

# Financial Frictions, Capital Misallocation, and Input-Output Linkages

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## Abstract

This paper analyzes the role of financial frictions in amplifying impulse responses to different types of shocks and shows that input-output linkages contribute to an increase in the impact of financial frictions on the aggregate economy. Financial frictions distort the allocation of capital and induce capital wedges, which are weak under total factor productivity (TFP) shocks but strong under uncertainty shocks. As a result, in standard models driven by TFP shocks, adding financial frictions dampens the impulse responses. On the other hand, financial frictions can drive aggregate TFP fluctuations and play a crucial role when uncertainty shocks hit the economy. Adding input-output linkages can further amplify the impact. In the model calibrated to U.S. data, I quantify the amplification effects of firm linkages, demonstrating that the addition of input-output linkages amplifies the effects of both TFP and uncertainty shocks. In particular, aggregate output drops an additional 84% under TFP shocks and an additional 40% under uncertainty shocks with input-output linkages. Furthermore, compared to other sectors, an increase in the dispersion of the return to capital in the Finance sector has the largest impact on aggregate output.

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# 1 Introduction

Since the Great Recession, macroeconomists have focused on the impact of financial frictions on business cycles. However, when financial constraints are applied to intertemporal variables, these models cannot generate empirically significant fluctuations<sup>1</sup>. This paper shows that 1) financial friction that impacts intertemporal variables is the key to generating significant fluctuation when second-moment (uncertainty) shocks hit the economy; 2) inter-firm trade linkages can further amplify the impact of financial frictions; and 3) there are dramatic differences in sectors sensitivities to a tightening of borrowing constraint. Compared to other sectors, an increase in the dispersion of the return to capital in the financial sector has the largest impact on aggregate output.

In this paper, I follow Townsend (1979) and consider financial friction to be the agency problem and the costly state verification. Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) (hereafter BGG) combine this agency problem with business cycles models. My model most closely follows the work of Christiano et al. (2014). The amplification mechanism is due to the fluctuation of entrepreneurs' net worth, which reflects their ability to borrow. When negative shocks hit the economy, net worth shrinks, and this decline leads to an increase in the interest rate spread and the default rate. Hence, credibility, investment, capital price, and output fall. The fall in capital price further reduces net worth. BGG call this mechanism the financial accelerator. However, opinions differ on the empirical significance of the financial accelerator. For example, Christensen and Dib (2007) show that the financial accelerator dampens the effect of supply shocks but amplifies those of investment demand shocks. Further, they find that the impact of the financial accelerator on output fluctuations is minor. Nolan and Thoenissen (2009) argue that the financial accelerator is important under shocks to entrepreneurs' net worth.

I show that adding financial frictions into an otherwise frictionless model dampens the effects of TFP shocks, and financial frictions play a quantitatively minor role under TFP shocks. However, in combination with uncertainty shocks, financial frictions play a crucial role and actually amplify the effects of these shocks. In the model I present, financial friction creates a wedge between the return to capital and the risk-free interest rate. As Cogley and Nason (1995) demonstrate, TFP shocks affect mostly intratemporal variables but have a weak internal propagation mechanism through capital. Hence, the main channel the financial friction works on, the capital wedge channel, is weak when a TFP shock hits the economy. If the capital wedge does not fluctuate much, adding frictions has a similar effect as raising capital adjustment costs and hence impedes responses under TFP shocks. However, an increase in uncertainty will raise the user cost of capital and the capital wedge.<sup>2</sup> This effect is amplified when distortion from financial friction is large.

This paper then shows that inter-firm trade linkages can further amplify the effects of shocks. That is, when firms purchase intermediate inputs from other firms, shocks hitting one firm can spread to its downstream customers and upstream suppliers. Input-output linkages have been used to answer mainly two types of questions in the literature. The first type relates to sectoral co-movement in business cycles. The second type of

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<sup>1</sup>See works like Chari, Kehoe and McGrattan (2007) and Schwartzman (2012)

<sup>2</sup>This model uses a convex capital adjustment cost. Thus there is no "wait and see" effect of uncertainty shocks in the model.

question considers the relative contributions of idiosyncratic shocks and aggregate shocks to aggregate output volatility. Recently, input-output linkages have been treated as a production network and used to discuss how different network structures affect aggregate volatility.<sup>3</sup> This paper focuses on a different aspect of input-output linkages: their amplification mechanism. Even though amplification due to input-output linkages is implicitly assumed in the literature, it has not been explicitly and rigorously analyzed.<sup>4</sup>

I extend the costly state verification model into multiple sectors (or, alternatively, I add financial frictions into a frictionless dynamic multi-sector general equilibrium model). Asymmetric information regarding entrepreneurs' return to capital and bankruptcy costs generates a wedge between the risk-free rate and return to capital. Technological nature differs in different industries, and, hence, industrial demand for borrowing, interest rate spreads, default rates, and capital wedges vary across sectors. I first use two simple economies to explain these ideas. I then develop the general model with costly state verification and calibrate it to U.S. data.

This paper also contributes to literature that attempts to formalize the idea that financial frictions affect aggregate TFP through resource misallocation. Sectors which have higher bankruptcy costs and a higher level of uncertainty suffer more from distortion, and are thus disproportionately susceptible to uncertainty shocks. When uncertainty increases, capital flows from more constrained firms to less constrained firms. This worsens the degree of capital misallocation and reduces the measured aggregate TFP.

#### *Quantitative Results:*

In the counterfactual exercises, I compare two model economies: a single-sector economy and a multi-sector economy with input-output linkages. They are both calibrated to match the same data moments and have the same steady state values with financial frictions. Hence, the major difference between these two economies is in the input-output linkages. The amplification of input-output linkages refers to the differences in impulse response between the two model economies. Adding input-output linkages drops aggregate output an additional 84% under TFP shocks and an additional 40% under uncertainty shocks. In the calibrated model, capital misallocation, which lowers aggregate TFP, contributes to 17% of the GDP drop when an uncertainty shock hits the economy. Furthermore, I examine the importance of each sector in terms of their impact on aggregate output. This is related to the observation that the onset of the Great Recession was the subprime mortgage crisis in the financial sector in the U.S. This led to bankruptcy of several financial institutions and eventually affected the entire economy. To examine the above observation that the financial sector has an important impact on the aggregate economy, I hit the model economy with idiosyncratic sectoral shocks and see which sector generates the largest drop in aggregate output. When hit by an idiosyncratic TFP shock (sectoral productivity decreases by 1%), the Manufacturing sector and FIRE<sup>5</sup> sector have the largest impact on aggregate GDP (aggregate GDP drops by 0.53% and 0.43%, respectively). The Manufacturing and FIRE sector both contribute to about 20% of final use

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<sup>3</sup>Please see Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012).

<sup>4</sup>Two exceptions are Jones (2013) and Bigio and La'O (2013).

<sup>5</sup>The FIRE sector consists of the Finance, Insurance, and Real Estate sectors.

share in aggregate output. Without linkages, a 1% drop in Manufacturing productivity can only cause aggregate GDP drops by 0.2%. In response to an idiosyncratic uncertainty shock, the aggregate output drop caused by the FIRE sector is the largest (0.32%), three times larger than the second sector, Manufacturing (0.09%).

*Literature:*

This paper relates to three strands of literature: those that focus on (i) financial frictions, (ii) input-output linkages, and (iii) uncertainty shocks. Costly state verification has often been used in DSGE models. My work most closely follows Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2014). Chen and Song (2013) provide a theory about how financial frictions drive TFP fluctuation through capital misallocation under news shocks. Literature regarding input-output linkages can be divided into two groups. The first group (Long and Plosser 1983; Shea 2002) asks questions about sectoral co-movement. The second group asks two primary questions. First, under what conditions can idiosyncratic shocks generate significant aggregate volatility similar to that generated by aggregate shocks (Acemoglu et al., 2012; Carvalho 2010; Durlauf 1993; Horvath 1998, 2000; Jovanovic 1987)? Second, what is the relative contributions of idiosyncratic shocks and aggregate shocks to business cycle fluctuations (Atalay 2014; Foerster et al. 2011)?

Recently, a small group of researchers has focused on a different aspect of input-output linkages: their amplification mechanism. Jones (2011, 2013) discusses this aspect, analyzing how the degree of misallocation gets amplified through input-output linkages. Regarding the combination of financial frictions and firm linkages, however, there is a paucity of research. Shourideh and Zetlin-Jones (2012) consider two types of firms, privately-held firms and publicly-held firms, and show that inter-firms linkages (intermediate inputs) amplify aggregate fluctuations induced by shocks on collateral constraints. Kalemli-Ozcan et al. (2014) argue that accounts payable and receivable provide incentives for firms to form a sustained production chain, and upstream firms have higher demand for working capital than downstream firms. They consider a single supply chain instead of a production network, and do not apply their results to a dynamic general equilibrium analysis. Neither paper considers a full production network.

Bigio and La'O (2013) is the first to combine financial frictions with production networks. Although my paper shares a similar framework, it differs from and advances Bigio and La'O (2013) in three important respects. First, the model in Bigio and La'O (2013) is static, and the shock responses are changes in steady state values. My model is dynamic and is, thus, more suitable to the study of business cycles. Second, by focusing on capital wedges, I can directly identify the degree of financial frictions from corporate bond spreads, while in Bigio and La'O (2013) the labor wedge cannot be independently identified from labor input shares. Instead, they try to identify labor wedges through indirect proxy. They argue that the fluctuation in cost-to-sales ratio of each sector is due to the variation in labor wedges, and they proxy the sectoral wedges of each year by dividing the cost-to-sales ratio to its largest value in the sample period. Third, the reason that Bigio and La'O (2013) skip capital is that financial constraints on intertemporal variables cannot generate empirically significant fluctuation. Hence, they focus on financial constraints on intratemporal variables, such as labor and intermediate inputs.

In contrast, I show that financial frictions on intertemporal variables can be important when we consider the effects of uncertainty shocks.

The impact of uncertainty shocks on the macroeconomy is analyzed by Bloom (2009) and Bloom et al. (2012). My finding that financial friction plays a tiny role under TFP shocks but is crucial under uncertainty shocks is consistent with Di Tella (2013) and Gilchrist et al. (2014). Di Tella (2013) shows that when risk-averse borrowers are able to sign complete contracts on the aggregate state of the economy, the balance sheet channel is null under TFP shocks. However, uncertainty shocks can drive balance sheet recessions even when borrowers can sign the complete contingent contracts. Gilchrist et al. (2014) analyze the relative importance of the two mechanisms induced by uncertainty shocks: the “wait and see” effects and credit spreads. Regarding investment dynamics, they show that financial shocks and uncertainty shocks can generate countercyclical credit spreads and procyclical leverage. These results are consistent with the data and counter to those implied by TFP shocks.

The rest of the paper proceeds as follows. I use two simple economies to explain the basic amplification mechanism of input-output linkages in Section 2. Section 3 develops the general model and equilibrium characterizations. Section 4 calibrates the model to U.S. data. Section 5 shows the result, and Section 6 concludes. The Appendix contains proofs of propositions and other technical materials.

## 2 Two Simple Economies

In this section, I use two simple economies to illustrate the basic idea behind the general model. Both economies consist of two sectors producing intermediate goods and a final goods production sector. In the first economy, Economy A (A for Autarky), two sectors produce goods by their own capital, and the final goods producer combines two intermediate goods to produce final goods (GDP). In the second economy, Economy L (L for linkages), sector 1 provides some of its goods as intermediate input to sector 2. Hence sector 1 is the common supplier in Economy L. I first introduce the two economies without any frictions, and then add frictions to see how they affect capital allocation and aggregate TFP.

### 2.1 The Benchmark Case

#### Economy A

In Economy A, there are two sectors (represented by firm 1 and firm 2) producing two intermediate goods,  $Y_1$  and  $Y_2$ :

$$\begin{aligned} Y_1 &= A_1 k_1^{\alpha_1}, \\ Y_2 &= A_2 k_2^{\alpha_2}, \\ Y &= Y_1^{\beta_1} Y_2^{\beta_2}. \end{aligned}$$

$Y$  is GDP (aggregation) and  $\beta_1 + \beta_2 = 1$ . Capital is homogeneous across sectors, so aggregate capital is  $K = k_1 + k_2$ .  $A_i$  is sectoral productivity. All markets are competitive.

