(x,z;w) &= B \text{ if } \pi(x,z;w)\geq B,\\ &= (1-\theta)\pi(x,z;w) \text{ if } \pi(x,z,w)<B.\end{aligned}$$

Two possible assumptions exist regarding the bankrutpcy costs represented by the pamameter θ : they could either represent real resource costs (e.g. legal costs), or they could represent a transfer.

XXX To elucidate the role of real effects of bankruptcy, we will solve this model under three assumptions regarding default costs: first, we assume that the default cost is a transfer

productivity of firms in default. The first assumption is that managers maximize profits and employment without constraint up to the default. The second is that firms are less productive if they will be in default. (The third is that firms are less productive is there is some chance that they will be in default.)

7.4.2 Case 1: bankruptcies are a pure transfer

The household first order conditions will be given by

$$\begin{aligned}&\max_{N\geq 0}U(C,N)\\ s.t. \quad : \quad & C=wN+\Pi,\end{aligned}$$

where Π is the total payout of firms to both equity holders and bond holders. The first order condition is simply:

$$\frac{U_2(C,N)}{U_1(C,N)}=w,$$

which given our assumptions about preferences implies

$$\gamma CN^{\phi}=w.$$

It is useful to decompose the total payout:

$$\begin{aligned}\Pi &= \int_0^{\infty}\left\{V(x,z;w)+D(x,z;w)\right\}dH(x)\\ &= \int_{x^*}^{\infty}\left(\pi(x,z;w)-B\right)dH(x)+\int_{x^*}^{\infty}BdH(x)+\int_0^{x^*}(1-\theta)\pi(x,z;w)dH(x)\\ &= \int_0^{\infty}\pi(x,z;w)dH(x)-\theta\int_0^{x^*}\pi(x,z;w)dH(x).\end{aligned}$$

Finally, we have equilibrium conditions in the labor market and in the goods market:

$$\int_0^{\infty}n(x,z;w)dH(x)=N,$$

and

$$C=Y=\int_0^{\infty}zxn(x,z;w)^{\alpha}dH(x).$$

7.4.3 Equilibrium

First, note that firms default if their productivity falls below a cutoff x^*, defined through:

$$\pi(x^*, z; w) = B.$$

This leads to:

$$x^* = \frac{1}{z} \frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)} \left(\frac{\alpha}{w}\right)^{\alpha}},$$

and the default rate is $H(x^*(z))$. Ceteris paribus, a higher aggregate productivity reduces the default rate, while a higher wage increases it. Second, note that all firms will pick the same employment since they have the same MPL and face the same wage. From the first-order condition, we have

$$\alpha \frac{y(x, z; w)}{n(x, z; w)} = w,$$

which can be aggregated as:

$$\alpha \int_{-\infty}^{\infty} y(x, z; w) dH(x) = w \int_{-\infty}^{\infty} n(x, z; w) dH(x),$$

or

$$\alpha Y = wN.$$

This equation, together with the first order condition for labor supply, yields

$$\gamma C N^{\phi} = w = \frac{\alpha Y}{N}.$$

Suppose first that bankruptcy costs are a transfer and do not repressent a real cost. Then, $C = Y$ and we obtain the usual result for the labor-only RBC model:

$$N = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1+\phi}},$$

i.e. employment is constant. The preferences are such that income and substitution effects offset each other.

Moreover, output satisfies

$$Y = z \times \overline{x} \times N^{\alpha},$$

where $\overline{x} = \left(\int_{-\infty}^{\infty} x^{1-\alpha} dH(x)\right)^{\frac{1}{1-\alpha}}$,[10] and hence

$$Y = z\overline{x}N^{\alpha} = z\overline{x}\left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha}{1+\phi}}.$$

In this case, **employment and output are independent of the level of outstanding debt**.

[10] This follows from

$$\begin{aligned} Y &= \int y = \int (zx)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} dH \\ &= \int (zx)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} dH \\ &= \int (zx)^{\frac{1}{1-\alpha}} \left(\frac{N^{1-\alpha}}{z\overline{x}}\right)^{\frac{\alpha}{1-\alpha}} dH \\ &= z\overline{x}N^{\alpha}. \end{aligned}$$

In particular, an implication of this equilibrium is that volatility of output is simply proportional to the volatility of the shock, and indeed equal:

$$\sigma(\log C) = \sigma(\log z),$$

and volatility is unaffected by any parameter, in particular the level of debt in the economy, or the bankruptcy loss pameter θ.

