Assessing the Impact of Central Bank Digital Currency on Private Banks

David Andolfatto*

*Federal Reserve Bank of St. Louis

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Abstract

In this paper, I investigate the impact of central bank digital currency (CBDC) on banks in a model where the banking sector that is not perfectly competitive. The theoretical framework combines the Diamond (1965) model of government debt with the Klein (1971) and Monti (1972) model of a monopoly bank. There are two main results. First, the introduction of interest-bearing CBDC increases financial inclusion and diminishes the demand for cash. Second, the introduction of interest-bearing CBDC need not disintermediate banks in any way and may, in fact, expand their depositor base if the added competition compels banks to raise their deposit rates.

1 Introduction

The recent surge of interest in cryptocurrencies has resurrected an old debate concerning the pros and cons of having a basic payment service operate through a public utility like the post office or perhaps even a central bank. In fact, postal savings systems have played a prominent role in many countries in the past and continue to do so in many countries to this day. The United

*The views expressed here are my own and should not be attributed to the Federal Reserve Bank of St. Louis or the Federal Reserve System. I thank Andrew Spewak for his assistance.
States has its own history in this regard. In particular, the U.S. Postal Savings System operated in the United States from 1911-67, apparently with considerable success, despite some severe legal restrictions on its business practices. Its main attractions were first, government-insured accounts, and second, widespread accessibility.¹

While postal banking is being taken seriously in the United States today,² its historical advantage in terms of ubiquitous locations seems increasingly irrelevant for the post-Internet world. Moreover, post offices—unlike central banks—are not typically charged with the authority to create currency. Because this is so, the other historical advantage of postal savings—government-insured accounts—is entirely unnecessary for central bank accounts. For these (and other) reasons, there is a growing interest in the concept of central bank digital currency (CBDC).³

It is important to be clear what one means by CBDC.⁴ After all, central banks already issue digital money in the form of reserves. In most jurisdictions, however, access to reserve accounts is restricted to depository institutions. The non-bank public is permitted to hold central bank liabilities only in the form of physical cash. In what follows, I define CBDC as central bank accounts accessible by the public in the same way deposit accounts are today. In the context of the United States, think of a technology that merges Fedwire with Treasury Direct. The latter entity permits any U.S. person to open an online account with the U.S. Treasury where interest-bearing treasury securities can be bought and sold (and in some cases transferred). The former entity offers U.S. depository institutions a real-time gross settlement (RTGS) system that processes close to $3 trillion worth of payments every day at negligible cost.

¹Government-insured accounts were particularly attractive in light of the general distrust of banks following the Panic of 1907 and prior to the establishment of the Federal Deposit Insurance Corporation. The U.S. postal savings system crested during the Great Depression. In 1934, it had $1.2 billion in assets—about 10% of the commercial banking system. Because postal savings banks (unlike commercial banks) faced ceilings on the deposit rates they could offer, the introduction of federal deposit insurance effectively spelled its demise.

²In particular, it is likely to form part of the Democratic Party’s economic agenda during the 2020 presidential campaign (Wack, 2018).


⁴See Berentsen and Schar (2018).
Assuming that a RTGS system is (or could also be made) available to private banks as well, what are the implications of a competing CBDC? The range of views on the matter are predictably quite mixed. Bordo and Levin (2017) cite a number of ways in which CBDC might improve the conduct of monetary policy. Ricks, Crawford and Menand (2018) argue that CBDC will improve financial inclusion and reduce the government’s implicit subsidy to banks. Moreover, because CBDC resembles narrow banking in some regard, they view it as likely to promote financial stability. Other commentators, however, are more sceptical of the proposal. Cecchetti and Schoenholtz (2017), for example, doubt the ability of a central bank to function efficiently at the retail level and doubt the potential for cost savings, given that the same technology is available to the private sector. The worry that CBDC competition will whittle away at the franchise value of banking, thereby distorting risk-taking incentives and otherwise increase the cost of funding. Their main concern, however, is that CBDC is likely promote financial instability. Since CBDC provides non-bank investors with an easily-accessible “flight-to-safety” vehicle, any hint of weakness in financial markets is evidently more likely to result in a self-fulfilling bank panic (Diamond and Dybvig, 1983).

It is of some interest, I think, to evaluate the various claims made about CBDC through the lens of different macroeconomic models. I am aware of only two such analyses at the present time. The first is by Barrdear and Kumhof (2016), who develop a rich dynamic general equilibrium model to study the macroeconomic effects of CBDC. In an experiment where a CBDC is introduced through the purchase of government bonds, lending and deposit rates decline, but by less than the decline in the policy rate. The lower real policy rate has the effect of stimulating macroeconomic activity and with it bank lending volumes and deposit levels, suggesting that if CBDC is introduced in this particular way, it could actually benefit incumbent banks.

The second theoretical analysis I am aware of is by Keister and Sanches (2018), who develop an analytically tractable dynamic general equilibrium model to study some of the macroeconomic implications of CBDC. A critical property of their model is a credit constraint that prevents banks from financing an efficient level of investment. An interest-bearing CBDC has two countervailing effects. On the one hand, it promotes efficiency in exchange because it lowers the opportunity cost of holding money, thereby increasing the demand for real money balances, which is generally too low from a social
perspective. On the other hand, it increases the funding costs of financially-constrained banks, thereby reducing the level of investment, which is already too low from a social perspective.

In this paper, I investigate the impact of CBDC on banks in a model where the banking sector is not perfectly competitive. The theoretical framework combines the Diamond (1965) model of government debt with the Klein (1971) and Monti (1972) model of a monopoly bank. The assumption of market power is arguably realistic in the present context, and it turns out to be important when investigating the likely impact of CBDC on the banking sector. I assume that both CDBC and bank deposits employ the same RTGS infrastructure and charge non-interest fees to cover costs. I also make a distinction between the interest paid on CBDC and the interest paid on reserves. The main results are twofold. First, an interest-bearing CBDC generally promotes financial inclusion (diminishes the demand for cash). Second, the introduction of CBDC need not disintermediate banks in any way—certainly not in their lending activities—and may, in fact, expand their depositor base if CBDC compels banks to raise their deposit rates.