Firms are owned by households. Households own capital and rent their capital in the capital market as in the conventional RBC framework. To simplify the model, there is no labor. Households receive capital income by renting capital and other profits from firm 1 and firm 2. Final goods producer maximizes its profits:

$$\max_{Y_1, Y_2} Y - p_1 Y_1 - p_2 Y_2,$$

and the corresponding first order conditions are

$$p_1 Y_1 = \beta_1 Y, \quad (2.1)$$

$$p_2 Y_2 = \beta_2 Y, \quad (2.2)$$

where prices  $p_1, p_2$  are in units of final goods, and the price of final goods is normalized to 1. Firm 1's problem is:

$$\max_{k_1} p_1 Y_1 - r_t k_1,$$

and the first order condition is:

$$\alpha_1 p_1 Y_1 / k_1 = r_t. \quad (2.3)$$

Similarly, firm 2's first order condition is:

$$\alpha_2 p_2 Y_2 / k_2 = r_t. \quad (2.4)$$

We want to consider the optimum allocation of capital. Denote  $k_1 = xK$ , where  $x$  is the allocation variable. From Eq (1) to (4),

$$\frac{k_1}{k_2} = \frac{x^*}{1 - x^*} = \frac{\alpha_1 \beta_1}{\alpha_2 \beta_2}.$$

So the optimum allocation is:

$$x^* = \frac{\alpha_1 \beta_1}{\alpha_1 \beta_1 + \alpha_2 \beta_2}. \quad (2.5)$$

The optimum ratio is exactly the ratio of the exponents of sectoral capital in the aggregate production function. For any allocation  $x$ , the aggregate output is:

$$Y = (A_1 k_1^{\alpha_1})^{\beta_1} (A_2 k_2^{\alpha_2})^{\beta_2} = A_1^{\beta_1} A_2^{\beta_2} (xK)^{\alpha_1 \beta_1} ((1-x)K)^{\alpha_2 \beta_2} = A(x) K^{\alpha_1 \beta_1 + \alpha_2 \beta_2}, \quad (2.6)$$

where  $A(x) = A_1^{\beta_1} A_2^{\beta_2} x^{\alpha_1 \beta_1} (1-x)^{\alpha_2 \beta_2}$  represents aggregate TFP.

In the optimal allocation, the marginal aggregate product of sectoral capital is the same across sectors. That is,

$$\frac{\partial Y}{\partial k_1} = \frac{\partial Y}{\partial k_2}.$$

Aggregate TFP is maximized when  $x = x^*$ . For any  $x$  deviating from  $x^*$ , there is capital misallocation and smaller aggregate TFP.

### Economy L

Now consider another economy similar to Economy A, but with intermediate input-output linkages. Sector 1 is the common supplier in this economy. It supplies its output to sector 2 and the final goods producer. Capital is homogeneous across sectors.

$$\begin{aligned} Y_1 &= A_1 k_1^{\alpha_1}, \\ Y_2 &= A_2 k_2^{\alpha_2} M_1^m, \\ Y &= X_1^{\beta_1} Y_2^{\beta_2}, \end{aligned}$$

and market clearing conditions

$$\begin{aligned} Y_1 &= M_1 + X_1, \\ Y &= C + I, \\ K &= k_1 + k_2. \end{aligned}$$

Again, denote  $k_1 = xK$ , and  $M_1 = \eta Y_1$ .  $x$  is the capital allocation variable, and  $\eta$  is the intermediate input allocation variable for  $Y_1$ . From FOCs, for any given  $\eta$ , the optimum allocation is

$$\frac{k_1}{k_2} = \frac{x^*}{1-x^*} = \frac{\alpha_1 \beta_1}{\alpha_2 \beta_2} \frac{1}{1-\eta}, \quad (2.7)$$

and  $\frac{\partial x^*}{\partial \eta} > 0$ . If there is no distortion on the allocation of intermediate input  $M_1$ , the optimum  $\eta$  is

$$\eta^* = \frac{m\beta_2}{\beta_1 + m\beta_2}.$$

Then,

$$\frac{k_1}{k_2} = \frac{\alpha_1 \beta_1 + m\alpha_1 \beta_2}{\alpha_2 \beta_2}. \quad (2.8)$$

As with Economy A, this allocation ratio is exactly the ratio of the exponents of sectoral capital in the aggregate production function. The marginal aggregate product of sectoral capital is the same across sectors under optimal allocation. The aggregate production function is

$$Y = A_L(\eta, x) K^{\alpha_1 \beta_1 + \alpha_2 \beta_2 + m\alpha_1 \beta_2}, \quad (2.9)$$

where

$$A_L(\eta, x) = C_L(m, \eta) A_1^{\beta_1 + m\beta_2} A_2^{\beta_2} x^{\alpha_1 \beta_1 + m\alpha_1 \beta_2} (1-x)^{\alpha_2 \beta_2},$$

and  $C_L(m, \eta) = (1-\eta)^{\beta_1} \eta^{m\beta_2}$ .

Comparing Economy A and Economy L, four remarks are in order.

- First, with intermediate input-output linkages, capital allocation now depends on intermediate input share (Eq(2.8)).
- Second, comparing  $A_L(\eta, x)$  and  $A(x)$ , productivity changes in the first sector  $A_1$  is amplified by an additional coefficient  $m\beta_2$ , but nothing changes in  $A_2$ .

- Third, note that  $\frac{\partial x^*}{\partial m} > 0$ . In the language of network theory,  $m$  can be interpreted as the out-degrees in a production network, or the “location” in a network. The interpretation is that the more common supplier one sector is (larger  $m$ ), the larger the fraction of capital that should be allocated to this sector. It deserves a large capital allocation because it has a higher aggregate marginal revenue of capital. Note that in an economy with input-output linkages, sectoral marginal product of capital ( $\frac{\partial Y_i}{\partial k_i}$ ) is not an appropriate measure for capital allocation. One should use marginal aggregate revenue of capital ( $\frac{\partial Y}{\partial k_i}$ ).
- Fourth, the effects of a change in  $m$  on TFP is threefold. First, it affects  $A_1$  term in  $A_L(\eta, x)$ . Second, it increases  $x$  and also the exponent term of  $x$  in  $A_L(\eta, x)$ . Third, it affects  $C_L(m, \eta)$ <sup>6</sup>.

In short, the out-degree of a sector affects capital allocation ( $x$ ), intermediate input allocation ( $\eta$ ), and sectoral productivity amplification in aggregate TFP. The net effect depends on parameter values.

## 2.2 Adding Frictions: Capital Wedges

Let’s consider the type of financial frictions that affect capital only. This results in capital wedges, which can be written in the form of capital tax. So capital rent becomes  $r/(1 - \tau_i)$ .

### Economy A

Firm 1’s problem is:

$$\max_{k_1} p_1 Y_1 - \frac{r k_1}{(1 - \tau_1)},$$

and the FOC is:

$$(1 - \tau_1) \alpha_1 p_1 Y_1 / k_1 = r.$$

Similarly, firm 2’s FOC condition is:

$$(1 - \tau_2) \alpha_2 p_2 Y_2 / k_2 = r.$$

the optimum allocation under distortion becomes

$$\frac{k_1}{k_2} = \frac{\hat{x}}{1 - \hat{x}} = \frac{(1 - \tau_1) \alpha_1 \beta_1}{(1 - \tau_2) \alpha_2 \beta_2}. \quad (2.10)$$

And TFP is

$$A(\hat{x}) = A_1^{\beta_1} A_2^{\beta_2} \hat{x}^{\alpha_1 \beta_1} (1 - \hat{x})^{\alpha_2 \beta_2}.$$

### Economy L

Similarly, with the same form of capital wedges, the optimum allocation under distortion becomes

$$\frac{k_1}{k_2} = \frac{\hat{x}}{1 - \hat{x}} = \frac{(1 - \tau_1) \alpha_1 \beta_1}{(1 - \tau_2) \alpha_2 \beta_2} \frac{1}{1 - \eta} = \frac{(1 - \tau_1) (\alpha_1 \beta_1 + m \alpha_1 \beta_2)}{(1 - \tau_2) \alpha_2 \beta_2}. \quad (2.11)$$

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<sup>6</sup>  $\frac{\partial \eta^*}{\partial m} = \frac{\beta_1 \beta_2}{(\beta_1 + m \beta_2)^2}$



It is now clear that both [intermediate input shares and distortions](#) affect the allocation variable  $x$ .

**Proposition 1.** *In Economy A and Economy L with distortions, without loss of generality, assume  $\tau_1 > \tau_2$ , then  $\hat{x} < x^*$ . And*

$$\begin{aligned} \frac{\partial \hat{x}}{\partial \tau_1} < 0, \quad \frac{\partial \hat{x}}{\partial \tau_2} > 0, \text{ and } \quad \frac{\partial \hat{x}}{\partial \tau_1} + \frac{\partial \hat{x}}{\partial \tau_2} < 0. \\ \frac{\partial TFP(\hat{x})}{\partial \tau_1} < 0, \quad \frac{\partial TFP(\hat{x})}{\partial \tau_2} > 0, \text{ and } \quad \frac{\partial TFP(\hat{x})}{\partial \tau_1} + \frac{\partial TFP(\hat{x})}{\partial \tau_2} < 0. \end{aligned}$$

The proof of the proposition is straightforward by taking partial derivatives. This proposition states the effects of aggregate TFP in response to idiosyncratic financial shocks and aggregate financial shocks. Consider the case of idiosyncratic shocks ( $\tau_i$  increases). Without loss of generality, assume sector 1 suffers from larger distortion than sector 2 ( $\tau_1 > \tau_2$ ). If  $\tau_1$  increases, sector 1 suffers more, and less capital should be allocated to this sector; hence  $\hat{x}$  decreases and aggregate TFP decreases. If, instead,  $\tau_2$  increases, distortions on sector 1 become relatively less severe, and more capital should be allocated to sector 1; hence  $\hat{x}$  increases and aggregate TFP increases. In summary, the effects of idiosyncratic shocks depend on the relative sectoral severity of distortions. In the case of aggregate shocks (both  $\tau_1, \tau_2$  increase a small amount), aggregate TFP decreases.

### 3 The General Model

The general model combines financial frictions through costly state verification and uncertainty shocks, as in Christiano et. al (2014), and the multi-sector RBC framework with input-output linkage, as in Long and Plosser (1983). Capital is homogeneous across sectors. This assumption allows capital to be reallocated easily across sectors. I don't consider the friction on capital reallocation<sup>7</sup>. The economy consists of households (workers and entrepreneurs),  $N$  intermediate goods producing sectors, a final goods producer, a capital producer, and a financial intermediary. Here I use the big household assumption, as in Christiano et al. (2014): a household contains differentiated workers and entrepreneurs of type  $1, \dots, N$ ; type  $i$  workers work in firms of sector  $i$ ; and type  $i$  entrepreneurs manage firms in sector  $i$ . In the following notation, all bold letters are vectors.

#### 3.1 Household Preference

The representative household's maximization problem is as follows:

$$\begin{aligned} \max_{c_t, h_t, D_{t+1}} \quad & \sum_{t=1}^{\infty} \tilde{\beta}^t \left\{ \frac{c_t^{1-\varrho}}{1-\varrho} - \frac{\epsilon h_t^{1+\epsilon/\epsilon}}{1+\epsilon} \right\} \\ \text{s.t.} \quad & c_t + D_{t+1} = w_t h_t + R_t D_t + \underbrace{\sum_{j=1}^N (1 - \kappa_j)(1 - \Gamma_j) Q_t R_{jt}^k k_{jt} - w_j^e}_{\text{capital income}}. \end{aligned}$$

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<sup>7</sup>Regarding capital reallocation, see Eisfeldt and Rampini (2006).

At each period, the representative household needs to decide consumption  $c_t$ , labor supply  $h_t$ , and deposits  $D_{t+1}$ . The last term in the budget constraint is the capital income transferred from entrepreneurs to the household. This transfer will be explained in the entrepreneur section.  $R_t$  is the risk-free rate known at  $t - 1$ .

The first order conditions for the representative household's choices on consumption and labor supply are:

$$\frac{h_t^{1/\epsilon}}{c_t^{-\varrho}} = w_t, \quad (3.1)$$

$$c_t^{-\varrho} = E_t \tilde{\beta} R_{t+1} c_{t+1}^{-\varrho}. \quad (3.2)$$

### 3.2 Technology

#### *Final goods producer*

The final goods producer purchases  $N$  intermediate goods  $X_{jt}$ , from all sectors to produce a final goods, which is used for consumption and investment. The market is competitive, and the price of the final goods is normalized to one. The technology is constant return to scale. The final goods producer maximizes its profits as follows:

$$\begin{aligned} \max_{X_{jt}} \quad & Y_t - \sum_{j=1}^N p_{jt} X_{jt}, \\ \text{s.t.} \quad & Y_t = \prod_{j=1}^N X_{jt}^{\beta_j}, \quad \sum_{j=1}^N \beta_j = 1. \end{aligned}$$

The corresponding first order condition is:

$$\beta_j Y_t = p_{jt} X_{jt}. \quad (3.3)$$

$\beta$  represents the final use share of each intermediate input.

#### *Capital producer*

After production of all goods in period  $t$ , entrepreneurs sell undepreciated capital to the capital producer. The capital producer combines existing capital and the final goods (investment) to produce new capital. This new capital is then repurchased by entrepreneurs in all sectors. The capital producer's problem is:

$$\begin{aligned} \max_{I_t} \quad & Q_t K_{t+1} - Q_t(1 - \delta)K_t - I_t, \\ \text{s.t.} \quad & K_{t+1} = \Phi(I_t/K_t)K_t + (1 - \delta)K_t. \end{aligned}$$

The first order condition implies

$$Q_t = \frac{1}{\Phi'(I_t/K_t)}. \quad (3.4)$$

Let  $\Phi(I_t/K_t) = I^\theta K^{-\theta}$ . This concave function represents convex capital adjustment cost as in Hayashi (1982). This capital adjustment cost function is used to pin down capital price  $Q_t$ .