7.4.4 Case 2: bankruptcies create real resource costs

Second, suppose now that bankruptcy costs are actually substracted from the resource constraint. Hence,

$$C = Y - \theta \int_{-\infty}^{x^*} \pi(x, z; w) dH(x).$$

Note that

$$\pi(x, z; w) = \Pi \times \frac{x^{1-\alpha}}{\int_{-\infty}^{\infty} x^{1-\alpha} dH(x)},$$

and $\Pi = (1-\alpha)Y$. Hence,

$$C = Y - \theta(1-\alpha)Y \frac{\int_{-\infty}^{x^*} x^{1-\alpha} dH(x)}{\int_{-\infty}^{\infty} x^{1-\alpha} dH(x)}.$$

The equilibrium is still determined by

$$\gamma C N^{\phi} = w = \frac{\alpha Y}{N},$$

leading to

$$N = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1+\phi}} (R(x^*))^{-\frac{1}{1+\phi}},$$

with

$$R(x) = 1 - \theta(1-\alpha) \frac{\int_{-\infty}^{x^*} x^{1-\alpha} dH(x)}{\int_{-\infty}^{\infty} x^{1-\alpha} dH(x)},$$

and x^* simulatneously determined by

$$\begin{aligned} x^* &= \frac{1}{z} \frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)} \left(\frac{\alpha}{w}\right)^{\alpha}} \\ &= \frac{1}{z} \frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)} \left(\frac{\alpha N}{\alpha z \overline{x} N^{\alpha}}\right)^{\alpha}} \\ &= \frac{1}{z^{1-\alpha}} \frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)} \overline{x}^{-\alpha} N^{1-\alpha}}, \end{aligned}$$

or

$$x^* = \frac{1}{z^{1-\alpha}} \frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)} \overline{x}^{-\alpha} \left(\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1+\phi}} (R(x^*))^{-\frac{1}{1+\phi}}\right)^{1-\alpha}}$$

$$x^* = \frac{1}{z^{1-\alpha}} \frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)} \overline{x}^{-\alpha} \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\alpha}{1+\phi}}} R(x^*)^{\frac{1-\alpha}{1+\phi}}.$$

This equation determines x^*. Note that $R' < 0$, $R(0) = 1 > 0$, so there is a unique solution. More bankruptcie induce higher labor supply and lower wage which reduces bankrutpcties. If ϕ goes to ∞ : inelastic labor supply. No feedback effect.

Finally,

$$
\begin{aligned}
Y &= z\overline{x}N^{\alpha} = z\overline{x}\left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha}{1+\phi}} R(x^*)^{-\frac{\alpha}{1+\phi}}, \\
C &= YR(x^*) = z\overline{x}\left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha}{1+\phi}} R(x^*)^{1-\frac{\alpha}{1+\phi}}
\end{aligned}
$$

Plot $C(z), Y(z), N(z)$.

Overall, The only effect of this on the equilibrium is to create a negative wealth effect that leads employment to rise as more firms fall into bankruptcy. This increase in labor supply pushes wages down and hence reduces the increase in bankruptcy that one would see otherwise.

$$
\begin{aligned}
\frac{\partial Y}{\partial B} &> 0 \\
\frac{\partial C}{\partial B} &< 0 \\
\frac{\partial N}{\partial B} &> 0
\end{aligned}
$$

Note that we now have

$$
\sigma(\log C) = \sigma(\log z) + \left(1 - \frac{\alpha}{1+\phi}\right)\sigma(\log R(x^*(z))...
$$

7.4.5 Case 3: firms in bankruptcy are less productive

7.4.6 Case 4: firms close to bankruptcy are less productive

We now consider the alternative assumption that firms are less productive if in bankrupty. Specifically, I assume that firms in default are less productive by a factor υ. This will reduce their labor demand, output supply, and profits.

Specifically, aggregate labor demand is now

$$
\begin{aligned}
N &= \int_{x^*}^{\infty} n(z,x;w)dH(x) + \int_0^{x^*} n(z,\upsilon x;w)dH(x) \\
&= \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}\left(\int_{x^*}^{\infty} x^{\frac{1}{1-\alpha}}dH(x) + \int_0^{x^*} x^{\frac{1}{1-\alpha}}\upsilon^{\frac{1}{1-\alpha}}dH(x)\right) \\
&= \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}\left(\overline{x}^{1-\alpha} - \left(1-\upsilon^{\frac{1}{1-\alpha}}\right)\int_0^{x^*} x^{\frac{1}{1-\alpha}}dH(x)\right)
\end{aligned}
$$

$$
\begin{aligned}
Y &= \int_{x^*}^{\infty} zxn(z,x;w)^{\alpha}dH(x) + \int_0^{x^*} zx\upsilon n(z,\upsilon x;w)^{\alpha}dH(x) \\
&= \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}}\left(\overline{x}^{1-\alpha} - \left(1-\upsilon^{\frac{1}{1-\alpha}}\right)\int_0^{x^*} x^{\frac{1}{1-\alpha}}dH(x)\right)
\end{aligned}
$$

Note that we still have $\alpha\frac{y}{n} = w$ for all firms.