The paper is organized as follows. In Section 2, I describe the physical structure of the economy—individual preferences, demographics, and available technologies. I describe the structure of monetary and fiscal policy in Section 3 and market structure and the timing of events in Section 4. Section 5 describes the mathematical restrictions that characterize optimal individual choices, including monopoly pricing by the bank sector. I characterize the stationary equilibrium and describe its properties in Section 6. Section 7 investigates the economic consequences of CBDC. Section 8 provides a discussion of how the model can be used to evaluate various claims made in commentaries concerning the likely costs and benefits of CBDC. Section 9 concludes.

2 The environment

I employ a version of the overlapping generations model developed by Diamond (1965). Let \( t = 1, 2, ..., \infty \) denote time. At any given date there is measure \( 2N \) each of young and old individuals. Young individuals live for two periods and there exists an initial old generation at \( t = 1 \) that lives for one period only. Individuals are divided evenly into two classes: workers and
firms. Hence, at any given date there are four broad groups of individuals, each of measure $N$, consisting of young and old workers, and young and old firms. In what follows, I normalize $N = 1$.

For simplicity, assume that all individuals have linear preferences defined over consumption when old.\(^5\) Let $(w_t, \hat{w}_t)$ denote the consumption (wealth) of a worker and firm, respectively, at date $t$. The young at date $t$ seek to maximize $w_{t+1} + \hat{w}_{t+1}$.

Young workers are endowed with a unit of time that produces $y$ units of output, either in the form of a consumption good or investment good. Young workers have heterogeneous levels of ability, indexed by $y > 0$, a parameter that is distributed across the population according to an exogenous cumulative distribution function $G(a) \equiv \Pr[y \leq a]$.

Young firms are endowed with a investment project that takes $k_t$ units of output at date $t$ and transforms it into $F(k_t)$ units of output at date $t + 1$. Assume that $F'' < 0 < F'$ where $F$ satisfies the Inada conditions.\(^6\) Firms are all identical.

The pattern of welfare-improving trade here is clear: young firms want young labor in exchange for the future output that can be produced by their joint effort. If parameters are such that the economy is dynamically inefficient, intergenerational transfers of resources from young workers to old workers may also be desirable.

### 3 Government policy

The government makes monetary transfers $Z_t$, collects tax revenues $T_t$, but makes no purchases. The primary deficit is therefore given by by $Z_t - T_t$. The total deficit (the primary deficit plus interest expense) is financed entirely with one-period, risk-free, nominal treasury debt. Let $D_{t-1}$ denote the stock

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\(^5\)This has the effect of rendering the consumption-saving choice a trivial matter: the young will choose to save all their income. Modeling the consumption-saving choice is possible but would not affect the qualitative nature of the results reported below.

\(^6\)The main difference with Diamond (1965) is the assumption that output is produced with two different technologies, instead of a single neoclassical production function that takes both capital and labor as inputs. In Diamond’s analysis, the real wage for workers is determined jointly with past investment decisions. In my set-up, the real wage for workers is exogenous.
of nominal government debt outstanding at date \( t - 1 \) scheduled to mature (turn into money) at date \( t \).

Government debt consists of three components, \( D_t = C_t + M_t + B_t \), where \( C_t \) represents physical cash (currency in circulation), \( M_t \) represents interest-bearing CBDC, and \( B_t \) represents digital, interest-bearing bonds. Let \( R_t^M \) and \( R_t^B \) denote the gross nominal interest rate earned on CBDC and bonds, respectively, over the time interval \( t \) to \( t + 1 \). I assume that cash earns zero interest, i.e., \( R_t^C = 1 \) for all \( t \). The government’s flow budget constraint is given by,

\[
Z_t + (R_{t-1}^B - 1)B_{t-1} + (R_{t-1}^M - 1)M_{t-1} = T_t + (D_t - D_{t-1}) \tag{1}
\]

for all \( t \geq 1 \) with \( D_0 > 0 \) in the hands of the initial old.

Assume that government transfers and taxes are lump-sum. In what follows, I assume that workers are the recipients of the transfer income \( Z_t \) and that firms pay the taxes \( T_t \), though this is not critical to the main results below.

There are several ways in which policy might be configured. I assume the following structure. First, tax revenue \( T_t \) is used solely to service the debt, so that

\[
T_t = (R_{t-1}^B - 1)B_{t-1} + (R_{t-1}^M - 1)M_{t-1} \tag{2}
\]

Second, all transfers \( Z_t \geq 0 \) are financed through new debt-issuance, so that

\[
Z_t = D_t - D_{t-1} \tag{3}
\]

This specification of policy will imply that inflation can only be the consequence of government spending (specifically, spending on transfers financed by new nominal debt). In particular, interest rate policy will have no (long-run) consequences on inflation.\(^7\) In what follows, I assume that \( Z_t = (\mu - 1)D_{t-1} \), so that \( D_t = \mu D_{t-1} \), with \( D_0 > 0 \) endowed to the initial generation.

While I view the central bank and treasury as a consolidated entity in what follows, I retain the convention of labeling the choice of \( R_t^M \) and \( R_t^B \) as monetary policy, and the choice of \( T_t \), \( Z_t \) as fiscal policy.

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\(^7\) It would be easy and perhaps of some interest to experiment with other policy configurations, but I leave this to the interested reader.
4 Market structure and timing

In reality, most individuals do not hold government debt directly, except in the form of physical cash. And so it will be in this economy. I assume that the digital, interest-bearing component of debt is held entirely by intermediaries. Because banks (both central and private) are the only intermediaries in this model economy, they will end up holding the entire interest-bearing component of government debt. In what follows, think of $B_{t-1}$ as bank reserves (interest-bearing accounts that private banks hold with the central bank) and think of $M_{t-1}$ as CBDC (interest-bearing accounts that non-bank entities hold with the central bank).

Workers—the natural savers in this economy—will want to accumulate securities in exchange for their labor. If they could, firms would want to issue private securities (claims to future inventory) to pay for the labor they need. I assume that workers will not accept firm-issued coupons as payment of labor. Workers will, however, accept both central and private bank deposit liabilities.