#### *Intermediate Goods Producer*

There are  $N$  intermediate goods producing sectors. Firms in sector  $j$  are managed by entrepreneurs of type  $j$ . Each firm's capital is owned by entrepreneurs and is bought one period ahead. With entrepreneurs' capital, firms need to decide how much labor to hire and how much intermediate inputs to buy from other sectors. After production, entrepreneurs receive capital income as capital rent,  $r_{j,t}^k$ . The technology is constant return to scale.

The firms' profit maximization problem is standard as follows:

$$\begin{aligned} \max_{l_{jt}, M_{ijt}, k_{jt}} \quad & p_{jt} Y_{jt} - w_t l_{jt} - r_{j,t}^k k_{jt} - \sum_{i=1}^N p_{it} M_{ijt}, \\ \text{s.t.}, \quad & Y_{jt} = A_{jt} (k_{jt}^{\alpha_j} l_{jt}^{1-\alpha_j})^{1-m_j} \prod_{i=1}^N M_{ijt}^{\gamma_{ij}}, \quad \sum_{i=1}^N \gamma_{ij} = m_j, \end{aligned}$$

where  $\alpha_j$  is capital share.  $M_{ijt}$  is intermediate goods produced in sector  $i$ , purchased as input by sector  $j$ . The exponent  $\gamma_{ij}$  represents input-output linkages.

*Input-Output Matrix:* Let  $\Gamma$  denote the input-output matrix of  $N$  sectors:

$$\Gamma = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,N} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N,1} & \gamma_{N,2} & \cdots & \gamma_{N,N} \end{pmatrix}.$$

$\Gamma$  can be treated as an adjacency matrix of the weighted production network, with each element representing the *directed* link from sector  $i$  to  $j$ . The  $j$ th row sum of  $\Gamma$  is called the weighted out-degree of sector  $j$ , which is the share of sector  $j$ 's output in the input supply of the entire economy. The larger the out-degree, the more common of a supplier this sector is. As I mentioned in Section 2, the larger the out-degree, the more capital that should be allocated to this sector to maximize profits. The  $j$ th column sum,  $m_j$ , is the total intermediate input share in sector  $j$ .

### 3.3 Costly State Verification

At the end of each period  $t$ , after production and wage payments, an entrepreneur in sector  $j$  sells his undepreciated capital to the capital producer and has net worth  $n_{j,t+1}^i$ . He needs to determine how much capital to buy for the next period  $Q_t k_{j,t+1}^i$ . If the value of capital is larger than his own net worth, he needs to borrow the amount  $Q_t k_{j,t+1}^i - n_{j,t+1}^i$ . As it turns out, in the equilibrium, every entrepreneur's net worth is insufficient to buy capital, and everyone needs to borrow from the financial intermediary.

However, after purchasing capital but before putting it into production at  $t+1$ , there is an idiosyncratic shock  $\omega_{j,t+1}^i$ , which converts one unit of purchased capital into  $\omega_{j,t+1}^i$

units of effective capital,  $\omega_{j,t+1}^i k_{j,t+1}^i$ . Assume  $\omega_{j,t+1}^i$  is drawn from a unit mean lognormal distribution with standard deviation  $\sigma_{j,t+1}$  independently and identically. This standard deviation,  $\sigma_{j,t+1}$ , is assumed to follow a stochastic process, and the innovation of it is called the uncertainty (volatility) shock. While the level of sectoral uncertainty  $\sigma_{j,t+1}$  is observable to everyone, the realization of  $\omega_{j,t+1}^i$  is private information. The lender must pay a cost ( $\mu_j$ ) to observe the realized return.

As in BGG, the optimal contract is a standard debt contract. The optimal contract specifies the leverage  $L_{j,t+1}^i = Q_t k_{j,t+1}^i / n_{j,t+1}^i$  and the threshold  $\bar{\omega}_{j,t+1}$ , such that for the lender, ex-ante

$$\bar{\omega}_{j,t+1} R_{j,t+1}^k Q_t k_{j,t+1}^i = z_{j,t+1}^i B_{j,t+1}^i,$$

where  $B_{j,t+1}^i$  is the amount borrowed from entrepreneur  $i$ , and  $z_{j,t+1}^i$  is the interest rate on the loan.  $R_{j,t+1}^k$  is the return on capital on  $t+1$  at sector  $j$ . For each individual entrepreneur, if realized  $\omega_{j,t+1}^i < \bar{\omega}_{j,t+1}$ , he defaults, and the bank pays  $\mu_j \omega_{j,t+1}^i R_{j,t+1}^k Q_t k_{j,t+1}^i$  to seize the entrepreneur's assets.  $\mu_j$  is an exogenous parameter representing liquidation (default, bankruptcy) costs, which differs across sectors. The total expected profit of sector  $j$  is  $R_{j,t+1}^k Q_t k_{j,t+1}$  (the sum of all entrepreneurs' profits in sector  $j$ ). For the lender, the sum of expected profits across sectors is equal to the deposit interest paid to households:

$$\sum_{j=1}^N \{ (1 - \mu_j) \int_0^{\bar{\omega}_j} \omega f_j(\omega) d\omega + \bar{\omega}_j [1 - F_j(\bar{\omega})] \} R_{j,t+1}^k Q_t k_{j,t+1} = R_{t+1} D_{t+1}.$$

Define  $\Gamma_j$  as the share of sector  $j$ 's profits going to the lender:

$$\Gamma_j(\bar{\omega}_j, \sigma_j) = \int_0^{\bar{\omega}_j} \omega f_j(\omega) d\omega + \bar{\omega}_j [1 - F_j(\bar{\omega})],$$

and  $\mu_j G_j$  the share of sector  $j$ 's profits as monitoring cost:

$$\mu_j G_j(\bar{\omega}_j, \sigma_j) = \int_0^{\bar{\omega}_j} \omega f_j(\omega) d\omega.$$

For the profits in sector  $j$ , entrepreneurs get  $(1 - \Gamma_j)$  share, and the bank gets  $(\Gamma_j - \mu_j G_j)$  share (net of monitoring cost).  $\Gamma_j(\bar{\omega}_{j,t+1}, \sigma_{j,t+1})$  and  $G_j(\bar{\omega}_{j,t+1}, \sigma_{j,t+1})$  are functions of the threshold  $\bar{\omega}_{j,t+1}$  and sectoral uncertainty  $\sigma_{j,t+1}$ .

After solving for the optimal contract<sup>8</sup> for each individual,

$$Q_t k_{j,t+1}^i = \psi(E_t R_{j,t+1}^k / R_{t+1}) n_{j,t+1}^i.$$

Aggregating the above equation over entrepreneurs in sector  $j$ , it turns out that sectoral capital only depends on sectoral net worth, and there is no need to keep track of individual net worth. Therefore, the sectoral leverage is a function of the expected spread between the risk-free rate and the sectoral return to capital:

$$L_{j,t+1} = \psi(E_t R_{j,t+1}^k / R_{t+1}).$$

---

<sup>8</sup>This contract problem is exactly the same as in BGG.

The total return on sectoral capital is:

$$R_{jt}^k = \frac{r_{j,t}^k + Q_t(1 - \delta)}{Q_{t-1}}, \quad (3.5)$$

where  $r_{j,t}^k = (1 - m_j)\alpha_j p_{jt} Y_{jt}/k_{jt}$  is the sectoral marginal revenue product of capital in the equilibrium. Equilibrium conditions from the optimal financial contract for all sectors are:

$$E_t \frac{R_{j,t+1}^k}{R_{t+1}} = \rho_j(\bar{\omega}_{j,t+1}, \sigma_{j,t+1}), \quad j = 1, \dots, N. \quad (3.6)$$

The banks' expected zero profit condition for each sector:

$$\frac{N_{j,t+1}}{Q_t k_{j,t+1}} = 1 - E_t \frac{R_{j,t+1}^k}{R_{t+1}} (\Gamma_j - \mu_j G_j). \quad (3.7)$$

At the end of period  $t$ , after production and the realization of profits but before borrowing, entrepreneurs transfer  $(1 - \kappa_j)$  share of their net worth back to their households. There are two reasons for the transfer. First, this makes entrepreneurs' net worth part of households' wealth. It is thus in the interest of the representative household to instruct its entrepreneurs to maximize expected net worth. Second, this setup ensure that entrepreneurs will not accumulate too much net worth and end up without the need to borrow<sup>9</sup>. In addition, households transfer a small amount of wealth back to entrepreneurs,  $w_j^e N_{j,t}$ , where  $w_j^e$  is set at 0.01. This small amount of wealth is used to ensure that every entrepreneur has positive net worth, mainly for those defaulted entrepreneurs. As a result, the law of motion for sectoral net worth is:

$$N_{j,t+1} = \kappa_j(1 - \Gamma_j(\bar{\omega}_{jt}, \sigma_{jt}))R_{jt}^k Q_{t-1} k_{jt} + w_j^e N_{j,t}. \quad (3.8)$$

For each sector, the relation between the interest rate spread ( $Z_j$  and  $R$ ) and the capital wedge (the spread between return to capital and the risk-free rate) is:

$$\frac{Z_{j,t+1}}{R_{t+1}} = \bar{\omega}_{j,t+1} \frac{R_{j,t+1}^k}{R_{t+1}} \frac{L_{j,t+1}}{L_{j,t+1} - 1}. \quad (3.9)$$

#### *Exogenous shocks*

Here I assume that sectoral productivity  $A_j$  and sectoral uncertainty  $\sigma_j$  follow AR(1) processes:

$$\ln(A_{j,t+1}) = \rho_a \ln A_{j,t} + \epsilon_{a,t+1}, \quad (3.10)$$

$$\ln \frac{\sigma_{j,t+1}}{\sigma_{j,ss}} = \rho_\sigma \ln \frac{\sigma_{j,t+1}}{\sigma_{j,ss}} + \epsilon_{\sigma,t+1}. \quad (3.11)$$

---

<sup>9</sup>BGG call  $\kappa$  the survival rate of entrepreneurs, based on the same technical reason.

### Market Clearing

There are five markets: capital goods, sectoral goods, final goods, labor, and credits.

$$K_t = \sum_{j=1}^N K_{jt}, \quad (3.12)$$

$$Y_{jt} = X_{jt} + \sum_{i=1}^N M_{jit}, \quad (3.13)$$

$$Y_t = C_t + I_t + \sum_{j=1}^N \mu_j G_j R_{jt}^K Q_{t-1} k_{jt}, \quad (3.14)$$

$$h_t = \sum_{j=1}^N l_{jt}, \quad (3.15)$$

$$D_{t+1} = \sum_{j=1}^N Q_t K_{j,t+1} - N_{j,t+1}. \quad (3.16)$$

## 3.4 Equilibrium Definition and Characterizations

The competitive equilibrium in the general model is defined as follows:

**Definition 1.** A competitive equilibrium consists of a sequence of vectors  $\{l_{jt}, K_{jt}, N_{jt}, Y_{jt}, M_{ijt}, X_{jt}, \bar{\omega}_{jt}, Y_t, C_t, h_t, I_t, D_t\}_{t=0}^{\infty}$  and a sequence of vectors of prices  $\{P_{jt}, R_{jt}^k, R_t, w_t, Q_t\}_{t=0}^{\infty}$  for  $j = 1, \dots, N$ , such that

1. Households maximize lifetime utility.
2. Entrepreneurs maximize expected profits.
3. Firms maximize profits.
4. All markets clear.

To solve for the equilibrium of the general model, it is useful to define some allocation variables. Let  $\mathbf{x}_t^k, \mathbf{x}_t^l$  be capital and labor allocation variables, so that at the equilibrium,

$$k_{jt} = x_{jt}^k K_t, \quad l_{jt} = x_{jt}^l h_t.$$

And define intermediate inputs allocation variables as  $\eta_{jit}$  for sector  $j$ , so that

$$M_{jit} = \eta_{jit} Y_{jt}, \quad X_{jt} = (1 - \eta_{jt}) Y_{jt},$$

where  $\sum_{i=1}^N \eta_{jit} = \eta_{jt}$ . Note that  $\eta_{ij}$  is the allocation for commodity use.  $Y_{jt} = X_{jt} + \sum_{i=1}^N M_{jit}$ .  $M_{jit}$  is the goods of sector  $j$  used to produce goods in sector  $i$ .  $X_{jt}$  is the goods of sector  $j$  used to produce the final goods (final use).

Finally, define the influence vector  $\{v_{jt}\}_{j=1}^N$  as:

$$v_{jt} \equiv \frac{p_{jt} Y_{jt}}{Y_t},$$

so that the  $j_{th}$  element of the influence vector is the ratio of sector  $j$ 's gross output to GDP. These allocation variables and the influence vector can be solved in the equilibrium in the following propositions. The proofs are in the Appendix.