The equilibrium is determined by the following equations: $C, N, Y, x^*, \overline{x}$

$$\begin{aligned}\overline{x} &= \left(\int_{x^*}^{\infty} x^{\frac{1}{1-\alpha}} dH(x) + \upsilon^{\frac{1}{1-\alpha}} \int_0^{x^*} x^{\frac{1}{1-\alpha}} dH(x)\right)^{1-\alpha} \\ &= \left(\int_0^{\infty} x^{\frac{1}{1-\alpha}} dH(x)\right)^{1-\alpha} \left(1 - (1-\upsilon^{\frac{1}{1-\alpha}})\frac{\int_0^{x^*} x^{\frac{1}{1-\alpha}} dH(x)}{\int_0^{\infty} x^{\frac{1}{1-\alpha}} dH(x)}\right)^{1-\alpha}\end{aligned}$$

$$Y = z\overline{x}N^{\alpha}$$

$$\gamma C N^{\phi+1} = \alpha Y$$

$$\begin{aligned}C &= Y - \theta \int_0^{x^*} \pi(z, x; w) dH(x) \\ &= Y\left(1 - \theta(1-\alpha)\frac{\upsilon^{\frac{1}{1-\alpha}} \int_0^{x^*} x^{\frac{1}{1-\alpha}} dH(x)}{\int_0^{\infty} x^{\frac{1}{1-\alpha}} dH(x)}\right) \\ &= Y.R(x^*; \upsilon)\end{aligned}$$

$$\gamma R(x^*; \upsilon) N^{1+\phi} = \alpha$$

$$\begin{aligned}x^* &= \frac{1}{z}\frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\left(\frac{\alpha}{w}\right)^{\alpha}} \\ &= \frac{1}{z}\frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\left(\frac{\alpha N}{\alpha z\overline{x}N^{\alpha}}\right)^{\alpha}} \\ &= \frac{1}{z^{1-\alpha}}\frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\,\overline{x}^{-\alpha}N^{1-\alpha}},\end{aligned}$$

or

$$x^* = \frac{1}{z^{1-\alpha}}\frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\,\overline{x}^{-\alpha}\left(\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{1+\phi}}(R(x^*))^{-\frac{1}{1+\phi}}\right)^{1-\alpha}}$$

$$Y = z\overline{x}N^{\alpha}\left(1 + \left(\upsilon^{\frac{1}{1-\alpha}} - 1\right)H(x^*(z))\right)^{1-\alpha}$$

$$R(u) = \int_u^{\infty} x^{\frac{1}{1-\alpha}} dH(x),$$

so that $R'(u) < 0$.

We have

$$N(z) = \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}} R(x^*(z)),$$

and

$$C(z) = Y(z) = \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} R(x^*(z)),$$

where the default cutoff is (as before):

$$x^*(z) = \frac{1}{z}\frac{B^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\left(\frac{\alpha}{w(z)}\right)^{\alpha}} = \frac{(B/z)^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\left(\frac{\alpha z}{w(z)}\right)^{\alpha}}.$$

The equilibrium is determined by the first-order condition,

$$\gamma \times \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}} R(x^*(z)) \times \left(\left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}} R(x^*(z))\right)^{\phi} = w(z)$$

$$\gamma \times \left(\frac{\alpha}{w}\right)^{\frac{\alpha+\phi}{1-\alpha}} z^{\frac{1+\phi}{1-\alpha}} \times R(x^*(z))^{1+\phi} = w(z)$$
$$\gamma \times \alpha^{\frac{\alpha+\phi}{1-\alpha}} z^{\frac{1+\phi}{1-\alpha}} \times R(x^*(z))^{1+\phi} = w^{\frac{1+\phi}{1-\alpha}}$$
$$\gamma \times \alpha^{\frac{\alpha+\phi}{1-\alpha}} z^{\frac{1+\phi}{1-\alpha}} \times R(x^*(z))^{1+\phi} = w^{\frac{1+\phi}{1-\alpha}}$$

— Substitability — higher bankruptcy leads to lower wealth – or quicks firms out of the market — no element for multiple equilibria

Suppose that the

Complementarities?

Equilibrium:

$$B \times C \times N^{\phi} = w$$
$$B \times \left(\frac{\alpha z}{w(z)}\right)^{\frac{\alpha+\phi}{1-\alpha}} \times R(x^*(z))^{1+\phi} = \frac{w(z)}{z}$$

Find $\widehat{w}(z) = w(z)/z$ s.t.:

$$B \times \alpha^{\frac{\alpha+\phi}{1-\alpha}} \times R\left(\frac{(B/z)^{1-\alpha}\,\widehat{w}(z)^{\alpha}}{(1-\alpha)^{(1-\alpha)}\,\alpha^{\alpha}}\right)^{1+\phi} = \widehat{w}(z)^{\frac{1+\phi}{\alpha+\phi}}$$

Suppose for instance that $H(x)$ is Pareto over $x \in (1, +\infty($; i.e.

$$R(u) = \kappa u^{-\gamma}, \text{ for } u \geq 1.$$

Note!! There are two B's here.

