Waiver Exam - Sample questions

1. Mister Tolstoy consumes bread (good 1) and pork (good 2) only and his utility function is $u(x_1, x_2) = \ln x_1 + 2 \ln x_2$. The price of bread is \$1 per kilogram and price of the pork is \$3 per kilogram. Mr. Tolstoy's income is \$27 per month.

   (a) Draw Mr Tolstoy's budget constraint. Find his monthly consumption of bread and pork.

   Suppose now that, due to the new economic plan implemented by the government, the production of pork has decreased rapidly. As an effect, the demand for pork became much greater than supply. The benevolent government has decided not to increase the price of pork but instead some rationing was introduced. Every citizen was given freely tickets which allowed him or her to buy 3 kilos of pork monthly. It was impossible to buy pork without tickets. It was strongly forbidden to sell or buy tickets, so unused tickets were worthless.

   (b) Draw Mr Tolstoy's budget constraint under new circumstances. Find his monthly consumption of bread and pork. Is he worse or better off than before?

   Suppose now that the law was somehow relaxed and it became possible to buy and sell (semi-legally) tickets for pork for the price \$1 per tickets which allowed to buy 1kg of pork. So, if consumer do not use some tickets, he/she can sell them.

   (c) Draw Mr Tolstoy's budget constraint under new circumstances. Find his monthly consumption of bread and pork. Is he selling or buying tickets for pork on secondary market? Is he worse or better off than in (b), in (a)?

   (d) Repeat the above analysis for Mrs Khudaya who's utility function is $u(x_1, x_2) = \ln x_1 + (1/2) \ln x_2$ and her monthly income is \$18.

2. A consumer has preferences over the single good x and other goods y represented by the utility function $u(x, y) = \ln(x) + y$. Let the price of x be p, the price of y be unity and let income be $m > 1$.

   (a) Derive the Marshallian demand for x and y.

   (b) Derive the indirect utility function.

   (c) Use the Slutsky equation to decompose the effect of an-own price change on the demand for x into an income and substitution effect.

   (d) Suppose that the price of x rises from p^0 to $p^1 > p^0$. Find CV, EV and change in the consumer surplus connected with price increase.

   (e) Illustrate your findings with two diagrams, one giving indifference curves and budget constraints, other giving Hicksian and Marshallian demands.

3. Suppose that an individual has a VNM utility function $u(x) = x^{1/2}$.

   (a) Calculate the Arrow-Pratt coefficients of absolute and relative risk aversion at the level of wealth w.

   (b) Calculate the risk premium for a gamble $(0.5 \circ 16, 0.5 \circ 4)$.

   (c) Calculate the risk premium for a gamble $(1/2 \circ 36, 1/2 \circ 16)$. Compare this result with the one in (b) and interpret.

4. A firm can produce a homogenous good y by using two technologies. First technology uses labor, L and capital K and is given by $y = \min\{K, L^{1/2}\}$. Second technology uses only labor and is given by $y = 0.5\sqrt{L}$. We normalize price of K to 1, price of L is w. The firm can use both technologies in any proportion.

   (a) In the short-run the amount of capital is fixed at the level 100 (Note that in in this case not more than 100 units of output can be produced by using first technology). In which proportions both technologies should be used to produce 50 units of output in a cheapest way? How this proportions depend on w?

   (b) The same question for 500 units of output.

   (c) In the long-run both factors are variable. Redo (a) and (b).

5. In some competitive industry the long-run total cost for every firm is given by $C(q) = A + q^2$ for $q > 0$ and $C(0) = 0$ where A is an annual licensing fee imposed by the government. Initially $A = 100$ and the industry is in the long-run competitive equilibrium with 100 firms.

   (a) Derive the supply functions for the firm and for the industry.

   (b) Find the equilibrium price, each firm's output and the industry output

   Now the demand rises, and new firms prepare to enter. The existing firms, however, influence the government to impose a "congestion surcharge" on the licensing fee of the following form – *a new firm entering the industry will be assessed a fee which is equal to the number of firms which are in the industry (including the new firm)*. Thus the 101-th firm pays 101, the 120-th firm pays 120, and in general the n-th firm pays n. These fees then become fixed for each firm for all succeeding years. The original firms continue to pay fees of 100 each.

   For the following questions ignore the discontinuities in the number of firms.

   (c) Suppose that the industry is in the long-run equilibrium with $n_0 > 100$ firms. What is the equilibrium price?

   (d) At any price higher than equilibrium price from (b), what is the long-run equilibrium number of firms in the industry?

   (e) Find the long-run industry supply curve.

6. A monopolistic firm with total costs TC $= 20q + 100$ has only two consumers buying the product. Consumer 1 has demand $q_1 = 20 - 0.25p$ and Consumer 2 has demand $q_2 = 60 - 0.75p$

   (a) Assume that the firm can implement third-degree price discrimination. Find optimal prices for both consumers, what is a difference between them?