**Proposition 2.** *In the competitive equilibrium, the influence vector  $\mathbf{v}$ , the intermediate inputs allocation matrix  $\boldsymbol{\eta}$ , and the labor allocation vector  $\mathbf{x}^l$  are constants across time.*

$$\begin{aligned}\mathbf{v} &= [I_N - \Gamma]^{-1} \beta, \\ \boldsymbol{\eta} &= \left[ \frac{1}{v} * v' \right] \circ \Gamma, \\ x_j^l &= \frac{(1 - m_j)(1 - \alpha_j)v_j}{\sum_j (1 - m_j)(1 - \alpha_j)v_j},\end{aligned}$$

where  $\circ$  is the Hadamard product for matrices (also known as the Schur product, an element wise product). Because there is no distortion on labor and intermediate inputs in the general model, there is no resource misallocation, and the optimal allocation only depends on technology parameters. This result is similar to the optimal allocation variable  $x^*$  in Section 2. However, the optimal capital allocation variable is time dependent. Denote sectoral marginal revenue product of capital as  $MRPK_{jt} \equiv \frac{\partial p_{jt} Y_{jt}}{\partial k_{jt}} = (1 - m_j)\alpha_j p_{jt} Y_{jt} / k_{jt}$ .

**Proposition 3.** *With frictions on capital, the capital allocation vector in the competitive equilibrium is:*

$$x_{jt}^k = \frac{\frac{(1 - m_j)\alpha_j v_j}{MRPK_{jt}}}{\sum_j \frac{(1 - m_j)\alpha_j v_j}{MRPK_{jt}}}.$$

This model features an aggregate production function, which consists of aggregate capital and labor.

**Proposition 4.** *In the competitive equilibrium with capital misallocation, the solution for the total production of aggregate final goods has the form:*

$$Y_t = \tilde{A}_t(x_l, x_{kt}, \eta) K_t^{\tilde{\alpha}} h_t^{1 - \tilde{\alpha}},$$

where

$$\begin{aligned}\ln(\tilde{A}_t) &= \boldsymbol{\beta}' \ln(1 - \boldsymbol{\eta}) + \mathbf{v}'(\mathbf{a}_t + \mathbf{c}_{yt}), \\ \tilde{\alpha} &= \mathbf{v}'[(1 - \mathbf{m}) \circ \boldsymbol{\alpha}], \\ c_{yjt} &= (1 - m_j)\alpha_j \ln x_{jt}^k + (1 - m_j)(1 - \alpha_j) \ln x_j^l + \sum_i \gamma_{ij} \ln \eta_{ij}.\end{aligned}$$

$K_t$  and  $h_t$  are aggregate capital and labor.  $\mathbf{a}_t$  is a vector of the logarithm of sectoral productivity,  $\ln(A_{jt})$ .  $\tilde{A}_t$  is the aggregate TFP, and it depends on the level of misallocation  $\mathbf{x}_t^k$ , labor allocation  $\mathbf{x}^l$ , and input-output linkages  $\gamma_{ij}$ . *Proposition 4* states that the fluctuation of aggregate TFP in the model comes from two resources: sectoral productivity,  $A_t$ , and capital allocation,  $\mathbf{x}_{kt}$ . In addition to conventional productivity-driven real business cycle models, this model shows that variance in the degree of capital misallocation can induce aggregate TFP fluctuation. The magnitude depends on parameters' values.

## 4 Data and Calibration

### *Data: the Input-Output Accounts*

In this section, I calibrate the general model to the U.S. input-output matrix at the two-digit NAICS level, which correspond to the direct requirement table at the sector level in the Input-Output Accounts in the Bureau of Economic Analysis (BEA). Since there is no government in the model, I exclude the government sector (NAICS 9). I also exclude the banking sector (NAICS 521-522) from the estimation of input-output coefficients ( $\gamma_{ij}$ ) because the model counterpart of the banking sector in the data is the financial intermediary, and the financial intermediary in the model does not involve production of goods. The number of sectors in the calibration is 14. Figure 1 is the Hinton diagram of the U.S. input-output matrix. The area occupied by a square is proportional to the intermediate input share's value.

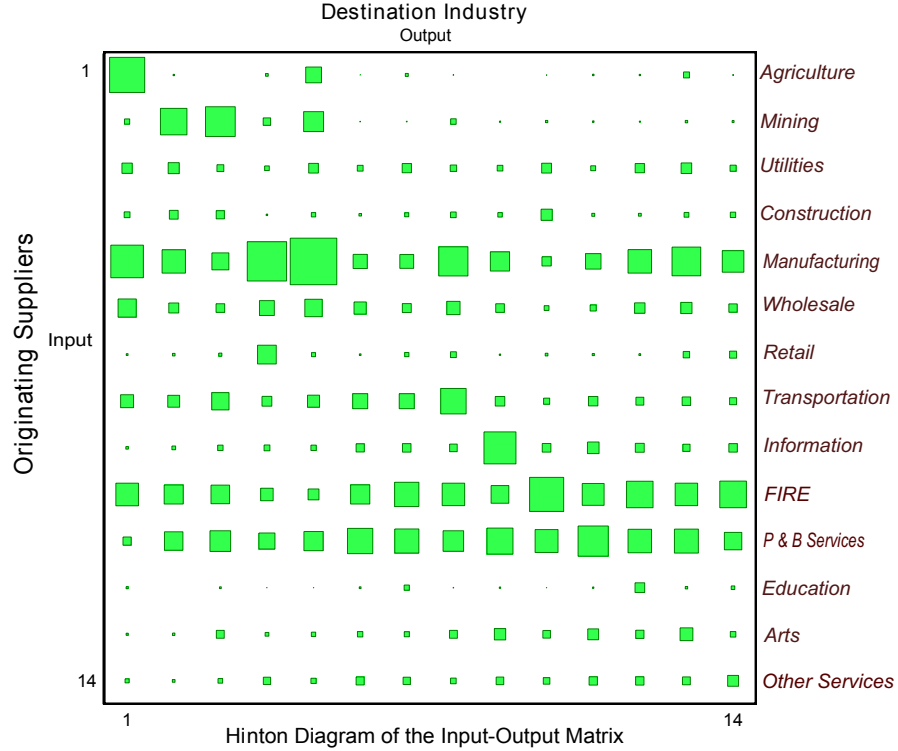


Figure 1: Hinton Diagram of U.S. Input-Output Matrix  $\Gamma$

The sum of column  $j$  of the input-output matrix ( $\Gamma$ ),  $\sum_i \gamma_{ij} = m_j$ , is the total intermediate inputs share in sector  $j$ . The sum of row  $j$  is called the weighted out-degree of sector  $j$  in network theory. That is, we can treat each sector as a node in a weighted production network, and  $\gamma_{ij}$  is the weighted degree between node  $i$  and  $j$ , which reflects the strength of the linkage from  $i$  to  $j$ .  $\gamma_{ij}$  measures the share of good  $i$  in the total intermediate input use of firms in sector  $j$ . The larger the out-degree one sector has, the more common this sector is as a supplier in the economy. In the U.S., the largest three common suppliers are the Manufacturing, FIRE, and Professional and Business sectors,



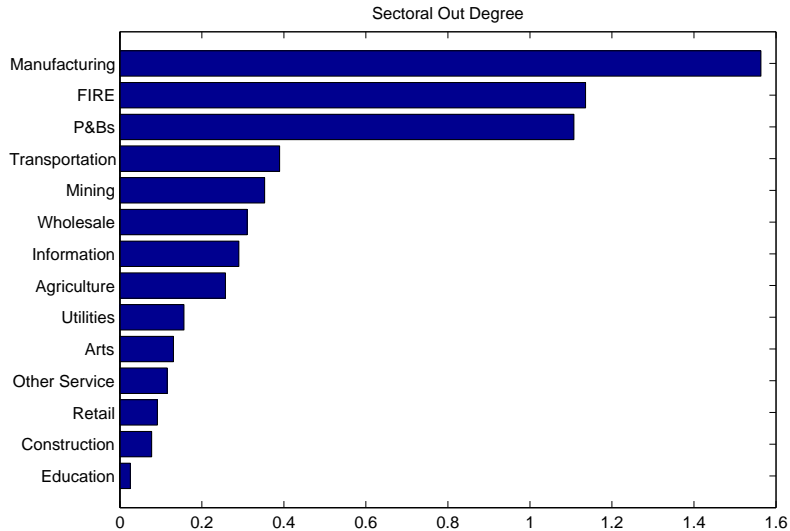


Figure 2: Sectoral Out-Degree

at the two-digit NAICS level. Figure 2 plots the weighted out-degree of each sector. The out-degree of the top three sectors is about three times larger than the out-degree of the fourth sector. This out-degree distribution exhibits a heavy tail at the high end, even at this broad level of disaggregation<sup>10</sup>.

After 1997, the BEA began to publish the input-output use tables and the direct requirement tables annually from 1998 to 2012. The following calibration estimates the cost shares of capital, labor, and intermediate inputs from the sample averages (across time) in the direct requirement tables for each sector.

The estimation of sectoral labor share ( $1 - \alpha_j$ ) is worth mentioning here. The BEA uses the same accounting procedure to calculate labor cost share in the Input-Output Accounts as in the National Income and Product Account (NIPA), which tends to underestimate the labor share, as pointed out in Krueger (1999) and Gomme and Rupert (2004). The direct requirement table in the BEA lists cost shares of intermediate inputs and shares of value added. Value added has three components: Compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The issue lies in the gross operating surplus. In the NIPA, gross operating surplus includes proprietors' income, rental income, corporate profits, net interest, business transfer payments, and consumption of fixed capital. Proprietors' income actually includes components of both labor and capital income, so the labor cost share in the NIPA is underestimated. Most studies assign two-thirds of proprietors' income as labor income from the NIPA, and the resulting value of the labor share ranges from 0.6 to 0.7.

However, the amount of proprietors' income is not listed in the I-O accounts. In the direct requirement table, gross operating surplus consists of the current surplus of government enterprises, consumption of fixed capital, current business transfer payments, and

<sup>10</sup>Acemoglu et al. (2012) exhibit the heavy tail on the out-degree distribution at the disaggregation level of 523 sectors.

other gross operating surplus. Other gross operating surplus includes a corporate component and a noncorporate component. The noncorporate component consists of proprietors' income without adjustments (IVA and CCA), proprietors' income with inventory valuation adjustment (IVA), rental income of persons without capital consumption adjustment (CCAdj), noncorporate capital consumption allowance, and noncorporate net interest. Since the I-O accounts only list the amount of the corporate and noncorporate parts of other gross operating surplus, the amount of proprietors' income is unknown. The literature assigns two-thirds of proprietors' income as labor income. Here, I assign one half of the noncorporate part of the other gross operating surplus as labor's income. So the method I used in dealing with the I-O accounts is:

Labor share = Compensation of employees + 1/2 of the noncorporate part of other gross operating surplus.

Capital share = Gross operating surplus + taxes on production and imports less subsidies - 1/2 of the noncorporate part of other gross operating surplus.

The aggregate labor cost share I estimated here is 0.58, which is smaller than the conventional value, but is consistent with the recent phenomenon of declined labor share, since the data period is from 1998 to 2012.

To calibrate the final use share  $\beta_j$ , I use data reported in the annual I-O use table after redefinitions from 1998 to 2012. Denote final use expenditure in dta as  $fe_{jt}$ , where  $t = 1998, \dots, 2012$ , and  $j = 1, \dots, 14$ . The final use expenditure I used is the sum of personal consumption expenditure, private fixed investment, and changes in private inventories reported in the use table. The use table also reports uses in exports, imports, and government expenditure. I ignore these other uses here<sup>11</sup>. Thus, the estimated final use share for each year is:

$$\hat{\beta}_{jt} = \frac{fe_{jt}}{\sum_{j=1}^{14} fe_{jt}}.$$

And the estimated final use share is the time average over the sample period (15 years):

$$\hat{\beta}_j = \frac{\sum_{t=1998}^{2012} \hat{\beta}_{jt}}{15}.$$

#### *Financial Data and Calibration Strategy*

For financial parameters, three types of financial data are used: sectoral corporate bond spreads, sectoral corporate bond default rates, and leverages. Sectoral leverage is calculated from *Compustat*. Although I exclude the banking sector, all other firms in the FIRE sector still partially serve the function of financial intermediary<sup>12</sup>. Hence their leverages are usually much higher than leverages of firms in other production sectors. However,

<sup>11</sup>The issue is that when import and export uses are taken into account, the final use of the Utilities sector is negative, since oil and gas are mostly imported. The model assumes that the value of  $\beta_j$  is larger than 0 because there are no imports and exports in the model.