Banks, both central and private, are financial intermediaries that transform illiquid securities into liquid payment instruments. The central bank transforms illiquid government debt into reserves for private banks and CBDC for workers and firms. Private banks transform illiquid government debt and private securities (firm-issued coupons) into deposit liabilities. I assume that all deposit liabilities are made redeemable for cash on demand and at par. The rationale for this contractual stipulation in the present context is that not all workers will end up having bank accounts (this will be endogenous). Those workers that do may be paid by deposit transfer, those that do not need to be paid in cash.\footnote{I want to stress that par redemption is an assumption here. I have not explored the circumstances under which cash may trade at a premium (or discount) relative to deposit money.}

Individuals wanting access to public/private banking services need to pay a one-time fixed cost of setting up the necessary accounts. Let $\phi > 0$ denote the cost accessing the banking system. For simplicity, I assume that $\phi$ takes the form of a “utility cost” representing spent time. Once they have bank access, individuals can borrow money at the lending rate $R^L_t$ or save money at the deposit rate $R^D_t$. If CBDC is available, they have the option of saving
money at the CBDC deposit rate $R_M^t$. Unbanked individuals must resort to making and receiving payments in cash. I also assume a utility cost of carrying cash that is proportional to the level of real cash balances carried from one period to the next. This will have the effect of discouraging people from holding large cash balances at very low (zero or less) nominal rates of interest.

The timing of events is as follows. Old workers enter period $t$ with the money (in the form of cash or deposits) they worked for and saved in the previous period. Old banked workers receive interest on their deposits. All old workers receive a money transfer $Z_t$ (in the form of a check for the unbanked and by direct deposit for those with bank accounts). Old workers spend all their money on goods and services.

Old firms enter period $t$ with bank debt, which they repay (interest and principal) at the end of the period. Firms use their physical capital (the previous period’s investment) to produce goods and services, a part of which they consume (profits), and the remainder which they sell for money. The money they acquire from sales is used to repay their bank loans and pay taxes $T_t$. Private banks use their monetary profit to purchase output for consumption.

Young firms enter period $t$ with an investment project in need of financing. Banks are the only source of financing and set the lending rate $R_L^t$. Firms pay the cost $\phi$ and open a bank account. The bank lends the firm money—which it creates *ex nihilo*—and credits the firm’s account by the amount of the money loan. The firm then spends the money on the good provided by workers in a competitive spot market. Workers without bank accounts are paid in cash (which firms can withdraw on demand and at par from their bank account). Workers with a bank account are paid by deposit transfers.

Young workers enter period $t$ and choose whether to access the banking system or not. Private banks set the deposit rate $R_D^t$ and the central bank sets the CBDC deposit rate $R_M^t$. Workers then sell their goods and services for money in a competitive spot market. Unbanked workers receive cash for their product, banked workers get paid by direct deposit and earn either the deposit rate $R_D^t$ or $R_M^t$, depending on whether they hold their money in private or central bank accounts.
5 Decision making

In what follows, let \( p_t \) denote the price-level at date \( t \) and let \( \Pi_{t+1} \equiv p_{t+1}/p_t \) denote the (gross) rate of inflation.

5.1 Firms

Assume that the cost of opening a bank account is sufficiently small that all firms choose to do so. A young firm chooses \( k_t \) to maximize \( \tilde{w}_{t+1} \) subject to

\[
\tilde{w}_{t+1} = F(k_t) - R_t^L \Pi_{t+1}^{-1} k_t - \tau_{t+1}
\]

(4)

where \( \tau_t \equiv T_t/p_t \). Investment demand \( k_t = k(R_t^L \Pi_{t+1}^{-1}) \) is characterized by,

\[
F'(k_t) = R_t^L \Pi_{t+1}^{-1}
\]

(5)

That is, investment is planned to a scale \( k_t \) such that a marginal addition yields an expected real return just equal to the expected real interest rate (the marginal cost of funding). Since there is a unit measure of young firms, \( k_t \) also represents the real aggregate demand for investment.

**Lemma 1** Investment demand \( k_t = k(R_t^L \Pi_{t+1}^{-1}) \) is decreasing in the inflation-adjusted lending rate \( R_t^L \Pi_{t+1}^{-1} \).

Note that the nominal value of newly created bank loans \( p_t k_t \) is credited to young firms’ bank accounts. Firms do not carry any of this money over to the next period. Instead, they spend it all in acquiring the goods and services \( k_t \) they need from young workers. Young workers who are unbanked are paid with cash. Young workers with bank accounts are paid by way of deposit transfers (young firm accounts debited, young worker accounts credited) within the banking system.

5.2 Workers

Consider next the situation faced by a type \( y \) worker. The parameter \( y \) here can be interpreted as a worker’s skill or human capital. Since workers do not value consumption when young, \( y \) also corresponds to their desired real
saving. Poor workers may not have enough savings to justify the cost of a bank account \( \phi \). These unbanked workers will spend their time accumulating cash. The real rate of return on cash is given by the inverse of the inflation rate, net of the cost of carrying cash. Assume that a worker carrying real cash balances \( y \) from one period to the next expends \( (1 - \theta)y \) utils of effort managing it (keeping it safe, etc.), where \( 0 \leq \theta < 1 \). The payoff for a young unbanked type \( y \) worker is therefore given by,

\[
w_{t+1}^u = \Pi_{t+1}^{-1} \theta y + z_{t+1}
\]

where \( z_t \equiv Z_t/p_t \).

Alternatively, a worker can access the banking system by incurring the utility cost \( \phi \). Doing so permits the worker to earn the interest rate \( R_t^D \) on deposits held in the bank and \( R_t^M \) on deposits held at the central bank. The only difference between bank deposits and CBDC is the rate of return that is offered, so the choice of account is determined solely by which offers a higher interest rate. Let \( R_t \equiv \max\{R_t^M, R_t^D\} \). The payoff for a young banked type \( y \) worker is therefore given by,

\[
w_{t+1}^b = \Pi_{t+1}^{-1} [R_t y - \phi] + z_{t+1}
\]

We can identify a type \( \hat{y} \) worker that is just indifferent between accessing the banking system or not by finding the income level \( \hat{y} \) that equates (6) and (7), i.e.,

\[
\hat{y}(R_t) = \left( \frac{\phi}{R_t - \theta} \right)
\]

High-income workers \( (y \geq \hat{y}) \) will find it desirable to access the banking system, while low-income workers \( (y < \hat{y}) \) find it too costly to do so and will therefore prefer to use cash.