Then this reads

$$B \times \alpha^{\frac{\alpha+\phi}{1-\alpha}} \times R\left(\frac{(B/z)^{1-\alpha}\,\widehat{w}(z)^{\alpha}}{(1-\alpha)^{(1-\alpha)}\,\alpha^{\alpha}}\right)^{1+\phi} = \widehat{w}(z)^{\frac{1+\phi}{\alpha+\phi}}$$
$$\frac{1+\phi}{\alpha+\phi}\Delta \log \widehat{w} = (1+\phi)\frac{R'(.)(.)}{R(.)}\left((1-\alpha)\left(\Delta \log B - \Delta \log z\right) + \alpha \Delta \log \widehat{w}\right)$$

$$\frac{1}{\alpha+\phi}\Delta \log \widehat{w} = (-\varepsilon)\left((1-\alpha)\left(\Delta \log B - \Delta \log z\right) + \alpha \Delta \log \widehat{w}\right)$$

$$\left(\frac{1}{\alpha+\phi} + \alpha\varepsilon\right)\Delta \log \widehat{w} = (-\varepsilon)(1-\alpha)\left(\Delta \log B - \Delta \log z\right)$$
$$\Delta \log \widehat{w} = \frac{(-\varepsilon)(1-\alpha)}{1+\alpha\varepsilon(\alpha+\phi)}\left(\Delta \log B - \Delta \log z\right)$$
$$\frac{\Delta \log \widehat{w}}{\Delta \log z} = \frac{(\varepsilon)(1-\alpha)}{1+\alpha\varepsilon(\alpha+\phi)} \underset{????}{<} \frac{1-\alpha}{\alpha}$$
$$\frac{(1-\alpha)}{1/\varepsilon+\alpha(\alpha+\phi)} \underset{????}{<} \frac{1-\alpha}{\alpha}$$

Without debt, this is a standard static RBC model: $R(0) = \kappa$; with debt, wages ae lower and

$$R(u) =$$

$$\theta \times \alpha^{\frac{\alpha+\phi}{1-\alpha}} \times \kappa \left(\frac{(B/z)^{1-\alpha}\, \widehat{w}(z)^{\alpha}}{(1-\alpha)^{(1-\alpha)}\, \alpha^{\alpha}} \right)^{-\gamma(1+\phi)} = \widehat{w}(z)^{\frac{1+\phi}{\alpha+\phi}}$$

$$\theta \times \alpha^{\frac{\alpha+\phi}{1-\alpha}} \times \kappa \left(\frac{(B/z)^{1-\alpha}}{(1-\alpha)^{(1-\alpha)}\, \alpha^{\alpha}} \right)^{-\gamma(1+\phi)} = \widehat{w}(z)^{\frac{1+\phi}{\alpha+\phi}+\alpha\gamma(1+\phi)}$$

An increase in z (or a decrease in B) increases $\widehat{w}$ and decreases $x^*(z)$.

Since

$$C = Y = z \left(\frac{\alpha}{\widehat{w}(z)} \right)^{\frac{\alpha}{1-\alpha}} \times R(x^*(z)),$$

under some conditions on $R(.)$ (and ϕ?), C and Y go up.

Statement about conditional volatility:

$$\sigma(\log C) = \sigma(\log z) + \frac{\alpha}{1-\alpha}\sigma$$

$$Y = \int_{x^*}^{\infty} zxn^{\alpha} dH(x)$$

Suppose productivity falls with x^*... then $z(x^*)$. Then can get multiple equilibria.

$M(s,s') = ...$

$$\sum_u P(s,u)w(u)^{1-\theta(s)}$$

$$d = 1_{ND}\left(y - wn - b - \eta\right) + 1_D \times 0$$

Agg Div

$$\begin{aligned} d &= \int 1_{ND}\left(y - wn - b - \eta\right) \\ &= \sum_x \int_{-\infty}^{\eta^*} y(x) - w(x)n(x) - b - \eta dH(\eta) \\ &= \left(\sum_x H(\eta^*)\left(y(x) - w(x)n(x) - b\right)\right) - \sum_x \int_{-\infty}^{\eta^*} \eta dH(\eta) \end{aligned}$$

value of firms that survive: buy at cost $v(x,\omega)$, then given $\eta < \eta^*(x)$, value next period is

$$H(\eta^*(x)) \sum_{\omega'} \sum_{x'} Q(x,x')v(x',\omega')$$