<sup>12</sup>For example, real estate companies can borrow from commercial banks and lend to their customers as mortgages.

this high leverage in the FIRE sector is not a correct empirical counterpart of the model, since the leverage in the model is an equilibrium result from default possibility, liquidation costs, and the expected return on capital. The common measure of leverage in the literature is *balance sheet leverage* (total assets divided by shareholders' equity), also known as total leverage, but this is not an appropriate measure here. Due to the fact that most financial firms use short-term debt to finance assets, and these liabilities are recognized as *operating liabilities* in accounting, I chose to use the *financial leverage*. It is defined as  $[1 + \text{financial debt}/\text{equity}(\text{SEQ})]$ . And financial debt is long-term debt (DLTT) plus debt in current liabilities (DLC)<sup>13</sup>.

Sectoral bond spreads are derived from the *TRACE* data set, from 2002 to 2013. I chose the spread between investment grade corporate bonds and the Federal Funds rate with 10 year maturity.

The corporate bond default rates are reported in Moodys' 2010 annual report on default.

Spreads, leverages, and default rates are then used as targets in Eq (3.6), (3.7), and default probability ( $F(\omega_j)$ ) in the model to pin down two vectors of parameters,  $\boldsymbol{\mu}$  and  $\boldsymbol{\sigma}$ , and a vector of endogenous thresholds  $\bar{\omega}$  at the steady state.  $\boldsymbol{\gamma}$  are then backed out from Eq (3.8) at the steady state. Capital wedges at the steady state can then be backed out from Eq (3.9) by using the sectoral corporate bond spreads.

### *Calibration Results*

For preference parameters, the coefficient of constant relative risk-aversion,  $\varrho$ , is set at 1.5, and the Frisch elasticity of labor supply,  $\epsilon$ , is set at 1. The value of the Frisch elasticity I chose is larger than the estimated value from micro literature (ranged from 0.1 to 0.4), but smaller than the estimated value from macro literature (ranged from 2 to 3). The value of 1 is appropriate when the effect of unemployment is taken into account as pointed in Bigio and La'O (2013). Table 1 lists all parameters' values and their targets. A bold letter indicates a vector. Values of a parameter vector are listed in Table 2 and 3. The autocorrelation parameters of the stochastic processes are set at 0.9 for all sectors in the counterfactual exercises in Section 5. The standard deviations of independent innovations are set at 0.0712.

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<sup>13</sup>Capitalized abbreviations in this section are Compustat mnemonics.

Parameter	Meaning	Target	value
Preference			
$\tilde{\beta}$	discount factor	3% Real Interest Rate (APR)	0.993
$\varrho$	coefficient of CRRA		1.5
$\epsilon$	Frisch elasticity of labor supply		1
Technology			
$\alpha$	capital shares	Direct Requirement Table from Input-Output Account	Table 2
$\Gamma$	Input-Output linkages (matrix)		
$\beta$	Final Use Share	Final Use in the BEA Use Table	
$\theta$	capital adjustment cost		0.9
$\delta$	capital depreciation rate		0.02
Financial parameters			
$\mu$	bankruptcy cost	Sectoral Default rates & Corporate bond spreads & Leverage	Table 3
$\sigma$	uncertainty		
$\kappa$	net worth transferring rate		

Table 1: Parameters and calibration targets

Sector	Capital Share	Labor share	Intermediate inputs share ( $m_j$ )
Agriculture	0.20	0.20	0.60
Mining	0.38	0.22	0.4
Utility	0.41	0.17	0.42
Construction	0.14	0.39	0.47
Manufacturing	0.13	0.21	0.66
Wholesale	0.32	0.38	0.30
Retail	0.27	0.40	0.33
Transportation	0.17	0.35	0.48
Information	0.27	0.26	0.47
FIRE	0.39	0.23	0.38
Professional and Business services	0.15	0.49	0.36
Education	0.08	0.51	0.39
Arts	0.19	0.38	0.43
Other services	0.18	0.50	0.32

Table 2: Technology Parameters

Sector	$\mu$	$\sigma$	$\kappa$
Agriculture	0.48	0.33	0.95
Mining	0.26	0.43	0.96
Utilities	1	0.22	0.94
Construction	0.61	0.29	0.94
Manufacturing	0.20	0.44	0.96
Wholesale	0.42	0.33	0.97
Retail	0.11	0.38	0.95
Transportation	0.45	0.27	0.95
Information	0.16	0.43	0.97
FIRE	1	0.22	0.95
Professional and Business services	0.30	0.49	0.96
Education	0.21	0.35	0.96
Arts	0.41	0.3	0.95
Other services	0.18	0.32	0.96

Table 3: Financial Parameters

With the parameters' values, the influence vector  $\mathbf{v} = [\mathbf{I}_N - \mathbf{\Gamma}]^{-1}\boldsymbol{\beta}$  is in Figure 3.

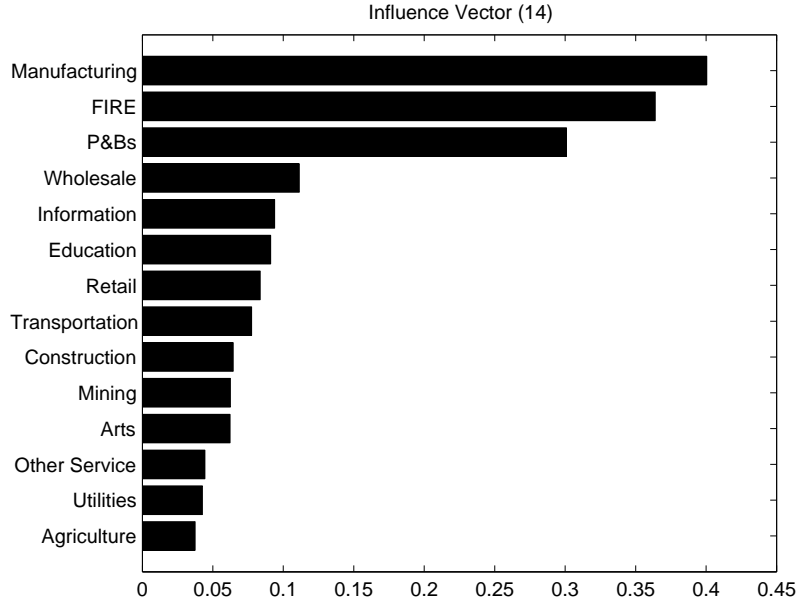


Figure 3: Influence Vector

## 5 Results

I first show that financial frictions play two distinct roles in amplification under different types of aggregate shocks and then discuss the amplification result from input-output linkages. I compare the amplification magnitudes of two economies with financial frictions: one has Input-Output linkages, and the other one does not have firm linkages. These two economies are calibrated to have the same aggregate moments.

### 5.1 Financial Frictions: Type of Aggregate Shocks Matters

In the model, the degree of financial frictions is mainly reflected by the bankruptcy cost parameter  $\mu$ . When there is no bankruptcy cost,  $\mu = 0$ , and the model goes back to the full information case. Risk-neutral entrepreneurs will borrow up until the expected return on capital is equal to the risk-free rate. For the following exercise, I vary the value of  $\mu$  from 0 to 1 (with 0.1 increment) and discuss how the dynamic changes with different types of shocks hitting the economy for a single sector case. In the following exercises, “amplification” refers to the comparison of impulse responses at the first period between different values of  $\mu$ . “Fluctuation” refers to the entire dynamic process in which variables go back to their steady states.

#### 5.1.1 TFP Shock

Figure 4 shows the impulse response of output, investment, labor, and capital wedge (the spread between return to capital and the risk-free rate) when a TFP shock (negative one standard deviation) hits the economy with different degrees of financial frictions. The larger the degree of financial frictions, the warmer the plot color is. Two remarks are in order. First, financial frictions mute the impulse response of output and labor under TFP shock. Second, financial frictions play a quantitatively minor role in amplification under TFP shock. The main reason is that with the particular financial friction putting on capital, the productivity shock has an endogenously weak effect on the capital wedge. In response to a 1% decrease in productivity, the capital wedge increases no more than 3 basis points. The main channel the financial friction works on, the capital wedge channel, is basically null when a TFP shock hits the economy. Hence the main difference between different values of  $\mu$  is the steady state values of capital wedges, which behave like exogenous tax on capital income. It thus acts as a mechanism similar to the capital adjustment cost, as discussed in Carlstrom and Fuerst (1997).

#### 5.1.2 Uncertainty Shock

Figure 5 shows impulse responses of output, investment, labor, and capital wedge when an aggregate uncertainty shock (one standard deviation, 10%) hits the economy with different degrees of financial frictions. When  $\mu = 0$ , the uncertainty shock plays no role in fluctuation. The impulse response functions are zero for all variables. When  $\mu$  increases from 0 to 1, the amplification and fluctuation grow larger and larger. The larger the degree of financial friction, the larger impulse response on capital wedges that an

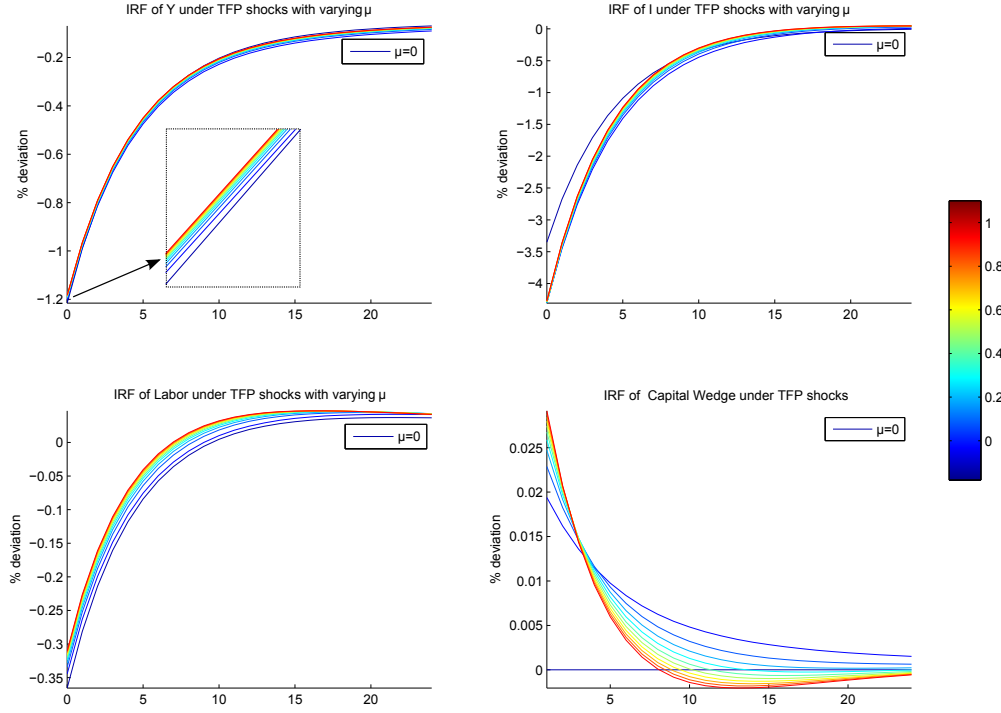


Figure 4: Impulse responses to TFP shocks with varying degree of financial frictions.

uncertainty shock can induce, and hence the larger the degree of amplification for other variables. We can see that the direction of impulse responses is totally opposite under two different types of aggregate shocks. In short, financial friction mutes the impulse responses under a TFP shock but reinforces the impulse responses under an uncertainty shock. Here, financial friction plays a crucial role in generating fluctuations under an uncertainty shock.

## 5.2 The Amplification from Input-Output Linkages

Consider another model economy in which there is a representative production sector (this is the single sector version of the general model), with the same type of financial friction described in Section 3. The parameters in the single-sector version are set to match the aggregate moments, as in the calibrated general model with firm linkages. That is, the single-sector version has aggregate leverage equal to 1.7, annual default rate at 3%, interest rate spread at 1.06%, consumption to GDP ratio equal to 0.75, and investment to GDP ratio equal to 0.2. The aggregate capital share is 0.42. The financial parameters are  $\mu = 0.2565$ ,  $\sigma = 0.3436$ , and  $\kappa = 0.9697$ . The main difference between the two economies is input-output linkages. Now consider that an exogenous shock hits two economies.