**Lemma 2** The cut off income level \( \hat{y}(R_t) \) is decreasing in the deposit rate \( R_t \) and is independent of the rate of inflation.

In words, a higher deposit rate induces more workers willing to bear the cost of accessing the banking system. A higher service fee \( \phi \) obviously discourages workers from using deposit money. A smaller inconvenience cost of using cash \( 1/\theta \) (a larger \( \theta \)) promotes the use of cash. The inflation rate
does not affect the decision to access the banking system or not because the rate of return on cash and deposit money are both affected in the same way by the rate of inflation.

The aggregate demand for real cash balances in this economy (currency in circulation) is given by,

\[ c(R_t) = \int_{\hat{y}(R_t)}^{\hat{y}(R_t)} y dG(y) \]  

Lemma 3 The demand for real cash balances \( c(R_t) \) is decreasing in the nominal deposit rate \( R_t \) and is independent of the inflation rate.

The demand for real cash balances also depends on the distribution of income \( G \). For example, if distribution \( \hat{G} \) stochastically dominates \( G \), then \( q_t \) is larger under \( \hat{G} \) than \( G \). The fact that the demand for real cash balances does not depend on inflation is sensitive to the fact that desired savings here does not depend on the real rate of interest. If we generalized preferences to allow for a non-trivial consumption-saving decision, then an increase in the expected rate of inflation would lower the real rate of interest faced by depositors and affect the demand for real money balances along an intensive margin. In particular, a lower real rate of interest is likely to result in lower desired saving by all workers, including those who save in the form of cash.

If the fraction \( G(\hat{y}) \) of young workers choose to operate with cash, then the remaining fraction \( 1 - G(\hat{y}) \) will choose to operate with deposit money. The aggregate demand for real deposit balances is given by,

\[ q(R_t) = \int_{\hat{y}(R_t)}^{\infty} y dG(y) \]  

Lemma 4 The demand for real deposit balances \( q(R_t) \) is increasing in the nominal deposit rate \( R_t \) and is independent of the inflation rate.

Define \( y \equiv \int_{0}^{\infty} y dG(y) \), the aggregate supply of real savings in this economy. From (9) and (10), we have

\[ y = c(R_t) + q(R_t) \]  

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That is, the nominal interest rate here simply determines the composition of real money balances between cash and deposits and not the total supply of savings.\(^9\) Whether deposits are held as bank deposits or CBDC depends here only on the relative magnitudes of \(R_t^D\) and \(R_t^M\).

### 5.3 Banks

Let me now consider the behavior of the banking sector, which I model here as a monopoly bank along the lines of Klein (1971) and Monti (1972). At the beginning of period \(t\), the bank has assets consisting of reserves \(B_t\) and loans \(p_t k_t\). It finances its asset portfolio entirely with deposit liabilities \(p_t \hat{q}_t\), so that its balance sheet constraint is given by,\(^10\)

\[
B_t + p_t k_t = p_t \hat{q}_t \tag{12}
\]

For a given interest rate structure, this balance sheet generates an expected profit equal to

\[
V_{t+1} = R_t^B B_t + R_t^L p_t k_t - R_t^D p_t \hat{q}_t \tag{13}
\]

Combining (12) with (13) and making explicit the dependence of \(k_t\) and \(q_t\) on the relevant interest rates, the bank’s objective can be expressed as,

\[
V_{t+1} = [R_t^L - R_t^B] p_t k_t (\Pi_{t+1}^{-1}) + [R_t^B - R_t^D] p_t \hat{q}(R_t^D) \tag{14}
\]

where

\[
\hat{q}(R_t^D) = \begin{cases} 
q(R_t^D) & \text{if } R_t^D \geq R_t^M \\
0 & \text{if } R_t^D < R_t^M
\end{cases} \tag{15}
\]

That is, I assume that if the deposit rate offered by banks weakly exceeds the deposit rate offered by CBDC \((R_t^D \geq R_t^M)\), then workers will hold all of their deposits with banks; i.e., \(\hat{q}(R_t^D) = q(R_t^D)\). If \(R_t^D < R_t^M\), then all deposits \(q(R_t^D)\) are held as CBDC. The bank is assumed to choose a lending rate \(R_t^L\) and a deposit rate \(R_t^D\) to maximize its value (14), taking as given the policy rates \(R_t^B\) and \(R_t^M\) and the behavior of depositors (15).

\(^9\)Again, this is sensitive to the specification of preferences. Endogenizing the saving rate will not alter any of the main conclusions that follow.

\(^{10}\)A meaningful role for equity finance could be introduced along the lines of Dermine (1986).
Suppose that $R_t^M$ is sufficiently low that it can be disregarded by the banking sector. Define $X_t$ as the solution to,

$$[R_t^B - X_t] q'(X_t) = q(X_t)$$  \hspace{1cm} (16)$$

which can be rearranged as,

$$X_t = \left[ \frac{\eta(X_t)}{1 + \eta(X_t)} \right] R_t^B$$  \hspace{1cm} (17)$$

where $\eta(X) \equiv Xq'(X)/q(X)$. Condition (17) characterizes the profit-maximizing deposit rate $R_t^D$ when $X_t > R_t^M$. Recall that by Lemma 4, $q'(R_t^D) > 0$. In words, a marginal increase in the deposit rate reduces profit by the level of deposits $q(R_t^D)$. On the other hand, it induces an increase in demand for deposits by the amount $q'(R_t^D)$ on which the bank earns the profit margin $[R_t^B - R_t^D]$. The optimal deposit rate just balances these two opposing effects.