   (b) Assume now that the firm is able to use first-degree price discrimination. Propose the first-degree price discrimination scheme for which the profits of the firm are maximized. How much every consumer buys? What are the profits of the firm?

   (c) Assume now that the firm is allowed to charge unique two- part tariff which is available to both consumers. Which kind of price discrimination is it? Find this two-part tariff. Compare marginal price in this two-part tariff with marginal costs of the firm.

   (d) Which of pricing schemes analyzed above is mostly preferred by consumer 1, consumer 2, the firm?

7. There are only two consumers and two goods in exchange economy. Consumer A has indirect utility function $v_A(p_1, p_2, m) = 2m/(2p_1 + p_2)$. Consumer B has expenditure function $e_B(p_1, p_2, \bar{u}) = \bar{u}\sqrt{p_1 p_2}$. Initially consumer A has a units of good 1 and consumer B has b units of good 2. Find competitive equilibrium of this economy. Under which condition it will exist? How much of every good will be traded in equilibrium?

8. There are two consumers, 1 and 2, in the economy with two consumption goods: guns, g and butter, b. Both consumers have the same utility function $u(g, b) = gb$. Guns and butter are produced by two firms which use only labor according to the production functions

$$g = \sqrt{l_g} \qquad \text{and} \qquad b = 0.5\sqrt{l_b}$$

   Both firms are owned by consumer 1, consumer 2 owns 200 units of labor.

   (a) Find the production possibility frontier for this economy. Illustrate.

   (b) Find competitive equilibrium.

   (c) Find competitive equilibrium if every consumer owns 100 units of labor and owns one firm.

   (d) How the change in ownership structure affects: 1) equilibrium prices, 2) equilibrium allocations?

   (e) Find the Pareto efficient allocations for this economy.

9. Consider the problem of determining by majority voting a tax level for wealth redistribution. Suppose there is an odd number of agents. Each agent has a level of wealth $w_i > 0$ and an increasing utility function over wealth levels. The mean wealth is $\overline{w}$ and the median wealth is w^*.

(a) Describe how the sign of the difference between $\overline{w}$ and w^* affects the distribution of wealth.

(b) Consider first a proportional tax rate $t \in [0,1]$ identical across agents. The set of alternatives for every agent is $[0,1]$, the set of possible levels of the tax rate. Tax revenues are redistributed uniformly across agents, the after-tax wealth of agent i is $(1-t)w_i + t\overline{w}$. Show that the preferences over $[0,1]$ of all agents are single peaked. Find the Condorcet winner t_c. How does it depend on $\overline{w}$ and w^*? Interpret.

(c) Suppose now that the taxation gives rise to a deadweight loss. Being very crude about it, suppose that a tax rate $t \in [0,1]$ decreases the pretax level of agent i's wealth to $(1-t)w_i$. Thus, the average tax receipts are $t(1-t)\overline{w}$ and the ex-post wealth level of agent i is $(1-t)^2 w_i + t(1-t)\overline{w}$. Show that preferences on wealth levels are again single peaked. Show then that $t_c \leq 1/2$. How does t_c depend on $\overline{w}$ and w^*? Compare with (b) and interpret.

10. Consider a public good problem with 2 agents. Agent i has utility function $a_i \ln(g_1+g_2)+x_i$ where g_i is the amount of the public good contributed by agent i and x_i is the amount of private consumption of agent i. The budget constraint of agent i is $x_i + g_i = w_i$ where w_i is the initial endowment of agent i. To make things simple, assume that $w_i > a_i$ for each agent.

    (a) Compute the Pareto-optimal level of public good provision for this economy.

    (b) Describe the voluntary contribution equilibria for different values of a_1 and a_2. Compare with (a).

    (c) Consider now a sequential game where agent 1 "leads" by contributing an amount g_1 and agent 2 "follows" knowing the amount of agent 1's contribution. Describe the subgame perfect Nash equilibria for different values of a_1 and a_2.

11. There are two risk-neutral workers and two jobs. One job offers wage w_1 and the other wage $w_2 < w_1$. Worker A first decides which job to apply to. Then worker B observes worker A's choice and makes his decision. If there is one applicant for a job, he is employed and receives the associated wage. If both workers apply to same job, then the firm randomly chooses between the two applicants, and he other is unemployed at wage 0.

    (a) Draw the extensive form of this game.

    (b) Write down the normal form and find all the pure strategy Nash-equilibria.

    (c) Now find the subgame perfect Nash equilibria. Explain carefully why some of the NE are not subgame perfect.

    (d) Now assume that the Worker B does not observe A's choice. Write down the extensive the normal forms of the game, and find all Nash Equilibria (including mixed strategy NE)

12. Consider a repeated Bertrand duopoly game (with demand $q = D(p)$ and costs $C(q) = cq$) in which firm j's strategy specifies what price p_{jt} it will charge in each period t as a function of the history of all past price choices by the two firms.