### Aggregate Uncertainty Shocks

Suppose uncertainty ( $\sigma_j$ ) in every sector increases by 10%; Figure 6 shows the impulse response functions of two economies. In the first period GDP declines by 0.65% in both

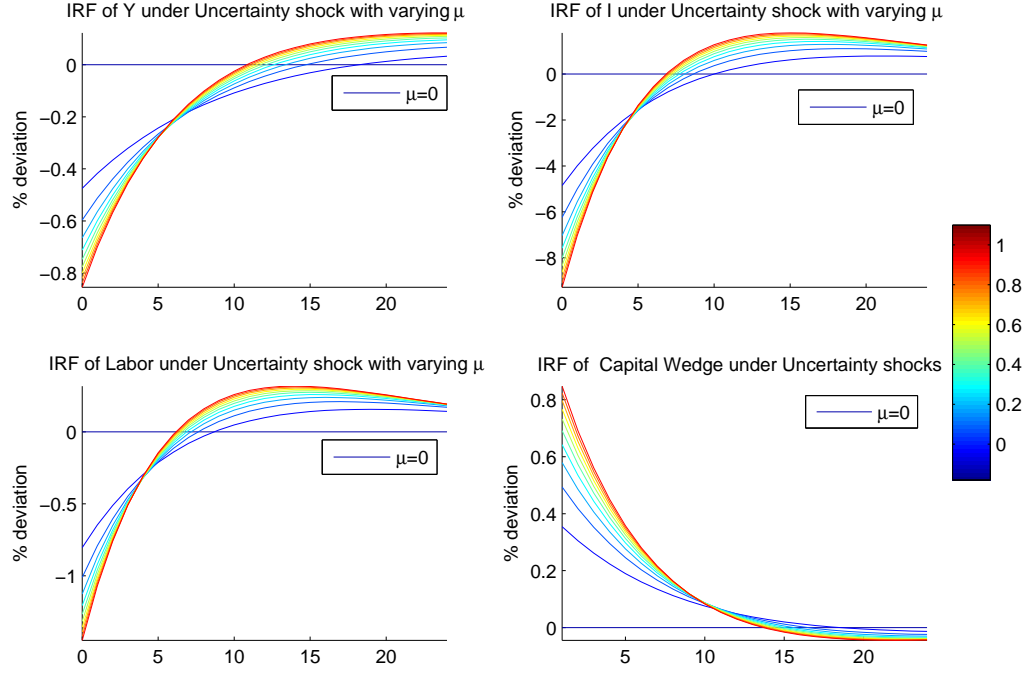


Figure 5: The impulse responses to uncertainty shocks with varying degrees of financial frictions.

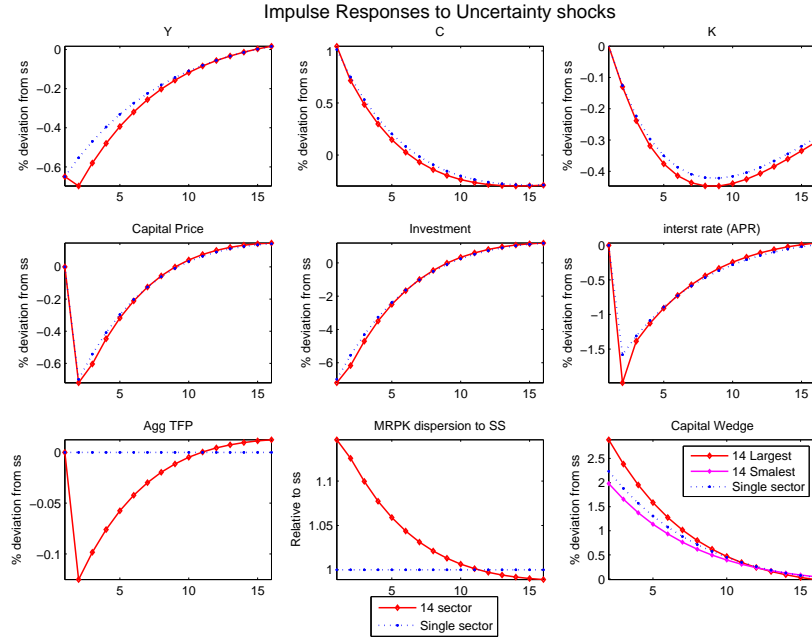


Figure 6: IRF of 14-sector vs. single-sector under uncertainty shocks

economies. Since capital is predetermined, the effects of capital misallocation will not show



up until the second period. In the second period, in the 14 sector economy, aggregate TFP drops 0.12% and GDP drops 0.69%, while in the single sector economy, GDP drops 0.55%. Most of this GDP drop difference comes from the sudden drop of aggregate TFP in the 14 sector economy. As uncertainty increases, capital flows from more constrained firms to less constrained firms, magnifies the degree of capital misallocation, and reduces aggregate TFP. The bottom middle panel in Figure 6 plots the standard deviation of MRPK distribution across sectors relative to its steady state value. When aggregate uncertainty increases, MRPK dispersion increases about 14%.

Regarding the impulse responses for other financial variables at the second period, the interest rate drops 2% (APR), the default rate (average) increases by 2.72% (mode), and spread increases about 1.12% (APR) in the 14 sector economy. In the single sector economy, the risk-free rate drops about 1.4% (APR), the credit spread increases 0.77% (APR), and the default rate increases 2.64% (APR).

	Single	14 Sector
$\Delta \ln Y$	-0.49%	-0.69%
$\Delta \ln C$	0.71%	0.73%
MRPK dispersion relative to SS	1	1.14
$\Delta r(\text{APR})$	-1.4%	-2%
Spread (APR)	0.78%	1.13%
Default rates (APR)	2.67%	2.77%

Table 4: Responses to an aggregate uncertainty shock at the second period

### Aggregate TFP Shocks

Suppose productivity ( $A_j$ ) in every sector decreases by 1%. Figure 7 shows impulse response functions of two economies.

In the 14 sector economy, aggregate output  $Y$  decreases by 2.18 %, and aggregate TFP decreases by 1.8%. The effect on aggregate TFP due to capital misallocation shows up at the second period in the 14 sector economy. The impulse response of aggregate TFP can be broken down into two parts: the response of sectoral productivity and capital misallocation, as in the following equation (log deviation):

$$\ln \tilde{A}_t = \mathbf{v}' \hat{A}_t + ((1 - m) \circ \boldsymbol{\alpha} \circ \mathbf{v})' \hat{x}_t^k.$$

At the second period, the aggregate TFP drops 1.48%. For this 1.48%, 1.46% comes from the productivity shock,  $\mathbf{v}' \hat{A}_{t=2}$ , and 0.02% comes from capital misallocation. That means that in response to productivity shocks, the fluctuation due to capital misallocation is negligibly small, and MRPK dispersion across sectors increases no more than 1%. In short, a TFP shock has little impact on the degree of capital misallocation.

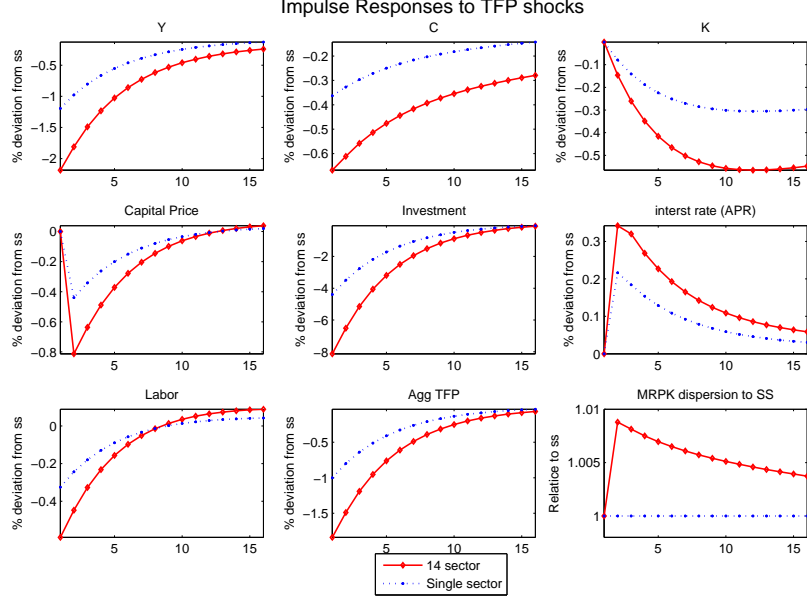


Figure 7: IRF of 14 sector vs. single sector under an aggregate TFP shock

	Single	14 Sector
$\Delta \ln Y$	-1.18%	-2.18%
$\Delta \ln C$	-0.39%	-0.67%
$\Delta \ln I$	-4%	-8.12%
MRPK dispersion relative to SS	1	1.009
$\Delta r(\text{APR})$	0.3%	0.34%
Spread (APR)	0.04%	0.08%
Default rates (APR)	0.12%	0.17%

	Single Benchmark	14 Benchmark
$\Delta \ln Y$	-1.08%	-1.9%

Table 5: Responses to an aggregate TFP shock

### 5.3 Sectoral Impacts

The following analysis focuses on individual sectoral responses and their impact on the aggregate economy.

**Question 1. Which sector suffers the most from distortion? What is the output gain if there is no capital misallocation?**

The correct measure for distortion here is the model implied capital wedge inferred from financial data (default rates, interest rate spreads, and leverages) in the steady state. I compare the difference between the optimal capital allocation in a frictionless model economy and that in the model economy with the financial friction described in Section 3. Figure 8 shows the result in descending order. The top panel is the model implied capital

wedges for each sector. The middle panel shows the sectoral capital allocation between two model economies. The bottom panel shows the deviation of capital allocation from its frictionless level. For example, the Construction sector is the most distorted (with the largest capital wedge); the proportion of capital allocated to the Construction sector is only 87% compared to its frictionless proportion. On the other hand, the Retail sector has the smallest capital wedge among the 14 sectors, and hence capital is over-allocated to the Retail sector, 22% larger than its frictionless proportion.

I then ask: what is the output gain if capital is allocated hypothetically to its frictionless level? The answer is that the steady state output will increase by 33%.

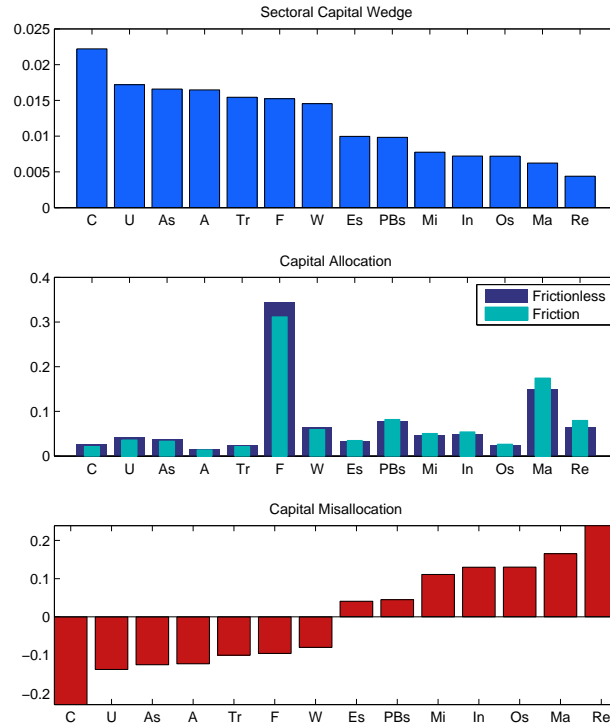


Figure 8: The Degree of Capital Misallocation in the Steady State. The sector order is: Construction, Utilities, Arts, Agriculture, Transportation, FIRE, Wholesale, Education, Professional and Business Services, Mining, Information, Other Services, Manufacturing, and Retail.

## Question 2. In response to aggregate shocks, which sector is most affected?

### *Aggregate TFP Shocks*

Suppose productivity in every sector decreases by 1%; which sector is most affected? The answer is in Figure 9. It turns out that manufacturing related industries, such as the Manufacturing, Agriculture, and Construction sectors, suffer the most from aggregate productivity shocks. The reason is that these sectors have large intermediate input cost

shares,  $m$ .

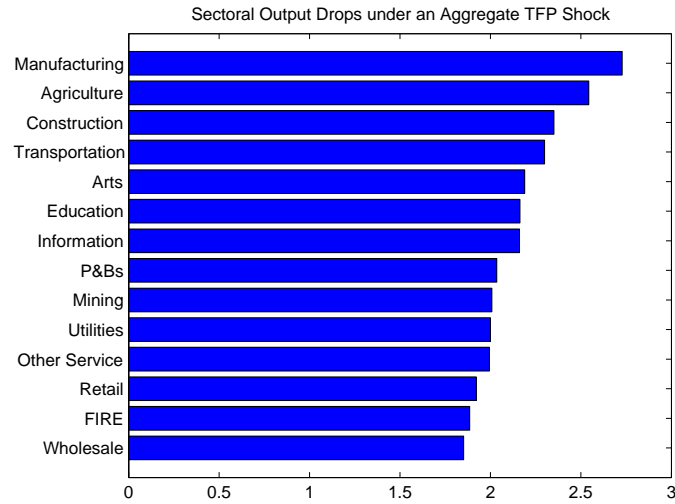


Figure 9: Sectoral Output Drop under an Aggregate TFP Shock

#### *Aggregate Uncertainty Shocks*

Suppose uncertainty in every sector increases by 10%; which sector is most affected? The answer is in Figure 10. The top three sectors are: FIRE, Utilities, and Agriculture.