**Lemma 5** If $X_t > R_t^M$, then the profit-maximizing deposit rate satisfies (17); i.e., $R_t^D = X_t < R_t^B$.

While the relation of the deposit rate $R_t^D$ to the IOR rate $R_t^B$ is not my main concern here, it is worth noting that the model does suggest that policy rate changes are passed through to deposit rates.\footnote{One can demonstrate that if $R_t^D q''(R_t^D) + 2q'(R_t^D) > 0$, then $R_t^D$ is increasing in $R_t^B$.} If the elasticity of deposit demand $\eta(X_t)$ is roughly invariant to the interest rate, then (17) suggests that the deposit rate will be roughly proportional to the IOR rate. It is of more interest to note how the profit-maximizing deposit rate here is unrelated to the choice of the lending rate and the size of the bank’s loan portfolio. This property of the model will be significant in what it implies about the effect of CBDC on bank lending behavior.

Suppose that the CBDC facility offers a yield $R_t^B > R_t^M > X_t$. As is clear from (14), banks have a strong incentive to retain deposits as long as the profit margin $[R_t^B - R_t^D]$ is strictly positive. Therefore, as long as $R_t^M < R_t^B$, banks will always have an incentive to at least match the interest paid on CBDC.

**Lemma 6** If $R_t^B > R_t^M > X_t$, where $X_t$ satisfies (16), then the profit-maximizing deposit rate is given by $R_t^D = R_t^M < R_t^B$.\footnote{One can demonstrate that if $R_t^D q''(R_t^D) + 2q'(R_t^D) > 0$, then $R_t^D$ is increasing in $R_t^B$.}
Lemma 6 raises the interesting possibility that very little activity may actually occur via CBDC if the main consequence of its availability is simply to discipline the deposit rate set by banks. Given (15), \( R^D_t = R^M_t \) implies that no deposits are held as CBDC, so that \( M_t = 0 \). (This, in turn, has potential budgetary consequences via condition 2.)

Note that CBDC need not permit universal access. And, in particular, just as non-banks are presently not permitted to use reserve accounts, imagine that banks are hypothetically not permitted to use CBDC accounts. Let me now investigate the implications of setting \( R^M_t > R^B_t \). In this case, banks would continue to play an important role in the credit market and payments system. That is, they would originate loans, create money, and facilitate payments. They would not, however, want to retain customer deposit accounts because the cost of retention \( R^M_t \) now exceeds the benefit \( R^B_t \). The best that banks can do in this case is let all deposits created on behalf of firms through their lending operations to flow through the payments system into CBDC accounts owned by (banked) workers. This money will, of course, flow back into the banking system when old (banked) workers purchase output from old firms. Money used to repay bank loans is effectively destroyed.

**Lemma 7** If \( R^M_t > R^B_t \), private banks retain zero deposits (all deposits will be held as CBDC) from one period to the next; i.e., \( M_t = p_t q(R^M_t) \).

Again, this situation has budgetary implications via condition (2). I will return to this later, but for now I turn to deposit-creation activity. From (14), we see that the profit-maximizing lending rate \( R^L_t \) satisfies,

\[
- \left[ R^L_t - R^B_t \right] k'(R^L_t \Pi^{-1}_{t+1}) \Pi^{-1}_{t+1} = k(R^L_t \Pi^{-1}_{t+1})
\]

This expression is more conveniently expressed in real terms. Define \( r^L_t = R^L_t \Pi^{-1}_{t+1} \) and \( r^B_t = R^B_t \Pi^{-1}_{t+1} \) and rewrite the expression above as,

\[
- \left[ r^L_t - r^B_t \right] k'(r^L_t) = k(r^L_t)
\]

(18)

In words, a marginal increase in the real lending rate increases profit by the loan level \( k(r^L_t) \). On the other hand, it reduces the demand for loans by the amount \( k'(r^L_t) < 0 \) (by Lemma 1) on which the bank earns the profit margin \( [r^L_t - r^B_t] \). The optimal lending rate just balances these two opposing effects. Note that the nominal lending rate moves in proportion to the expected rate of inflation; i.e., \( R^L_t = r^L_t \Pi_{t+1} \).
Without any significant loss of generality, consider the class of investment return functions that satisfy $F'(k)k = \alpha F(k)$, where $0 < \alpha < 1$. Then condition (18) implies that the profit-maximizing lending rate satisfies a simple markup condition,

$$r_t^L = (1/\alpha)r_t^B$$

Lemma 8 Assume $F'(k)k = \alpha F(k)$, with $0 < \alpha < 1$. Then the profit-maximizing lending rate is given by $R_t^L = (1/\alpha)R_t^B$.

An interesting property of the profit-maximizing lending rate here is how it does not depend on the CBDC interest rate, at least, to the extent that the IOR rate $R_t^B$ is set independently of $R_t^M$. This remains true even if $R_t^M > R_t^B$, the case for which banks retain no deposits.

6 Stationary equilibrium

The purpose of this section is to collect the restrictions on economic behavior derived above and use them to deduce how this economy functions, for a given policy configuration, as it operates into the indefinite future without disturbance. The experiments below will involve examining the long-term consequences of different policy configurations.

In a stationary equilibrium, all real variables, ratios, and rates, remain constant over time.\(^{12}\) Time subscripts are dropped since they are no longer necessary, except for nominal variables not already expressed as rates.

To begin, let me assume a world without CBDC; or, equivalently, that the conditions of Lemma 5 hold. Define the real value of government debt $d_t \equiv D_t/p_t$. By stationarity, $d_t = d$. It therefore follows that the equilibrium rate of inflation in a stationary economy is given by $\Pi = \mu$; i.e., the rate at which the nominal debt grows over time.

\(^{12}\)I am assuming that a stationary state exists, that it is unique, and that it is stable. Existence is generally easy to establish. Uniqueness and stability takes a little more work. A related paper (Andolfatto and Martin, 2018) shows how an appropriately-designed Taylor rule can establish uniqueness and stability in an environment similar to the one being studied here.
Lemma 9  Given that monetary policy pegs nominal rates of interest and
given that taxes are used to finance the interest expense of debt, the inflation
rate is determined by the rate of nominal debt-issuance ($\mu$) used to financed
government transfers.

The equilibrium lending rate continues to be described by Lemma 8 i.e.,
$R_L = (1/\alpha)R^B$. Given a rate of inflation $\mu$, the equilibrium real lending
rate is determined by $r^L = R^L/\mu$. Note that the nominal lending rate here is
independent of the inflation rate, which implies that fiscal policy can influence
the real rate of interest relevant for financing capital expenditures.