    (a) Find subgame perfect Nash equilibrium of the game in which firms compete a finite number of times T.

    (b) Assume now that game is played indefinitely. Consider the following trigger (or reversion strategy):

    - play the monopoly price p^m in period 1.
    - In each period $t > 1$ play p^m if in every previous period both firms have charged price p^m; otherwise charge price equal to the marginal cost c.

    Find the values of discount factor δ for which the strategies described above constitute a subgame perfect Nash equilibrium of the infinitely repeated Bertrand duopoly game. What are the profits of both firms when the above strategies are played? How δ depends on demand function and on costs?

    (c) Let p^* be any price from the interval $[c, p^m)$. Consider the following strategy.

    - play p^* in period 1.
    - In each period $t > 1$ play p^* if in every previous period both firms have charged price p^*; otherwise charge price equal to the marginal cost c.

    Find the values of discount factor δ for which the strategies described above constitute a subgame perfect Nash equilibrium of the infinitely repeated Bertrand duopoly game. What are the profits of both firms when the above strategies are played? How δ depends on demand function, price p^* and costs?

    (d) Suppose now that there are $n > 2$ firms playing Bertrand game. Repeat analysis from (a)-(c). In particular, how the values of discount factor δ in (b) and (c) depend on n?

13. Consider the following variant of the Battle of the Sexes: Bruce (player 1) is unsure whether Sheila (player 2) prefers to go out with him or prefers to avoid her, while Sheila knows Bruce's preferences — he definitely wants to go out with her. Specifically, suppose Bruce thinks that with probability 1/2 the game is

		Sheila L	Sheila R
Bruce	U	2,1	0,0
Bruce	D	0,0	1,2

and with probability 1/2 it is

		Sheila L	Sheila R
Bruce	U	2,0	0,2
Bruce	D	0,1	1,0

Sheila knows which game is being played.

(a) Model this as a Bayesian game; that is write down the action sets, type sets, prior beliefs and conditional utilities. What constitutes a strategy for Bruce, for Sheila?

(b) Construct a strategic form for this game and find all the pure-strategy Bayesian Nash equilibria.

14. Your friend, Mr. Hu, is the only seller of chinese nuts in the CEU. It was discovered recently that chinese nuts are good for brain work, so they become quite popular in the CEU. Mr. Hu has two kinds of customers, students and professors. If a package of q kilos of nuts is offered for price \$t, the utility of typical student and professor is

$$U_{st} = (10q - q^2) - t, \qquad \text{and} \qquad U_{pr} = 2(10q - q^2) - t,$$

respectively. The reservation utility level for both types is zero. The cost of one kilo of nuts for Mr. Hu is \$2.

After successfully passing MI1 course, you advised Mr. Hu that he can create packages aimed exclusively for students and for professors (he can easily identify students by asking for student card). Actually you said him which packages should be offered for student and which for professor.

(a) Find optimal offer of Mr. Hu's for students and for professors.

After a lot of complaints from professors (especially from Econ Dept), Mr. Hu is not allowed by the CEU to sell package only to given group, so he offers both packages from (a) to everybody

(b) What package professors buy in this case? Is Mr. Hu better off than in (a)?

After taking MI2 course you advised Mr. Hu that he can offer two different packages available for everybody and increase his profits compared to (b). Actually, Mr. Hu knows that the chance that the buyer is a professor is equal to 1/4, and the chance that the buyer is a student is equal to 3/4.

(c) Formulate Mr. Hu's problem under new circumstances. Explain which constraints are binding and which can be neglected (you do not have to prove that). Solve this problem. Illustrate. Compare with (a). Which group is better off compared with (a)?

(d) After complaints from students, the CEU allowed a new vendor of chinese nuts, Mr. Lu, to operate here (his costs are the same as Mr. Hu). Actually, advised by your colleague, he is selling a package of 4 kilos for \$23. Are students to switch to him, what about professors?

(e) As a result, a harsh price war emerged between Mr. Hu and Mr. Lu. As a microtheorist you would like to know if this war will end with some kind of equilibrium. Clearly the competition is Bertrand type but there are two markets: one for students and another for professors. Can you predict which packages and for which price will be offered for both groups in equilibrium?

15. Consider a following moral hazard problem. A worker can exert two effort levels e. Good effort level ($e = 2$) induces a production error with probability 0.25. Bad effort level ($e = 0$) induces a production error with probability 0.75. Worker's utility function is $U(w, e) = \sqrt{w} - e$ where w is his wage rate. Production errors are observable, hence can be introduced into the worker's contract, but effort cannot. The product obtained is worth 20 if there are no errors and 0 otherwise. The manager is risk-neutral and has utility function $u(x, w) = x - w$ where x is the value of the product. Assume that the worker has reservation utility equal to $U_0 = 1$. Calculate the optimal contract and the effort the manager desires both under conditions of symmetric information and asymmetric information on the worker's behavior. Show all steps of your solution.