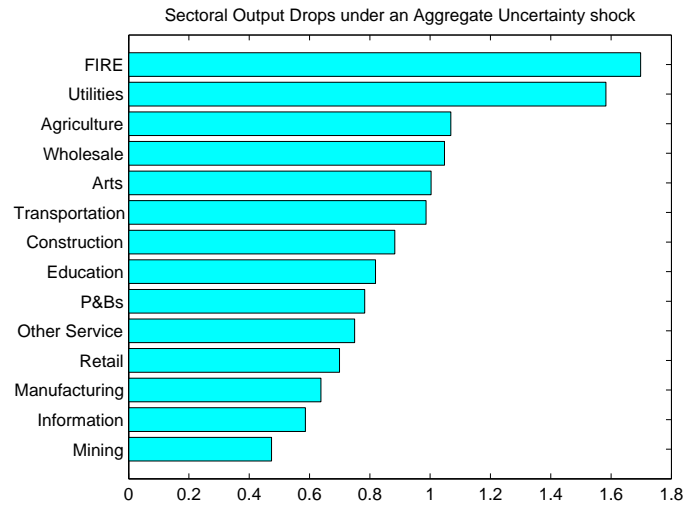


Figure 10: Sectoral Output Drop under an Aggregate Uncertainty Shock

This answer depends crucially on sectoral financial parameters. The result depends on the interaction of three financial parameters: uncertainty level  $\sigma$ , bankruptcy cost  $\mu$ , and transferring rate  $\kappa$ . The FIRE and Utilities sectors suffer most because they have largest bankruptcy cost, and are most vulnerable to fluctuations in uncertainty level.

Next, I am going to analyze each individual sector's impact on aggregate output, to find out which sectors are key sectors during the recession.

**Question 3. In response to idiosyncratic shocks, which sector has the largest impact on aggregate output?**

*Idiosyncratic TFP Shock*

Suppose productivity ( $A_j$ ) decreases by 1% in a single sector; how much does aggregate output drop? The answer is in Figure 11. The most “influential” three sectors are Manufacturing, FIRE, and Professional and Business services. The order in Figure 11 coincides with the order of the influence vector,  $\mathbf{v}$  (Figure 3), which is the ratio of gross sectoral output to GDP. Note that the influence vector includes two elements: input-output linkages and the final use shares. The reason that the Manufacturing, FIRE, and Professional and Business services sectors have largest impact on aggregate output is because they are common suppliers with large final use shares.

This feature is intuitive, as productivity drops directly affect sectoral gross output and affect GDP through input-output linkages.

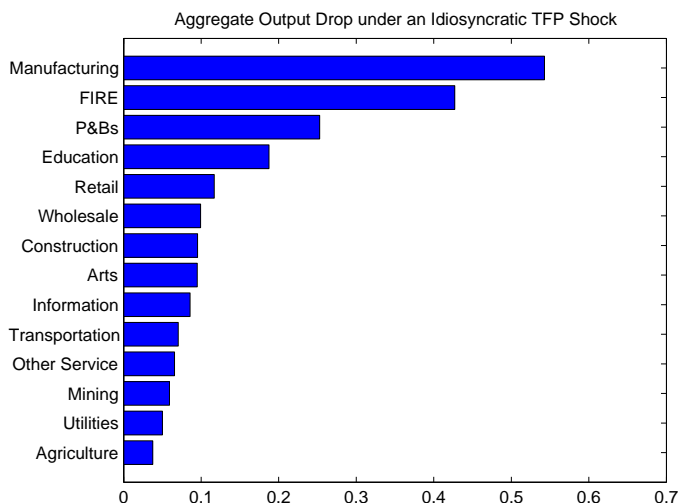


Figure 11: Aggregate Output Drop under an Idiosyncratic TFP Shock

*Idiosyncratic Uncertainty Shock*

Suppose uncertainty ( $\sigma_j$ ) increases by 10% in a single sector; how much does aggregate output drop? The answer is in Figure 12. The three most “influential” sectors are the FIRE, Manufacturing, and Wholesale sectors. Compared to the answer in TFP shocks, the FIRE sector now has the largest impact on aggregate output when its uncertainty level increases. That’s because the FIRE has the one of the three common suppliers, which has the largest bankruptcy cost and are most susceptible to an increase in uncertainty level.

The answers to Question 3 imply that the Manufacturing and FIRE sectors are the two most important sectors in terms of their impact on aggregate economy.

## 6 Conclusion

This paper demonstrates an important implication of input-output linkages: their amplification effects. I show how capital misallocation affects the aggregate measured productiv-

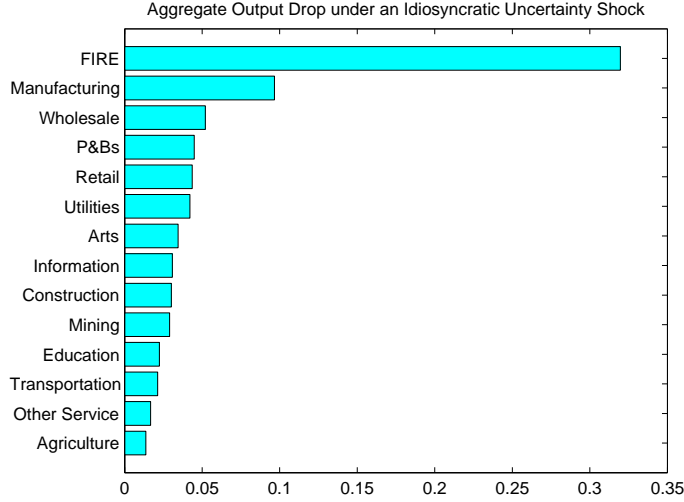


Figure 12: Aggregate Output Drop under an Idiosyncratic Uncertainty Shock

ity and how inter-firm linkages amplify the degree of capital misallocation and responses to shocks.

Although the U.S. financial market is well-developed and the degree of capital misallocation is small compared to other countries, I show that with the addition of input-output linkages, an increase in borrowing costs has a large impact on aggregate fluctuations and magnifies the degree of capital misallocation. In turn, this reduces the aggregate measured productivity significantly. Furthermore, by combining data on corporate bond spreads, default rates, and industrial financial leverages, the model shows substantial differences in industries sensitivities to an increase in uncertainty levels. Firms in the FIRE and Utilities sectors are most vulnerable to fluctuations in borrowing costs. Compared to other sectors, an increase in the dispersion of the return to capital in the FIRE sector has the largest impact on aggregate output. These findings describe how small economic shocks can generate large aggregate fluctuations, as was observed in the 2008 financial crisis.

One future direction would be to add distortions on labor and intermediate inputs. This would induce three types of resource misallocation, and the degree of misallocation would be larger than in the current model. If these types of distortions were taken into consideration, the level of distortions would enter into the formula of allocation variables  $(\mathbf{x}^k, \mathbf{x}^l, \boldsymbol{\eta})$ , and the magnitude of aggregate TFP variations due to varying distortions could become larger, which could help further explain the underlying reason behind aggregate TFP variations. However, the main challenge for this direction is the identification problem, since without observable data on the financial constraints of labor and intermediate goods, these distortions are not identified. To overcome this issue, some indirect proxies are needed, as Bigio and La'O (2013) have used in a static environment. However, future research must examine a dynamic setting in order to more fully investigate business cycles.

# Appendix

## A Model Equations

Households' problem:

$$\begin{aligned} \max_{c_t, h_t} \quad & \sum_{t=1}^{\infty} \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\epsilon h_t^{1+\epsilon/\epsilon}}{1+\epsilon} \\ \text{s.t.} \quad & c_t + D_{t+1} = w_t h_t + R_t D_t + \underbrace{\sum_{j=1}^N (1-\gamma_j)(1-\Gamma_j) R_{jt}^k k_{jt} - w_{jt}^e}_{\text{transfer}} \end{aligned}$$

FOC:

$$\frac{h_t^{1/\epsilon}}{c_t^{-\sigma}} = w_t, \quad (\text{A.1})$$

$$c_t^{-\sigma} = E_t \beta R_{t+1} c_{t+1}^{-\sigma} \quad (\text{A.2})$$

Technology: Final goods producer:

$$\begin{aligned} \max_{X_{jt}} \quad & Y_t - \sum_{j=1}^N p_{jt} X_{jt}, \\ \text{s.t.} \quad & Y_t = \prod_{j=1}^N X_{jt}^{\beta_j}, \quad \sum_{j=1}^N \beta_j = 1. \end{aligned} \quad (\text{A.3})$$

FOC:

$$\beta_j Y_t = p_{jt} X_{jt}. \quad (\text{A.4})$$

Entrepreneurs in sector  $j$ 's problem can be decomposed into intratemporal part and intertemporal part. Intratemporal problem (at time  $t$ ) is:

$$\begin{aligned} \max_{l_{jt}, M_{ijt}} \quad & p_{jt} Y_{jt} - w_t l_{jt} - \sum_{i=1}^N p_{it} M_{ijt} \\ \text{s.t.}, \quad & Y_{jt} = A_{jt} (k_{jt}^{\alpha_j} l_{jt}^{1-\alpha_j})^{1-m_j} \prod_{i=1}^N M_{ijt}^{\gamma_{ij}}, \quad \sum_{i=1}^N \gamma_{ij} = m_j. \end{aligned} \quad (\text{A.5})$$

FOCs:

$$(1-\alpha_j)(1-m_j)p_{jt}Y_{jt} = w_t l_{jt}, \quad (\text{A.6})$$

$$\gamma_{ij} p_{jt} Y_{jt} = p_{it} M_{ijt}. \quad (\text{A.7})$$

Intertemporal problem FOCs

$$E_t \frac{R_{j,t+1}^k}{R_{t+1}} = \rho_j(\omega_{t+1}, \sigma_{t+1}), \quad j = 1 \sim N. \quad (\text{A.8})$$

Banks' zero profit condition for each sector

$$\frac{N_{j,t+1}}{Q_t k_{j,t+1}} = 1 - \frac{R_{j,t+1}^k}{R_t} (\Gamma_j - \mu_j G_j) \quad (\text{A.9})$$

$$R_{jt}^k = \frac{(1 - m_j) \alpha_j p_{jt} Y_{jt} / k_{jt} + Q_t (1 - \delta)}{Q_{t-1}} \quad (\text{A.10})$$

Low of motion for network:

$$N_{j,t+1} = \gamma_j (1 - \Gamma_j (\omega_{jt}, \sigma_{jt})) R_{jt}^k Q_{t-1} k_{jt} + w_{jt}^e \quad (\text{A.11})$$

Capital producer's problem:

$$\begin{aligned} \max_{K_{t+1}, I_t} \quad & Q_t K_{t+1} - Q_t (1 - \delta) K_t - I_t \\ \text{s.t.} \quad & K_{t+1} = \Phi(I_t / K_t) K_t + (1 - \delta) K_t. \end{aligned} \quad (\text{A.12})$$

FOC:

$$Q_t = \frac{1}{\Phi'(I_t / K_t)}. \quad (\text{A.13})$$

Market clearing conditions:

$$K_t = \sum_{j=1}^N K_{jt} \quad (\text{A.14})$$

$$Y_{jt} = X_{jt} + \sum_{i=1}^N M_{jit} \quad (\text{A.15})$$

$$Y_t = C_t + I_t + \sum_{j=1}^N \mu_j G_j R_{jt}^K Q_{t-1} k_{jt}, \quad (\text{A.16})$$

$$h_t = \sum_{j=1}^N l_{jt} \quad (\text{A.17})$$

$$D_{t+1} = \sum_{j=1}^N Q_t K_{j,t+1} - N_{j,t+1} \quad (\text{A.18})$$

Shocks:

$$\ln(A_{j,t+1}) = \rho_{aj} \ln A_{j,t} + \epsilon_{aj} \quad (\text{A.19})$$

$$\ln \frac{\sigma_{j,t+1}}{\sigma_{j,ss}} = \rho_{\sigma_j} \ln \frac{\sigma_{j,t+1}}{\sigma_{j,ss}} + \epsilon_{\sigma_j} \quad (\text{A.20})$$



## B Proofs of Propositions

**Proof of *Proposition 2*:**

**Influence Vector  $\mathbf{v}$ :**

From Eq (A.4) and (A.7):

$$p_{jt} = \frac{\beta_j Y_t}{X_{jt}}, \quad M_{ijt} = \frac{\gamma_{ij} p_{it} Y_{it}}{p_{jt}}.$$

Multiply both sides of Eq (A.15) by  $p_{jt}$ , and plug in the above two terms:

$$\begin{aligned} p_{jt} Y_{jt} &= p_{jt} X_{jt} + \sum_{i=1}^N p_{jt} M_{jit}, \\ \Rightarrow p_{jt} Y_{jt} &= \beta_j Y_t + \sum_{i=1}^N \gamma_{ij} p_{it} Y_{it}, \\ \Rightarrow \frac{\beta_j Y_t}{X_{jt}} Y_{jt} &= \beta_j Y_t + \sum_{i=1}^N \gamma_{ij} \frac{\beta_i Y_t}{X_{it}} Y_{it} \end{aligned}$$

Define the influence vector as  $v_{jt} \equiv \frac{p_{jt} Y_{jt}}{Y_t} = \frac{\beta_j Y_{jt}}{X_{jt}}$ , then the above equation becomes:

$$v_{jt} Y_t = \beta_j Y_t + \sum_{i=1}^N \gamma_{ij} v_{it} Y_t.$$

Divide  $Y_t$  on both sides:

$$v_{jt} = \beta_j + \sum_{i=1}^N \gamma_{ij} v_{it}.$$

Write in the vector form:

$$\mathbf{v}_t = \boldsymbol{\beta} + \boldsymbol{\Gamma} \mathbf{v}_t.$$

That is, the influence vector is independent of time, and

$$\mathbf{v} = [\mathbf{I}_N - \boldsymbol{\Gamma}]^{-1} \boldsymbol{\beta}.$$

**Intermediate inputs allocation matrix  $\boldsymbol{\eta}$ :**

The definition of  $\boldsymbol{\eta}$  is:  $M_{jit} = \eta_{jit} Y_{jt}$ , and  $X_{jt} = (1 - \eta_{jt} Y_{jt})$ , where  $\eta_{jt} = \sum_{i=1}^N \eta_{jit}$ , the column sum. From Eq (A.7):

$$\begin{aligned} \gamma_{ij} p_{jt} Y_{jt} &= p_{it} M_{ijt} = p_{it} \eta_{ijt} Y_{it}, \\ \Rightarrow \frac{p_{it} Y_{it}}{p_{jt} Y_{jt}} &= \frac{\gamma_{ij}}{\eta_{ijt}}. \end{aligned}$$

Since

$$\frac{p_{it} Y_{it}}{p_{jt} Y_{jt}} = \frac{v_i}{v_j},$$

we have

$$\eta_{jit} = \frac{\gamma_{ij}v_j}{v_i}.$$

So  $\eta_{ji}$  is independent of time, and it can be written in the vector form as:

$$\boldsymbol{\eta} = [\frac{1}{\mathbf{v}} * \mathbf{v}'] \circ \boldsymbol{\Gamma},$$

where  $\circ$  is the Hadamard product, an element wise matrices product.