Lemma 10  An increase in the inflation target ($\mu$) lowers the real lending
rate and stimulates investment spending. An increase in the IOR rate ($R^B$)
leads to a proportionate increase in the profit-maximizing lending rate ($R^L$)
and lowers investment spending.

The fact that investment increases with inflation follows from condition
(5), which is essentially a Mundell-Tobin effect; see Mundell (1963) and Tobin
(1965). Thus, the model has a mechanism to generate outcomes consistent
with empirical evidence rejecting the Fisher effect (a one-for-one relation be-
tween inflation and the nominal interest rate). The effect of increasing the
policy rate $R^B$ for a given rate of inflation similarly induces a portfolio sub-
stitution effect. That is, if the real rate of return on government debt is made
more attractive, investors substitute out of private capital into government
securities. In the model here, an increase in the IOR rate leads banks to
curtail their lending to firms.

With $r^L$ determined, the equilibrium level of real investment is given by
$k(r^L)$. To finance this level of investment, at each date $t$ young firms need
to borrow $p_t k(r^L)$ dollars. This amount of “inside money” is created by
the banking system in the act of lending. Thus, the total money supply

\footnote{Of course, the model also has a mechanism to generate outcomes consistent with
the Fisher effect (for example, suppose that $R^B = \mu$). On another matter, one could
legitimately question the quantitative impact that inflation has on investment and the
marginal product of capital. On the other hand, “capital” here might alternatively be
replaced by employment in a model where recruiting activities constitutes and investment
expenditure (Andolfatto, 1996). Finally, another mechanism through which inflation may
affect real bond yields is through its effect on liquidity premia; see Fried and Howitt (1983).}
in period $t$ is given by currency in circulation $C_t$ plus total bank deposit liabilities $B_t + p_t k(r^L)$. Note that the monetary base here corresponds to the public debt $D_t = C_t + B_t$. One can also identify the broad money aggregate $M_1 t \equiv C_t + p_t k(r^L)$, though in this model, the economically relevant money supply is given by $D_t + p_t k(r^L)$. Over the course of period $t$, this money is spent on the goods and services available for sale, $y \equiv \int_0^\infty ydG(y)$. Therefore, the market-clearing condition

$$D_t + p_t k(r^L) = p_t y$$  \hspace{1cm} (20)

must hold at each date $t \geq 1$. Since $D_t$ is determined by policy, condition (20) determines the equilibrium price-level.

I want to take some time to study equation (20) as a theory of the price-level. Condition (20) can be expressed in the following manner,

$$p_t = \left[ \frac{1}{y - k(r^L)} \right] D_t$$  \hspace{1cm} (21)

That is, the equilibrium price-level is proportional to the quantity of “outside” money $D_t$ in the economy. The proper interpretation of $D_t$ here, I think, is the level of public debt existing on the balance sheet of the banking sector (including money funds and the central bank). The denominator in (21) represents the demand for real (outside) money balances $d(r^L) = y - k(r^L)$. Thus, the model states that for a given demand for real balances, an increase in the supply of debt monetized by the banking sector and spent (here) on transfers that are subsequently spent on goods and services will put upward pressure on the price-level. The mechanism here is consistent with the statement that an increase in “aggregate demand” puts upward pressure on the price-level.\footnote{Note that the lump-sum money transfers $Z_t$ here are neutral (but not superneutral). This is not a general property of the model and has more to do with the way money is injected into the economy. A lump-sum injection of money to young firms, for example, is non-neutral. This is despite the fact that all prices are flexible.}

Note that equation (21) does not imply that we should observe the price-level moving in proportion to the debt as a matter of empirical observation. The price-level also depends on money demand. Anything affecting the supply of saving ($y$) or the demand for investment ($k$) will have an effect on the price-level here for a given supply of debt, $D_t$.\footnotetext{Note that the lump-sum money transfers $Z_t$ here are neutral (but not superneutral). This is not a general property of the model and has more to do with the way money is injected into the economy. A lump-sum injection of money to young firms, for example, is non-neutral. This is despite the fact that all prices are flexible.}
Consider the effect of increasing the IOR rate $R^B$. For a given inflation target, this increases the real interest rate on government debt $r^B = R^B / \mu$. This change in the policy rate is passed through as a higher real lending rate $r^L$ (Lemma 8), which has the effect of depressing investment demand $k(r^L)$. The flip side of this latter effect is an increase in the demand for real money balances $d(r^L)$, which exerts a disinflationary pressure (the price-level declines). Note that because the deposit rate $R^D$ remains unaffected, the amount of currency in circulation $C_t$ remains unaffected as well. However, the broad money aggregate $M1_t \equiv C_t + p_t k(r^L)$ declines along with the decline in bank lending $p_t k(r^L)$.

The implications of the IOR rate are summarized by condition (16), i.e.,

$$[R^B - R^D] \cdot q'(R^D) = q(R^D)$$

(22)

From (17), a constant elasticity of deposit demand to the deposit rate implies that an increase in the policy rate $R^B$ induces an increase in the profit-maximizing deposit rate $R^D$. From Lemma 4 and equation (11), the effect of an increase in $R^B$ is to reduce the currency-to-deposit ratio. That is, at the margin, more workers are motivated to access the banking system, given the higher rate of return now available on their bank deposits. Banks are motivated to increase the deposit rate because doing so increases their deposit base, which they use to take advantage of the higher IOR rate.

Define $v_{t+1} \equiv V_{t+1}/p_{t+1}$, the real value of bank sector monopoly profit. Using (14), we can write,

$$v(R^B, \mu) = \left[ \frac{R^L - R^B}{\mu} \right] \cdot k \left( \frac{R^L}{\mu} \right) + \left[ \frac{R^B - R^D}{\mu} \right] q(R^D)$$

(23)

where, recall, $R^L$ and $R^D$ depend on the policy parameters $R^B$ and $\mu$. Consider the effect of increasing the IOR rate. By the Envelope Theorem, the effect on bank profit depends on the sign of $b(R^B, \mu) = q(R^D) - k(R^L/\mu)$, where $b_t \equiv B_t/p_t$. Recall that $B_t$ represents the interest-bearing debt held as assets in the banking sector. There is nothing here that prevents this number from being negative. In this latter case, the banking system is borrowing reserves and an increase in the IOR rate has the effect of diminishing bank profit. The opposite is true when the banking system is flush with reserves.

**Lemma 11** An increase in the IOR rate increases/decreases monopoly bank profit if bank reserves are positive/negative.
Finally, one can compute steady-state welfare for young firms and workers (banked and unbanked). Since \( M = 0 \) here, the tax revenue necessary to finance the interest expense of debt using (2) is,

\[
\tau = (R^B - 1)b(R^B, \mu) / \mu
\]  

(24)

Note that the tax (24) may in fact be a transfer if either ( \( R^B < 1 \) and \( b > 0 \) ) or ( \( R^B > 1 \) and \( b < 0 \) ). A negative IOR rate here is possible because of the assumed cost of holding cash over time. In particular, from (8) we see that the nominal interest rate \( R^B - 1 \) is bounded below by \( \theta - 1 \), where \( 0 < \theta < 1 \). With \( \tau \) determined by (24), steady-state welfare for firms is given by,

\[
\hat{w} = (1 - \alpha) F(k) - \tau
\]  

(25)

using (4) and the restrictions \( r^L = F'(k) \) and \( \alpha F(k) = F'(k)k \).

To compute the welfare for workers, we need to first derive the equilibrium transfer that is financed through seigniorage. Manipulating condition (3), we have

\[
z = [1 - 1/\mu]d(r^L)
\]  

(26)

where, recall, \( d(r^L) = y - k(r^L) \), \( r^L \equiv R^L/\mu \), and \( R^L \) is the profit-maximizing lending rate. Using (8), the equilibrium measure of unbanked workers is given by \( G(\hat{y}(R^B)) \), where \( R^D \) is the profit-maximizing deposit rate. Note that the measured of unbanked workers is decreasing in \( R^B \). For an unbanked worker with real savings \( y < \hat{y} \), welfare is given by

\[
w^u(y) = \theta y / \mu + z
\]  

(27)

For a banked worker with real savings \( y \geq \hat{y} \), welfare is given by,

\[
w^b(y) = \left[r^D y - \phi / \mu\right] + z
\]  

(28)

7 Central Bank Digital Currency

What are the consequences of introducing a CBDC that yields interest rate \( R^M \) larger than the profit-maximizing deposit rate offered by banks? There are two cases to consider. The first involves the conditions described in Lemma 6, i.e., \( R^B > R^M \). The second involves the conditions described in Lemma 7, i.e., \( R^M > R^B \).
The apparatus now in place makes evaluating the implications of CBDC relatively straightforward. Consider first the case for which $R^B > R^M$. By condition (15), it is clear that the profit-maximizing deposit rate in this case matches (marginally exceeds) the interest paid on CBDC; i.e., $R^D = R^M$. This must be the case as banks will continue to make the riskless profit margin $[R^B - R^M]$ as long as $R^B > R^M$.

If banks match (slightly exceed) the interest offered on CBDC, the take-up rate for CBDC will remain zero. Since $M_t = 0$ for all $t$, the budgetary consequences are the same as if CBDC did not exist. The effect of CBDC in this case is simply to compels banks to compete more aggressively for deposits. The welfare of very poor workers (27) remains unaffected. However, because the deposit rate is now higher, the number of unbanked workers declines. The welfare of all banked workers (28) is now higher.

How does CBDC affect bank lending activity and the welfare of firms? By Lemma 8, the profit-maximizing lending rate is independent of the interest paid on CBDC. This is because the opportunity cost of bank lending is given by the IOR rate, not the CBDC rate. Thus, CBDC does not lead to bank disintermediation in any dimension. In fact, since the number of unbanked individuals declines, CBDC actually increases the scale of inter-mediation (bank lending remains unaffected, but deposits increase). Bank profits decline because of a lower profit margin on deposits $[R^B - R^M]$.

Condition (21) suggests that the price-level remains invariant to $R^D = R^M$. The increase in the deposit rate alters the composition of the debt between currency and bonds, but this has no price-level consequences. There is, however, an added fiscal burden that here is borne by firms (though, in general, could be borne by the economy more broadly). In particular, the government budget constraint (2) suggests that tax revenue must rise to accommodate the higher interest expense of the debt, both because the interest rate is now higher and also because more of the debt is being held in the form of interest-bearing money rather than cash.

Proposition 1 The introduction of a competitive CBDC in a monopolistic bank sector has no effect on bank lending rates, no effect on bank lending activity, increases the market deposit rate, expands the deposit

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15 Of course, I am assuming that CBDC has the same fixed cost as bank deposits and that there are no subsidies that may encourage CBDC take-up over bank deposits.
base (decreasing the number of unbanked individuals), and reduces bank monopoly profits.

Suppose that the central bank continues to raise the interest rate it pays on CBDC to the point where \( R^M > R^B \). In this case, the return banks earn on deposits at the central bank earn less than the deposit rate they must pay workers to retain their deposits. As it never makes sense for banks to offer a deposit rate higher than the IOR rate, banks in this model willfully let the deposits they create flow to the central bank; again, see condition (15).

Since all deposits are in this case held at the central bank, we have \( M_t = p_t q(R^M) \). From the banking sector’s balance sheet constraint (12), we have \( B_t = -p_t k(r^L) \); that is, the bank is borrowing reserves from the central bank. The way this works is as follows. The banking sector still originates \( p_t k(r^L) \) dollars in loans for young firms, which is deposited in their accounts at the beginning of the period. Throughout the period, firms spend this money by converting it into cash (for unbanked workers) and by converting it into CBDC (for banked workers). The conversion into cash and CBDC is performed using borrowed reserves.

Note that in this case we have \( D_t = C_t + M_t + B_t \), with \( M_t > 0 \) and \( B_t < 0 \). Since \( M_t = p_t q(R^M) \) and \( B_t = -p_t k(r^L) \), we have

\[
D_t = p_t c(R^M) + p_t q(R^M) - p_t k(r^L)
\]

where, by (11), \( c(R^M) + q(R^M) = y \), so that the expression above reduces to the market-clearing condition (20). Thus, increasing the interest rate on CBDC has the effect of reducing the currency-to-deposit ratio, but otherwise leaving the banking sector’s lending operations unaffected—at least, to the extent that banks can freely borrow at the IOR rate.