#### **Labor allocation variable $x^l$ :**

Sum Eq (A.6) over sectors and use the definition of  $\mathbf{v}$ :

$$\frac{(1 - \alpha_j)(1 - m_j)p_{jt}Y_{jt}}{l_{jt}} = w_t = \frac{\sum_{j=1}^N (1 - \alpha_j)(1 - m_j)v_j Y_t}{h_t}$$

Let  $l_{jt} = x_{jt}^l h_t$ , then

$$\begin{aligned} x_{jt}^l &= \frac{(1 - \alpha_j)(1 - m_j)p_{jt}Y_{jt}}{\sum_{j=1}^N (1 - \alpha_j)(1 - m_j)v_j Y_t} \\ &= \frac{(1 - \alpha_j)(1 - m_j)v_j}{\sum_{j=1}^N (1 - \alpha_j)(1 - m_j)v_j}. \end{aligned}$$

Thus the labor allocation variable  $x_j^l$  is independent of time.

#### **Proof of *Proposition 3*:**

#### **Capital allocation variable $x_t^k$ :**

From Eq (A.10), the marginal revenue product of capital is defined as:

$$\text{MRPK}_{jt} = \frac{(1 - m_j)\alpha_j p_{jt}Y_{jt}}{k_{jt}} = Q_{t-1}R_{jt}^k - Q_t(1 - \delta).$$

At the equilibrium, the capital rent  $r_{jt}^k$  is equal to  $\text{MRPK}_{jt}$ .

Define the capital allocation variable  $x_{jt}^k$  be that  $k_{jt} = x_{jt}^k K_t$ , and substitute  $p_{jt}Y_{jt} = v_j Y_t$  in the above equation, we get

$$\frac{k_{jt}}{k_{it}} = \frac{x_{jt}^k}{x_{it}^k} = \frac{r_{it}^k(1 - m_j)\alpha_j v_j}{r_{jt}^k(1 - m_i)\alpha_i v_i}.$$

Since  $\sum_{j=1}^N x_{jt}^k = 1$ ,

$$x_{jt}^k = \frac{\frac{(1-m_j)\alpha_j v_j}{r_{jt}^k}}{\sum_j \frac{(1-m_j)\alpha_j v_j}{r_{jt}^k}}.$$

**Proof of Proposition 4:**

Take logarithm on Eq (A.5). Denote  $\ln Y_{jt} = y_{jt}$ . Substitute  $k_{jt}$  with  $x_{jt}^k K_t$ ,  $l_{jt}$  with  $x_j^l h_t$ , and  $M_{ijt}$  with  $\eta_{ij} Y_{it}$  we get:

$$\begin{aligned}
y_{jt} &= \ln A_{jt} + (1 - m_j) \alpha_j \ln(x_{jt}^k K_t) + (1 - m_j)(1 - \alpha_j) \ln(x_j^l h_t) + \sum_{i=1}^N \gamma_{ij} \ln(\eta_{ij} Y_{it}), \\
\Rightarrow y_{jt} &= \ln A_{jt} + \underbrace{(1 - m_j) \alpha_j \ln x_{jt}^k + (1 - m_j)(1 - \alpha_j) \ln x_j^l + \sum_{i=1}^N \gamma_{ij} \ln \eta_{ij}}_{\text{denote this as } c_{yjt}} + \underbrace{(1 - m_j) \alpha_j}_{\text{denote this as } \delta_{kj}} \ln K_t \\
&\quad + \underbrace{(1 - m_j)(1 - \alpha_j) \ln h_t}_{\text{denote this as } \delta_{lj}} + \sum_{i=1}^N \gamma_{ij} \ln Y_{it}, \\
\Rightarrow y_{jt} &= \ln A_{jt} + c_{yjt} + \delta_{kj} \ln K_t + \delta_{lj} \ln h_t + \sum_{i=1}^N \gamma_{ij} y_{it}.
\end{aligned}$$

Denote  $\ln A_{jt}$  as  $a_{jt}$ . The vector form of the above equation is

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{a}_t + \mathbf{c}_{yt} + \boldsymbol{\delta}_k \ln K_t + \boldsymbol{\delta}_l \ln h_t + \boldsymbol{\Gamma}' \mathbf{y}_t, \\
\Rightarrow \mathbf{y}_t &= [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} (\mathbf{a}_t + \mathbf{c}_{yt} + \boldsymbol{\delta}_k \ln K_t + \boldsymbol{\delta}_l \ln h_t).
\end{aligned} \tag{B.1}$$

Use the aggregate production function (A.3), and replace  $X_{jt}$  with  $(1 - \eta_j) Y_{jt}$ . So  $Y_t = \prod_{j=1}^N ((1 - \eta_j) Y_{jt})^{\beta_j}$ . Take logarithm, and use Eq (B.1). The vector form of the production function in logarithm is the following:

$$\begin{aligned}
\ln Y_t &= \boldsymbol{\beta}' \ln(1 - \boldsymbol{\eta}) + \boldsymbol{\beta}' \mathbf{y}_t, \\
\Rightarrow \ln Y_t &= \underbrace{\boldsymbol{\beta}' \ln(1 - \boldsymbol{\eta}) + \boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} (\mathbf{a}_t + \mathbf{c}_{yt})}_{\text{denote these terms as } \ln \tilde{A}_t} + \underbrace{\boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} \boldsymbol{\delta}_k}_{\text{denote this as } \tilde{\alpha}} \ln K_t + \boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} \boldsymbol{\delta}_h \ln h_t.
\end{aligned} \tag{B.2}$$

Note that  $\tilde{\alpha} = \boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} \boldsymbol{\delta}_k$  is a constant, not a vector. And  $\ln \tilde{A}_t$  is a single time-varying variable

Since the vector sum,  $\boldsymbol{\delta}_h + \boldsymbol{\delta}_k = \mathbf{1} - \mathbf{m}$ , we have the sum:

$$\boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} \boldsymbol{\delta}_k + \boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} \boldsymbol{\delta}_h = 1,$$

be a constant of 1 (not vector). Thus, Eq (B.2) can be written as

$$\ln Y_t = \ln \tilde{A}_t + \tilde{\alpha} \ln K_t + (1 - \tilde{\alpha}) \ln h_t,$$

and from Proposition 2,  $\boldsymbol{\beta}' [\mathbf{I}_N - \boldsymbol{\Gamma}']^{-1} = \mathbf{v}'$ . This completes the proof.

## C Strategy to compute the Steady State

$9+8N+N^2$  equations for  $9+8N+N^2$  unknowns (additionally, there are also 2 N shocks), a messy nonlinear system to solve for steady states.

1. From 3N equations on financial contracts, given parameters  $\mu_j, \sigma_j$ , solve for  $R_j^k, \omega_j, L_j, \rho_j(\omega_j)$  at SS. From  $R_j^k = (1 - m_j)\alpha_j p_j Y_j / Q k_j + (1 - \delta)$ . Define  $c_{kj} = Q(R_j^k - (1 - \delta)) = \alpha_j p_j Y_j / k_j$ . At this stage, we know values of  $c_{kj}$ . So  $k_j = (1 - m_j)\alpha_j p_j Y_j / c_{kj}$ .
2.  $Y_{jt} = X_{jt} + \sum_{i=1}^N M_{jit}$ . Define intermediate and final use variables  $\eta_{ji}$  for sector  $j$ , so that  $M_{ji} = \eta_{ji} Y_j$ ,  $X_j = (1 - \eta_j) Y_j$ , where  $\sum_{i=1}^N \eta_{ji} = \eta_j$ . Define influence vector  $v_j = \frac{p_j Y_j}{Y}$ . Then

$$v = [I_N - \Gamma]^{-1} \beta \quad (C.1)$$

$$\eta = [\frac{1}{v} * v'] \circ \Gamma \quad (C.2)$$

3. Solve for  $x_k, x_l$ . Let  $sum_{xk} = \sum_j \frac{(1-m_j)\alpha_j v_j}{c_{kj}}$ , and  $sum_{xl} = \sum_j (1 - m_j)(1 - \alpha_j)v_j$ ,

$$x_{kj} = \frac{\frac{(1-m_j)\alpha_j v_j}{c_{kj}}}{sum_{xk}} \quad (C.3)$$

$$x_{lj} = \frac{(1 - m_j)(1 - \alpha_j)v_j}{sum_{xl}} \quad (C.4)$$

4. Combine  $p_j Y_j = v_j Y$  and  $k_j$  from step 1, we get  $k_j = \frac{(1-m_j)\alpha_j v_j Y}{c_{kj}}$ , so

$$\sum_j k_j = K = [\frac{\sum_j (1 - m_j)\alpha_j v_j}{\sum_j c_{kj}}] Y = \varphi_k Y \quad (C.5)$$

and similarly,

$$h = \frac{\sum_j (1 - m_j)(1 - \alpha_j)v_j}{w} Y = \varphi_h Y \quad (C.6)$$

so  $\ln K = \ln \varphi_k + \ln Y$ , and  $\ln h = \ln \varphi_h + \ln Y$ . At this stage,  $\varphi_k$  is known, while  $\varphi_h$  depends on wage  $w$ .

5. Solve for wage. Define  $\delta_k = (1 - m) \circ \alpha$ , and  $\delta_h = (1 - m) \circ (1 - \alpha)$ , take log on sectoral production technology, and write in vector form, we get

$$y = [I_N - \Gamma']^{-1} (a + c_y + \delta_k \ln K + \delta_h \ln h) \quad (C.7)$$

$$= [I_N - \Gamma']^{-1} (a + c_y + \delta_k \ln \varphi_k + \delta_h \ln \varphi_h) + 1_{N \times 1} \ln Y \quad (C.8)$$

, where  $c_{yj} = (1 - m_j)\alpha_j \ln x_{kj} + (1 - m_j)(1 - \alpha_j) \ln x_{lj} + \sum_i \gamma_{ij} \ln \eta_{ij}$ .

Take log on  $Y = \prod (1 - \eta_j)^{\beta_j} Y_j^{\beta_j} \implies \ln Y = \beta' \ln(1 - \eta) + \beta' y$ , plug in y vector, and since  $\beta' 1_{N \times 1} = 1$ ,  $\ln Y$  on both sides cancel out, we get

$$\beta' \ln(1 - \eta) + \beta' [[I_N - \Gamma']^{-1} (a + c_y + \delta_k \ln \varphi_k + \delta_h \ln \varphi_h)] = 0 \quad (C.9)$$

, and wage is the only unknown in the above equation. This solves wage.

6. Solve for output  $Y$ . From the resource constraint and the Euler equation,

$$Y = C + I + \sum_{j=1}^N \mu_j G_j R_j^K Q k_j = (\varphi_h Y)^{-1/\epsilon\sigma} w^{1/\sigma} + \varphi_k Y \delta^{1/\theta} + \left( \sum_j \mu_j G_j R_j^K Q x_{kj} \right) \varphi_k Y$$

$$\Rightarrow [1 - \varphi_k \delta^{1/\theta} - \left( \sum_j \mu_j G_j R_j^K x_{kj} \right) Q \varphi_k] Y = (\varphi_h)^{-1/\epsilon\sigma} w^{1/\sigma} Y^{-1/\epsilon\sigma}$$

rewrite as  $AY = BY^{-1/\epsilon\sigma}$

$$Y = \left( \frac{B}{A} \right)^{\frac{\epsilon\sigma}{\epsilon\sigma+1}}$$

Then other variables can be solved easily.

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