There is a fiscal implication to consider. From the government budget constraint (2) and the fact that \( M_t = p_t q(R^M) \) and \( B_t = -p_t k(r^L) \), we can derive,

\[
\tau = (1/\mu) \left[ (R^B - 1)(-k (R^L/\mu)) + (R^M - 1)q(R^M) \right] \quad (29)
\]

The difference with (29) in comparison with (24) is that the government now earns interest at rate \( R^B \) on the reserves it lends to banks, which is income that is offset by the interest it now pays at rate \( R^M \) on CBDC.
7.1 Reserve requirement

The fact that $R^M$ has no effect on bank lending activity when $R^M > R^B$ depends on the central bank being willing to lend reserves at the policy rate $R^B$. Suppose that the banking system is not permitted to borrow reserves. This imposes an additional constraint on the bank’s constrained maximization problem; namely, $B_t \geq 0$.

If the constraint $B_t \geq 0$ binds, then the balance sheet constraint (12) implies that the banking system must hold deposits equal to the amount of loans it creates, $p_t k(r^L)$. This, in turn, implies that banks must offer a deposit rate at least equal to the interest paid on CBDC, i.e., $R^D = R^M > R^B$. Bank profit in this case is given by $V_{t+1} = [R^L_t - R^M_t] p_t k(R^L_t / \mu)$. The profit-maximizing lending rate in this case does depend on the interest paid on CBDC. But again, this is only true for the case in which $R^M > R^B$.

8 Discussion

In this section, I use the model developed above to evaluate a number of claims that have been made in regard to the likely economic consequences of CBDC.

To begin, both critics (Cecchetti and Schoenholtz, 2017) and supporters (Ricks, Crawford, and Menand, 2018) have envisaged the need for an expanded central bank balance sheet to accommodate a CBDC. But to the extent that the main benefit of CBDC works through inducing a higher return on deposits net of costs, the analysis above suggests that CBDC can work in the manner intended even with zero take-up. This of course assumes, quite reasonably I think, that both CBDC and bank deposits are processed through the same RTGS system. Cecchetti and Schoenholtz (2017) fear a large flow of deposits from uninsured bank accounts to insured CBDC accounts. This fear, however, seems based on the questionable assumption that banks would not raise the deposit rates they offer to retain deposits. Moreover, one could make the argument that bank deposits are already de facto insured against crisis events (Grey, forthcoming).

But what if, for one reason or another, we witnessed a large migration to CBDC from bank deposits? Cecchetti and Schoenholtz (2017) claim that
the cost of private banking is likely to rise as a result. They note that money market mutual funds have already eroded the “franchise value” of banking and that CBDC may serve to eliminate it. Ricks, Crawford and Menand (2018, pg. 6) claim, on the other hand, that such a migration should not be expected to affect the quantity or cost of credit in the broader economy, citing strong empirical evidence of a disconnect between bank lending and deposit rates; see Figure 1.16

Figure 1

The theoretical analysis above supports Ricks, Crawford and Menand (2018) over Cecchetti and Schoenholtz (2017). As the model above demonstrates, creating deposits—what Grey (forthcoming) refers to as credit laundering—can be accomplished quite separately from the business of retaining deposits—at least, in world where the central bank is targeting the interest rate and where banks are permitted to borrow needed reserves. The opportunity cost of funds for banks is the IOR rate, not the CBDC rate (unless the two rates are wedded in some manner). For a given loan demand schedule, a monopoly bank optimizes its lending rate in relation to the IOR rate. From

16See also Driscoll and Judson (2013).
(19), we see that this decision has nothing to do with the CBDC rate. Cecchetti and Schoenholtz (2017) are correct, however, in claiming that banking sector’s franchise value (monopoly profit) is compromised. But this is does not necessarily result in an increase in funding cost that implicitly must be passed on to debtors.

It is also of some interest to note the very different views on how CBDC may influence financial stability. As Cecchetti and Schoenholtz (2017) point out, CBDC is related to narrow banking proposals, which are often advanced on the grounds of enhancing financial stability. Cecchetti and Schoenholtz (2017) are sceptical, however. Their main concern is that CBDC provides investors with a “flight-to-safety” instrument, the mere availability of which could be destabilizing (Bryant, 2005). Of course, as they point out, such instruments are already available in the form of cash and treasury debt. The main difference with CBDC is its apparent superiority and widespread availability as a “flight to safety” vehicle. The run-inducing incentives put in place by CBDC would, by their reckoning, require an heroic expansion of lending by the central bank in a financial crisis. On the other hand, it is not clear why the credible threat of such an intervention would not be sufficient to discourage bank runs in the first place (Andolfatto, Berentsen, Martin, 2017).

9 Conclusion

The analysis above suggests the main benefit of CBDC will accrue to depositors in jurisdictions where banks use their market power to keep deposit rates depressed relative to what would prevail in a more competitive setting. The model predicts that CBDC is likely to increase financial inclusion. It also offers the striking conclusion that CBDC need not have any impact on bank lending operations–banks are not disintermediated. The main adverse consequences are to be felt by banks in the form of lower monopoly profits. The fiscal authority too may have to bear the burden of a higher interest expense on its debt, but this may be a cost worth bearing if it leads to greater financial inclusion and stimulates household saving and capital formation.

These conclusions are the implications that follow from a highly abstract and provisional model and so should naturally be viewed with caution. The model abstracts from risk. There is no role for bank capital. There is no
moral hazard. All of the debt is held in the banking sector. Only banks make loans, and so on. On the other hand, the model features a bank sector with pricing power, banks that issue deposit liabilities redeemable in cash (and CBDC). The monetary authority follows an interest rate rule. The general equilibrium must be consistent with policy, and so on. The modeling framework is sufficiently simple to permit many interesting extensions worth exploring.

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The question this raises is whether modifying any of these abstractions might reasonably be expected to change any of the main conclusions derived above. Of course, this can only be answered by undertaking the necessary research.
10 References


